# Gaussian beam propagation in a weakly nonlinear medium: A geometrical optics approach

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A geometrical optics method for solving wave propagation in weakly nonlinear, weakly inhomogeneous media is presented and applied to the propagation of a two-dimensional Gaussian beam. A complex distorted phase $\phi$ is obtained. This complex phase is used to describe both the zeroth-order amplitude and the nonlinear rays along which the energy propagates. This approach recovers standard self-focusing and self-blooming effects in a quadratically nonlinear medium.

## I. INTRODUCTION

The nonlinear propagation of electromagnetic waves in media with intensity-dependent indices of refraction has been studied extensively.[1,2] For high-power beams various nonlinear phenomena such as self-focusing[3,4] and mode locking[5] can occur. These phenomena are of long and continuing interest. Self-focusing, for example, is of great importance in studying the heating of plasmas by intense microwaves.[6]

In this paper we present a geometrical optics method for solving the problem of electromagnetic wave propagation in a weakly nonlinear, weakly inhomogeneous medium. Geometrical optics has been successfully applied to wave propagation in linear weakly inhomogeneous media.[7] However, much less has been done on the application of geometrical optics to wave propagation in nonlinear media.

We make use of a perturbation method devised by Choquet-Bruhat.[8] The solution is expressed as an asymptotic expansion in terms of a small parameter $\delta$, which is also a measure of the period of the wave. The method is applicable if the nonlinear term is of the order of $\delta$ or less. It reduces the problem of wave propagation in a nonlinear medium to two decoupled differential equations. The eikonal equation and the amplitude transport equation are solved separately. Other methods such as Whitham's variational technique[9] provide two coupled equations for the phase and the amplitude.

The plan of the paper is as follows. In Sec. II we derive the ray and the amplitude transport equations. These equations are applied in Sec. III to the propagation of a Gaussian beam in a quadratically nonlinear medium. As in the case of a Gaussian beam propagating in a linear medium, complex rays are used.[11,10] We summarize our results in Sec. IV.

## II. DERIVATION OF RAY AND TRANSPORT EQUATIONS

We start with Maxwell's equations in an inhomogeneous isotropic nonlinear medium,

$$\nabla\times\mathbf{B}=\frac{1}{c^2}\{\epsilon_1(\mathbf{x})\partial_t\mathbf{E}+\epsilon_2(\mathbf{x})\partial_t[f(\mathbf{E})\mathbf{E}]\}\ , \tag{1}$$

$$\nabla\times\mathbf{E}=-\partial_t\mathbf{B}\ . \tag{2}$$

We assume the medium to be weakly inhomogeneous and weakly nonlinear. The electric and magnetic fields are expressed as asymptotic expansions in terms of a small parameter $\delta$ that measures the period of the wave

$$\mathbf{u}=\sum_{n=0}^{\infty}\delta^n\mathbf{u}_n(\mathbf{x},t,\phi(\mathbf{x},t)/\delta)\ , \tag{3}$$

where $\mathbf{u}$ is a six-dimensional vector whose components are the components of $\mathbf{E}$ and $\mathbf{B}$, respectively, $\phi$ is a phase function, and $\phi/\delta$ is a fast varying term and is henceforth referred to as $\theta$. Note that in linear geometrical optics the $n$th term of the expansion can be expressed in the form $\delta^n\mathbf{u}_n(\mathbf{x},t)\exp(\phi/\delta)$. However, this form cannot be generally used in our case.

We now use Eq. (3) in Eqs. (1) and (2). The operators $\partial_{x_i}$ and $\partial_t$ are replaced by $\partial_{x_i}+\delta^{-1}\partial_{x_i}\phi\partial_\theta$ and $\partial_t+\delta^{-1}\partial_t\phi\partial_\theta$, respectively. The nonlinear term is assumed to be of the order of $\delta$ and is also expanded in a Taylor's series in terms of $\delta$:

$$f(\mathbf{E})=f(\mathbf{E}_0)+\delta\sum_i E_{i,1}\partial_{E_i}f+\delta^2\sum_i E_{i,2}\partial_{E_i}f+\frac{\delta^2}{2}\sum_{i,j}E_{i,1}\partial_{E_i}\partial_{E_j}f+\cdots\ , \tag{4}$$

where the derivatives are evaluated at $\mathbf{E}=\mathbf{E}_0$.

