Radio Science, Volume 22, Number 4, Pages 449–454, July–August 1987

New electromagnetic resonance effects associated with cavity-backed apertures

Richard W. Ziolkowski

Engineering Research Division, Lawrence Livermore National Laboratory, California

(Received October 15, 1986; revised February 10, 1987; accepted February 10, 1987.)

A number of canonical problems that describe the coupling of electromagnetic fields through apertures into enclosed regions have been solved. The scattering data generated by these cavity-backed aperture solutions share a common property: they are dominated by resonance features closely connected to the presence of the cavity. The behavior of these cavity-backed aperture resonance features has been characterized for variations in frequencies, angles of incidence, look angles, aperture sizes, and interior object properties. It has been found that the scattering cross sections contain information about the interior of the cavity.

1. INTRODUCTION

A number of canonical problems that describe coupling through apertures into enclosed regions have been solved recently with the generalized dual series (GDS) approach and have been reported elsewhere. These include the two-dimensional solutions to the scattering of E- and H-polarized plane waves from an empty infinite circular cylinder having an infinite axial slot [*Ziolkowski*, 1985*a*; *Johnson and Ziolkowski*, 1984] and from one that encloses an infinite concentric [*Ziolkowski et al.*, 1984; *Ziolkowski and Grant*, 1987] or off-set impedance cylinder [*Ziolkowski and Schmucker*, 1986], and the three-dimensional solutions to the plane wave scattering from an empty open spherical shell with a circular aperture [*Ziolkowski and Johnson*, 1987; *Ziolkowski*, 1985*b*], and from an open spherical shell with a circular aperture enclosing either a concentric metallic or dielectric sphere [*Ziolkowski et al.*, 1986; *Ziolkowski et al.*, 1987]. These problems have been studied extensively to determine the effects on the aperture coupling and scattering of variations in the polarization, frequency, angle of incidence, aperture size, and interior object characteristics. The GDS solutions are systematic and inherently contain the behavior near the rim of the aperture required by Meixner's edge conditions. They can handle small to large ratios of cylinder or sphere radius to wavelength and arbitrary angles of incidence without additional special considerations. The results from the two-dimensional slit cylinder and the three-dimensional open spherical shell problems have been correlated to determine which of the coupling/scattering effects are or are not shared by these distinct geometries, hence, which of them might be extrapolated to more general configurations.

Because they describe coupling via apertures into enclosed regions and scattering from structures having edges, curvature, and nontrivial interior configurations, the importance of these canonical electromagnetic cavity-backed aperture (CBA) problems can not be overstated. They provide a basic means with which fundamental aperture coupling and scattering physics can be studied in detail; they can be used to construct and/or validate approximate coupling or scattering models that can be applied to more general apertures and scattering objects; and they aid in the development of improved numerical techniques especially near the edges of the apertures or scatterers where discontinuities appear and where those methods may encounter difficulties. Moreover, accurate canonical solutions of this type provide standards to which general purpose numerical code results can be compared. For example, the two-dimensional slit cylinder problems have proved to be valuable for EMP investigations. The locations of field hotspots near the interior object or the current peaks induced on an interior wire are being studied as a function of all of the problem parameters. The three-dimensional open spherical shell problems involve a finite scatterer for which experimental data can be obtained [*Chang and Senior*, 1969].