We proceed as in standard linear geometrical optics and equate terms of order $\delta^{-1}$. The following equations are obtained:

$$\nabla\phi\times\partial_\theta\mathbf{B}_0=\frac{\epsilon_1}{c^2}\partial_t\phi\partial_\theta\mathbf{E}_0\ , \tag{5}$$

$$\nabla\phi\times\partial_\theta\mathbf{E}_0=-\partial_t\phi\partial_\theta\mathbf{B}_0\ . \tag{6}$$

Using Eq. (6) to express $\mathbf{B}_0$ in terms of $\mathbf{E}_0$, Eq. (5) yields

$$M\cdot\partial_\theta\mathbf{E}_0=0\ , \tag{7}$$

where the elements of the matrix $M$ are

 

$$M_{ij}=(\nabla\phi)^2\delta_{ij}-\partial_{x_i}\phi\partial_{x_j}\phi-\frac{\epsilon_1}{c^2}(\partial_t\phi)^2\delta_{ij} .$$

Equation (7) has a nontrivial solution if the determinant of the matrix $M$ vanishes. This provides a partial differential equation for the phase $\phi$:

$$(\nabla\phi)^2=\frac{\epsilon_1}{c^2}(\partial_t\phi)^2 . \quad (8)$$

The preceding equation is the same as the eikonal equation of linear geometrical optics.[7] It can be transformed from a second-order partial differential equation to a set of first-order ordinary differential equations along a set of characteristics referred to in geometrical optics as the rays

$$\frac{d\phi}{ds}=0 , \quad (9)$$

$$\frac{d\mathbf{x}}{ds}=\gamma\nabla\phi , \quad (10)$$

$$\frac{dt}{ds}=-\gamma\frac{\epsilon_1}{c}\partial_t\phi , \quad (11)$$

where $s$ is a function of the arclength along a ray; $\gamma$ is arbitrary.

Equating terms of order $\delta^0$, the following equations are obtained:

$$\nabla\phi\times\partial_\theta\mathbf{E}_1+\partial_t\phi\partial_\theta\mathbf{B}_1=-\nabla\times\mathbf{E}_0-\partial_t\mathbf{B}_0 , \quad (12)$$

$$\nabla\phi\times\partial_\theta\mathbf{B}_1-\frac{\epsilon_1}{c^2}\partial_t\phi\partial_\theta\mathbf{E}_1=-\nabla\times\mathbf{B}_0+\frac{\epsilon_1}{c^2}\partial_t\mathbf{E}_0+\frac{\epsilon_2}{c^2}h(\mathbf{E}_0)\partial_t\phi\partial_\phi\mathbf{E}_0 , \quad (13)$$

where

$$h(\mathbf{E}_0)=f(\mathbf{E}_0)+[\partial_{\mathbf{E}_0}f(\mathbf{E}_0)]\cdot\mathbf{E}_0 .$$

Equations (12) and (13) can be rewritten in matrix form

$$\left[A^{(t)}\partial_t\phi+\sum_i A^{(i)}\partial_{x_i}\phi\right]\partial_\theta\mathbf{u}_1=-\left[A^{(t)}\partial_t\mathbf{u}_0+\sum_i A^{(i)}\partial_{x_i}\mathbf{u}_0+\frac{\epsilon_2}{c^2}h(\mathbf{E}_0)\partial_t\phi\partial_\phi\mathbf{u}_0'\right] , \quad (14)$$

where $\mathbf{u}_0'$ is given by

$$u'_{0,i}=u_{0,i},\quad i\le 3 ,$$

$$u'_{0,i}=0,\quad i>3 .$$

$A^{(t)}$ and $A^{(i)}$ are symmetric $6\times6$ matrices given by

$$A^{(t)}_{ij}=\frac{\epsilon_1}{c^2}\delta_{ij},\quad i\le 3 ,$$

$$A^{(t)}_{ij}=\delta_{ij},\quad i>3 ,$$

$$A^{(1)}_{26}=A^{(1)}_{62}=-A^{(1)}_{35}=-A^{(1)}_{53}=1 ,$$

$$A^{(2)}_{34}=A^{(2)}_{43}=-A^{(2)}_{16}=-A^{(2)}_{61}=1 ,$$

$$A^{(3)}_{15}=A^{(3)}_{51}=-A^{(3)}_{24}=-A^{(3)}_{42}=1 .$$

All other entries of the $A^{(i)}$ matrices are zero.

From Eqs. (5) and (6), it is evident that

$$\left[A^{(t)}\partial_t\phi+\sum_i A^{(i)}\partial_{x_i}\phi\right]\partial_\theta\mathbf{u}_0=0 . \quad (15)$$

Therefore we can express $\mathbf{u}_0$ in the following form:

$$\mathbf{u}_0=a(\mathbf{x},t,\theta)\mathbf{R}(\mathbf{x},t) , \quad (16)$$

where $\mathbf{R}$ is the right null vector of the matrix

$$N=A^{(t)}\partial_t\phi+\sum_i A^{(i)}\partial_{x_i}\phi .$$

From Eq. (15), the determinant of $N$ is zero. This provides the eikonal equation (8) that was obtained earlier by setting the determinant of the $3\times3$ matrix $M$ equal to zero.

The ray equations (10) and (11) can be expressed in terms of the matrices $A^{(t)}$, $A^{(i)}$, and $N$:

$$\frac{dx_i}{ds}=\mathbf{L}A^{(i)}\mathbf{R} , \quad (17)$$

$$\frac{dt}{ds}=\mathbf{L}A^{(t)}\mathbf{R} , \quad (18)$$

where $\mathbf{L}$ is the left null vector of $N$.

Substituting Eq. (16) for $\mathbf{u}_0$ and multiplying Eq. (14) by $\mathbf{L}$ gives

$$\mathbf{L}\cdot\left[A^{(t)}\mathbf{R}\partial_t+\sum_i A^{(i)}\mathbf{R}\partial_{x_i}\right]a+\mathbf{L}\cdot\left[A^{(t)}\partial_t\mathbf{R}+\sum_i A^{(i)}\partial_{x_i}\mathbf{R}\right]a+\frac{\epsilon_2}{c^2}h(\mathbf{E}_0)\mathbf{L}\cdot\mathbf{R}'\partial_t\phi\partial_\phi a=0 , \quad (19)$$

where

$$R_i'=R_i,\quad i\le 3 ,$$

$$R_i'=0,\quad i>3 .$$

Note that

$$\mathbf{L}\cdot\left[A^{(t)}\mathbf{R}\partial_t+\sum_i A^{(i)}\mathbf{R}\partial_{x_i}\right]=\partial_s . \quad (20)$$

Therefore we can express Eq. (20) in the following form:

$$\partial_s a+Ph(\mathbf{E}_0)\partial_\phi a+Qa=0 , \quad (21)$$

where

$$P=\frac{\epsilon_2}{c^2}\mathbf{L}\cdot\mathbf{R}'\partial_t\phi ,$$

$$Q=\mathbf{L}\cdot\left[A^{(t)}\partial_t\mathbf{R}+\sum_i A^{(t)}\partial_{x_i}\mathbf{R}\right] .$$

Equation (21) is a first-order quasilinear differential equation that describes the evolution of the zeroth-order amplitude.

In the linear case the second term of Eq. (21) is zero and the linear solution is

$$a=g(\xi,\phi/\delta)G(\xi,s) , \tag{22}$$

where $\xi$ is a parameter that characterizes a given ray. The linear phase $\phi$ is determined by solving the eikonal equations (9)–(11). The function $g$ gives the wave profile and is chosen to make the solution agree with the initial conditions. The function $G$ describes the evolution of the amplitude along the ray due to the inhomogeneity of the medium, and to changes in the ray geometry. It is given by

$$G=\exp\left[-\int_0^s ds'Q\right] . \tag{23}$$

In contrast when the nonlinearity is present, it becomes difficult to solve Eq. (21) along the linear characteristics. However, along the nonlinear characteristics defined by

$$\frac{d\phi}{ds}=Ph(\mathbf{E}_0) \tag{24}$$

the transport equation (21) takes the same form as in the linear case

$$\frac{da}{ds}=-Qa . \tag{25}$$

This system, as we will now show, can be readily solved. The amplitude is given by

$$a=g(\xi,\phi'/\delta)G(\xi,s) , \tag{26}$$

where $G$ is again given by (23) but $\phi'$ is the nonlinear or distorted phase that is invariant along the nonlinear characteristics. We use (26) in (24) and integrate, with respect to $s$, to obtain the following equation in which $\phi'$ is defined implicitly:

$$\phi'=\phi-\int_0^s ds'P(\xi,s')h[g(\xi,\phi'/\delta)G(\xi,s')\mathbf{R}''] , \tag{27}$$

where $\mathbf{R}''$ is a three-dimensional vector whose components are equal to the components of $\mathbf{R}$ that correspond to $\mathbf{E}$. We have taken the initial condition for the distorted phase to be the linear phase $\phi=\phi'$. The integral is taken along the linear rays. Equation (27) can be solved by iteration. Higher-order terms $\mathbf{u}_1,\mathbf{u}_2,\ldots,$ can now be obtained successively by solving linear systems of differential equations.

To summarize, the linear phase $\phi$ is determined by solving the linear eikonal equation. Once $\phi$ is determined, $\mathbf{R}$ is found by solving a linear system of algebraic equations given by Eq. (15). The amplitude is given by Eq. (16), where $a$ is defined by Eq. (26). The function $G$ is obtained from Eq. (23). The function $g$ is chosen to agree with the initial conditions. It depends on $\phi'$, the distorted phase, which is found by iteration from Eq. (27). The method is subject to the standard restrictions of linear geometrical optics. In addition, $\epsilon_2 h(\mathbf{E})/\epsilon_1 \le O(\delta)$.

## III. APPLICATION TO GAUSSIAN BEAMS

We apply the method presented earlier to a two-dimensional Gaussian beam propagating in a quadratically weakly nonlinear medium. For simplicity we assume the medium to be homogeneous with $\epsilon_1=1$:

$$\epsilon_{\rm NL}=1+\epsilon_2\langle E^2\rangle , \tag{28}$$

where $\langle E^2\rangle$ represents the time average of the square of the electric field. A quadratically nonlinear response can be produced by a number of effects, such as the electronic polarization of the atoms or molecules of the medium. For an optical beam propagating in glass, $\epsilon_2\approx 7\times10^{-22}$ $\mathrm{m^2/V^2}$.[12] In our case, the glass medium is considered weakly nonlinear if the power of the optical beam is of the order of a few megawatts.

At $z=0$, the electric field is given by

$$E_x=E_0\exp\left[\frac{-k_0y^2}{2b}-i\omega t\right] , \tag{29}$$

where $k_0=\omega/c$; $b$ is any length characteristic of the aperture field. Since the nonlinearity is independent of time, the time harmonic solution $\exp(-i\omega t)$ applies everywhere.

For a two-dimensional Gaussian beam there are three field components $E_x$, $B_y$, and $B_z$. Therefore the matrices $A^{(t)}$, $A^{(i)}$, and $N$ reduce to $3\times3$ matrices. We use $(1/ik_0)^n$ as the expansion parameter $\delta$. The fast varying linear phase $\theta$ is expressed as $\theta=ik_0[\psi(y,z)-ct]$. The eikonal equation (8) reduces to

$$(\partial_y\psi)^2+(\partial_z\psi)^2=1 . \tag{30}$$

The left null vector is given by

$$\mathbf{L}=\beta\sqrt{c/2}\left[1,\frac{1}{c}\partial_z\psi,\frac{-1}{c}\partial_y\psi\right] , \tag{31}$$

where $\beta$ is arbitrary. We choose $\beta=1$. Since the matrix $N$ is symmetric, $\mathbf{R}$ is the transpose of $\mathbf{L}$. The amplitude transport equation (21) gives

$$\partial_\tau a+\tfrac{1}{2}(\partial_y^2\psi+\partial_z^2\psi)a-\frac{i\epsilon_2c}{4}\langle a^2\rangle\partial_\phi a=0 , \tag{32}$$

where $\tau$ is an arclength along the ray. Note that if the nonlinear term is neglected the solution reduces to that of linear geometrical optics.[7] Along the nonlinear characteristics, the amplitude transport equation reduces to

$$\frac{da}{d\tau}=-\tfrac{1}{2}(\partial_y^2\psi+\partial_z^2\psi)a . \tag{33}$$

The distorted (nonlinear) phase equation (27) gives

$$\psi'=\psi+\frac{\epsilon_2}{2}\int_0^\tau d\tau\langle\mathbf{E}^2\rangle . \tag{34}$$

Note that if there is no field variation in the transverse direction, the solution is

$$E_x = E_0 \exp\left[ i k_0 z \left[ 1 + \frac{\epsilon_2 E_0^2}{4} \right] - i\omega t \right] . \quad (35)$$

This result can also be obtained from the wave equation by neglecting the term of order $(\epsilon_2 E_0^2)^2$.

Following the method of Keller and Streiffer,[11] the eikonal equation (30) for the Gaussian beam is solved by extending both the phase and the rays analytically to complex space.

The following set of equations are obtained for complex $y$ and $z$ along the rays

$$y = \eta + (\partial_\eta \psi)\tau , \quad (36)$$

$$z = [1 - (\partial_\eta \psi)^2]^{1/2} \tau , \quad (37)$$

where $\eta$ is complex and is the value of $y$ at $z=0$. From Eq. (29), the complex phase at $z=0$ is equal to $i\eta^2/2b$. Equations (36) and (37) can then be expressed as

$$y = \eta + \frac{i\eta}{b}\tau , \quad (38)$$

$$z = \left[ 1 + \frac{\eta^2}{b^2} \right]^{1/2} \tau . \quad (39)$$

The rays are straight lines in complex space. Using (38) we can express $\eta$ in terms of $y$ and $\tau$:

$$\eta = \frac{y}{1 + (i\tau/b)} . \quad (40)$$

Eliminating $\eta$ in (39):

$$z = \left[ 1 + \left[ \frac{y}{b + i\tau} \right]^2 \right]^{1/2} . \quad (41)$$

We are interested in the value of $\tau$ for $y$ and $z$ real. We write $\tau$ as $\tau_R + i\tau_I$, where $\tau_R$ and $\tau_I$ are the real and the imaginary parts of $\tau$, respectively. It is then straightforward to show that Eq. (41) reduces to two coupled algebraic equations:

$$2y^2 \tau_R \tau_I = [2b\tau_R(-\tau_I b + z^2 - \tau_R^2 + 3\tau_I^2) + 4\tau_R\tau_I(\tau_R^2 - \tau_I^2) - 2\tau_R\tau_I z^2] , \quad (42)$$

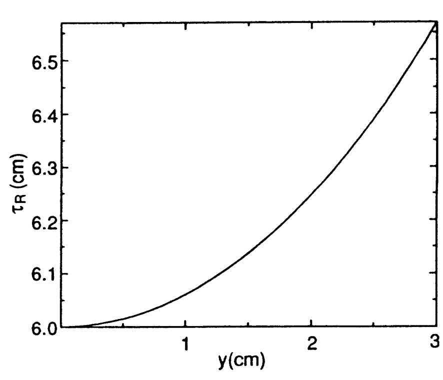


FIG. 1. $\tau_R$ vs $y$ at $z=6$ cm for $b=2$ cm.

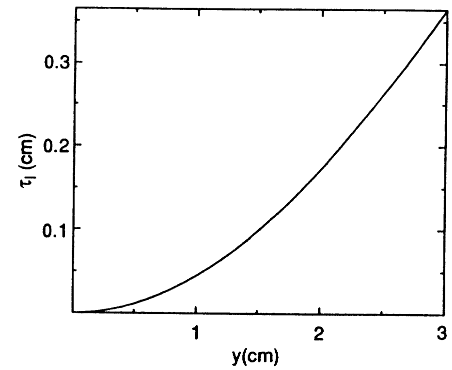


FIG. 2. $\tau_I$ vs $y$ at $z=6$ cm for $b=2$ cm.

$$y^2(\tau_R^2 - \tau_I^2) = [b^2(z^2 - \tau_R^2 + \tau_I^2) + 2b\tau_I(3\tau_R^2 - z^2 - \tau_I^2) + z^2(\tau_I^2 - \tau_R^2) + (\tau_R^2 - \tau_I^2) - 4\tau_R^2\tau_I^2] . \quad (43)$$

Equations (42) and (43) were solved numerically for $\tau_R$ and $\tau_I$ by an iterative scheme for different real points

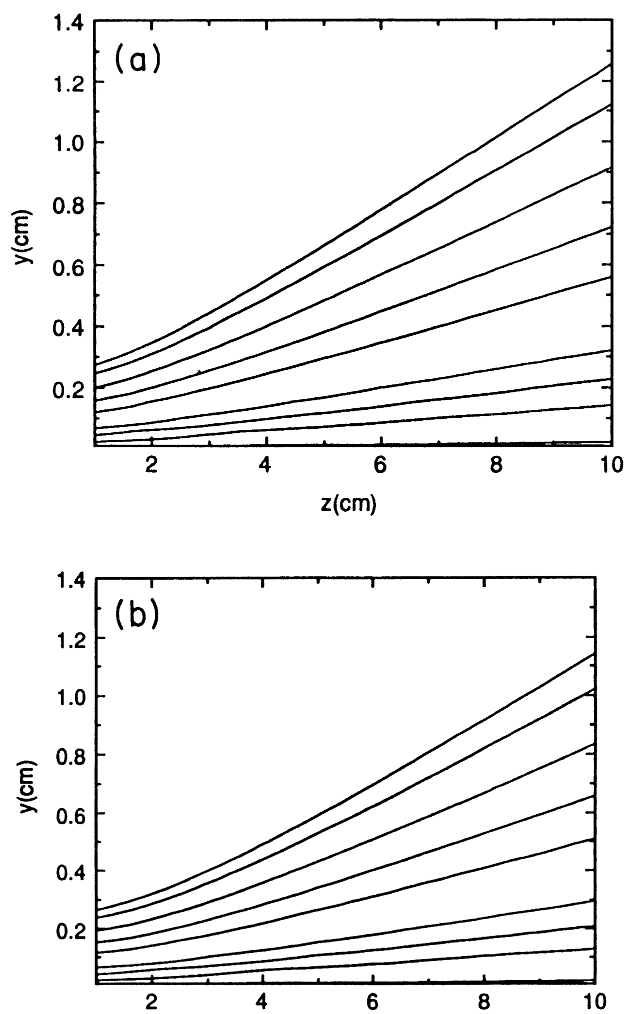


FIG. 3. (a) Linear contours $k=10$ cm; $b=2$ cm. (b) Nonlinear contours $k=10$ cm; $b=2$ cm; $\epsilon_2 E^2 = 0.2$.

$(y,z)$. Plots of $\tau_R$ versus $y$ and $\tau_I$ versus $y$ at a given $z$ are shown in Figs. 1 and 2. The linear phase $\psi$ is then determined by

$$\psi=\frac{i\eta^2}{2b}+\tau \ , \tag{44}$$

where $\eta$ can be obtained from Eq. (40).

To calculate the distorted phase in Eq. (34) we reparametrize the rays in terms of a real variable $\alpha$. This simplifies the process of carrying out the integral. The ray equations can be expressed as

$$y_R=\eta_R+(y_f-\eta_R)\alpha \ , \tag{45}$$

$$y_I=\eta_I(1-\alpha) \ , \tag{46}$$

$$z_R=z_f\alpha \ , \tag{47}$$

$$z_I=0 \ , \tag{48}$$

where the subscripts $R$ and $I$ refer to the real and imaginary parts, respectively. The subscript $f$ refers to the end point; that is, the real point where we are calculating the distorted phase. $\alpha$ varies from 0 to 1.

We need to relate $\tau$ to $\alpha$. Equating $y_R+iy_I$ obtained from Eqs. (45) and (46) to Eq. (38), it is straightforward to show

$$\tau=\frac{b}{\eta_R^2+\eta_I^2}[-\eta_I y_f+i(\eta_R^2+\eta_I^2-\eta_R y_f)]\alpha \ . \tag{49}$$

The zeroth-order electric field is extended by analytic continuation to complex space. It is expressed along a given ray as

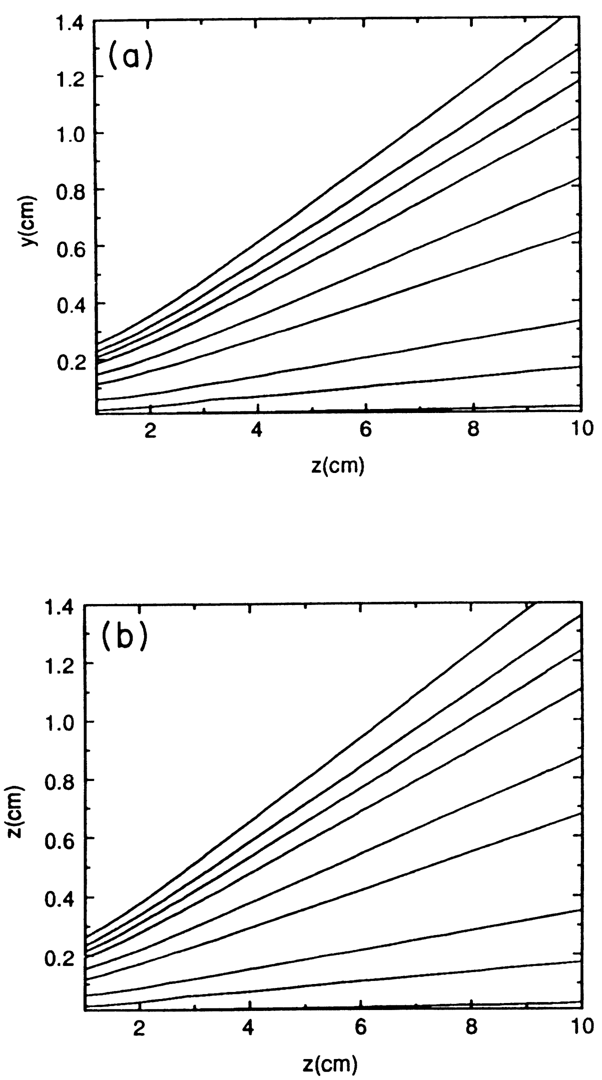


FIG. 4. (a) Linear contours $k=10$ cm; $b=1.5$ cm. (b) Nonlinear contours, $k=10$ cm, $b=1.5$ cm, $\epsilon_2E^2=-0.25$.

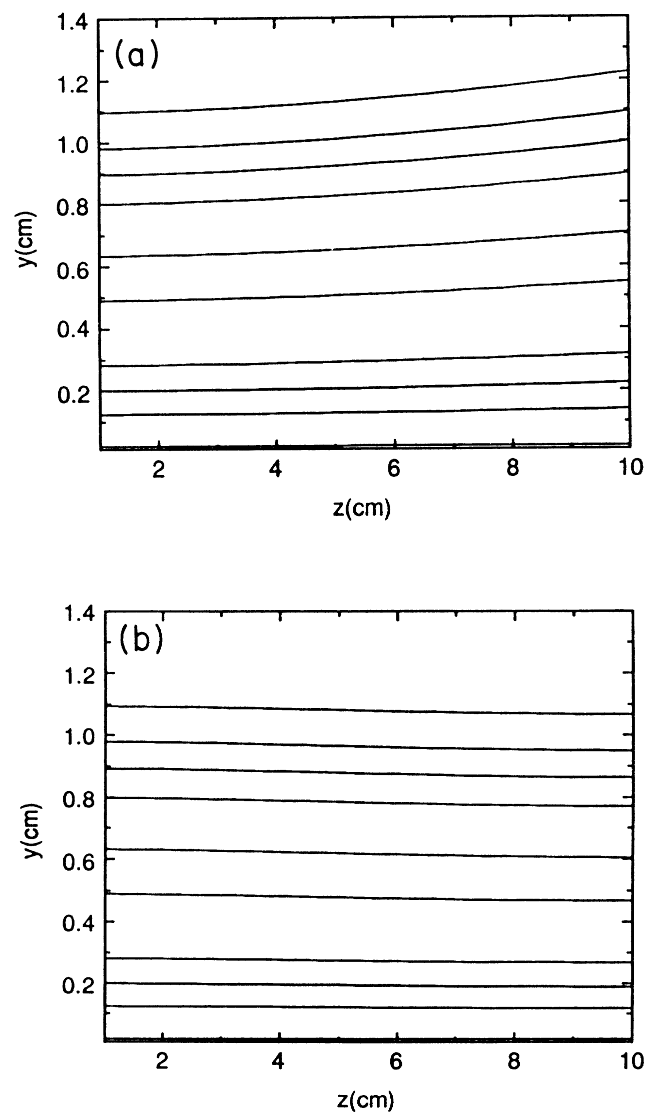


FIG. 5. (a) Linear contours, $k=5$ cm, $b=20$ cm. (b) Nonlinear contours, $k=5$ cm, $b=20$ cm, $\epsilon_2E^2=0.8$.

$$E_{x,0}[y(\alpha),z(\alpha),t]=E_0\left\{\frac{J(\eta,0)}{J[\eta,\tau(\alpha)]}\right\}^{1/4}\exp\{ik_0[\psi_R'(\alpha)+i\psi_I'(\alpha)]-i\omega t\}\ , \quad (50)$$

where $\psi_R'$ and $\psi_I'$ are the real and the imaginary parts of the distorted phase $\psi'$, respectively. $J(\eta,\tau)$ is the solution to Eq. (33) and is given by the Jacobian of the transformation from the ray coordinate $(\eta,\tau)$ to $(y,z)$:

$$J[\eta,\tau(\alpha)]=[1+(\eta/b)^2]^{-1/2}\{1+(\eta/b)^2+[i\tau(\alpha)/b]\}\ . \quad (51)$$

The term $\langle E^2\rangle$ is then expressed along a given ray as:

$$\langle E^2\rangle=\tfrac{1}{2}E_0^2 v(\alpha)v^*(\alpha)\exp[-2k_0\psi_I'(\alpha)]\ , \quad (52)$$

where $v(\alpha)=\{J(\eta,0)/J[\eta,\tau(\alpha)]\}^{1/4}$ and $v^*$ is its complex conjugate. The product is computed numerically.

Separating real and imaginary parts, the distorted phase equation gives

$$\psi_R'(\alpha)=\psi_R(\alpha)-\left[\frac{\epsilon_2E_0^2}{4}\right]\frac{b\eta_I y_f}{(\eta_R^2+\eta_I^2)}\int_0^\alpha v(\alpha')v^*(\alpha')\exp[-2k_0\psi_I'(\alpha')]d\alpha'\ , \quad (53)$$

$$\psi_I'(\alpha)=\psi_I(\alpha)+\left[\frac{\epsilon_2E_0^2}{4}\right]\frac{b(\eta_R^2+\eta_I^2-\eta_R y_f)}{(\eta_R^2+\eta_I^2)}\int_0^\alpha v(\alpha')v^*(\alpha')\exp[-2k_0\psi_I'(\alpha')]d\alpha'\ . \quad (54)$$

Equations (53) and (54) are solved by an iterative scheme that computes the distorted phase at meash points along $\alpha$. The distorted phase at the endpoint, where both $y$ and $z$ are real, is determined by carrying out the integral to $\alpha=1$. This process is repeated for different real endpoints $(y,z)$.

In the linear case, the phase paths defined by $\psi_I=$const, represent the direction of energy propagation. We plot these contours for different values of $k$ and $b$ in Figs. 3(a)–5(a). We plot the corresponding nonlinear phase paths, defined by $\psi_I'=$const, in Figs. 3(b)–5(b). In Fig. 3(b), $\epsilon_2$ is greater than zero. The focusing effect reduces the diffraction shown in Fig. 3(a). In Fig. 4(b), $\epsilon_2$ is less than zero. The self-blooming effect increases the diffraction shown in Fig. 4(a). Finally, in Fig. 5(b), the focusing effect produced by a positive nonlinearity turns the slight diffraction shown in Fig. 5(a) into a very slight focusing. For stronger nonlinearities, this method is not valid.

## IV. CONCLUSION

In geometrical optics, the zeroth-order amplitude of an electromagnetic wave propagating in a weakly nonlinear, weakly inhomogeneous medium is described by a quasi-linear first-order partial differential equation. This equation is solved along a set of characteristics that defines the distorted phase of the wave. We computed the complex distorted phase of a two-dimensional Gaussian beam propagating in a quadratically nonlinear medium. This phase describes the phenomena of self-focusing and self-blooming. We plan next to study weakly nonlinear wave propagation in dispersive media such as plasmas.

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