Paper number 7S0206.
0048-6604/87/007S-0206$08.00

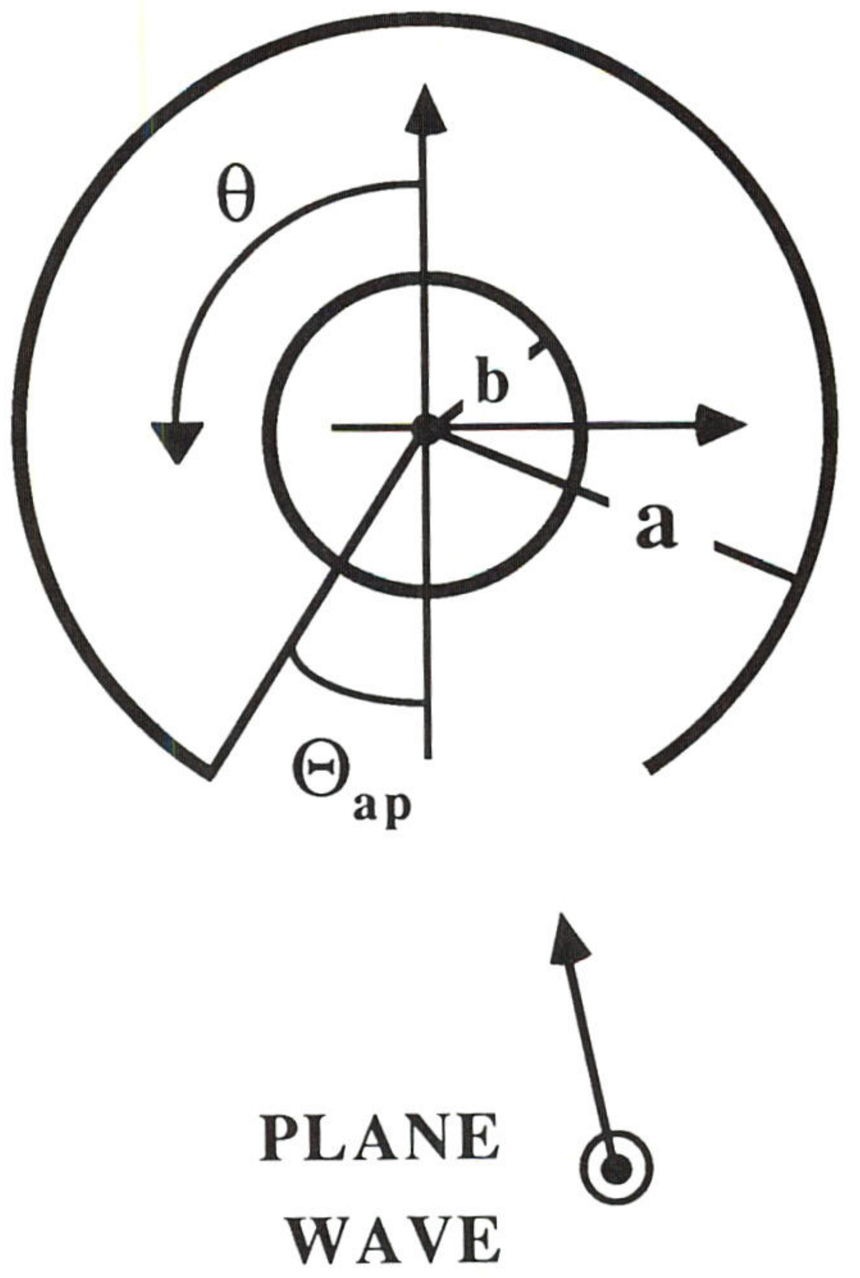

Fig. 1. The generic configuration of the scattering of a plane wave from a cavity-backed aperture with an interior load is depicted.

2. RESULTS

It has been found that resonance features corresponding to the presence of the interior cavity dominate all of the aperture coupling and the scattering results. These include the currents induced on the interior conductors and the open scatterer, the fields in the aperture, the energy captured by the open cavity, and the scattering cross sections. The locations in frequency of the cavity-backed aperture (CBA) resonances and the resultant current and field patterns at those values can be identified with corresponding closed-cavity resonance locations and patterns.

A cross section of the generic problem configuration is shown in Figure 1. The open scatterer has radius a, the interior object has radius b. Half the angular extent of the aperture is measured by the angle Θ_{ap}. The angle θ is measured from the center of the metal.

Plate 1 shows the radar cross sections resulting from a plane wave normally incident upon an open spherical shell enclosing various interior loads. The open shell has a radius $a = 1.0$ m and a circular aperture with an angular extent of $\Theta_{ap} = 10°$. The incident plane wave is directed into the aperture along the axis of symmetry of the open shell. The red curve represents the case where the open shell is empty. The blue curve represents the case where the open shell encloses a concentric, perfectly conducting metallic interior sphere whose radius $b = 0.3$ m. The green curve represents the case where the open shell encloses a concentric, dielectric interior sphere. This dielectric sphere is homogeneous and lossless with a relative permittivity of $\varepsilon_r = 3.0$ and has a radius $b = 0.3$ m. The black curve represents the corresponding closed sphere results and is given for comparison and reference.

The open sphere results are closely correlated with the closed sphere results at lower ka values except for the presence of the CBA resonance features. As ka increases (wavelength decreases) so that the wave can begin to sense the presence of the aperture and the interior cavity, the deviation of the open sphere from the closed sphere results increases. For the empty interior case the peaks of the antiresonance features occur at $ka = 2.7402$, 3.8591, 4.4877, and 4.9470. These are slightly lower than the corresponding closed cavity resonance locations: 2.744, 3.870, 4.493, and 4.973; due to the detuning of the cavity by the aperture. The peaks of the antiresonance features in the metal and dielectric inner sphere cases occur, respectively, at $ka = 2.4100$, 3.7829, 4.9290, 5.0361, and 5.3411; and $ka = 2.5825$, 3.8219, 4.2708, 4.9388, and 5.3258. For the empty open sphere these CBA features correspond to the TM_{11}, TM_{21}, TE_{11}, and TM_{31} modes; for the loaded open spheres they correspond to the TM_{11}, TM_{21}, TE_{11}, TM_{31}, and TM_{12} modes. These mode assignments were made by tracking the resonance locations from the corresponding closed sphere to the present open sphere cases. [*Ziolkowski et al.*, 1986; 1987]

The total differential or scattering cross sections (the bistatic cross sections integrated over all of the 4π solid angles) are plotted in Plate 2 for the same cases given in Plate 1. The minima of the CBA resonance features are located at $ka = 2.7403$, 3.8596, 4.4882, and 4.9490 for the empty sphere case; at $ka = 2.4101$, 3.7834, 4.9311, 5.0364, and 5.3416 for the case with an interior metallic sphere, and at $ka = 2.5826$, 3.2844, 4.2711, 4.9408, and 5.3262 for the case with an interior dielectric sphere. These minima occur at the maxima of the energy captured in the open sphere (i.e., the total field energy in the spherical volume $r \leq a$).

The resonance peaks in these cross-sections are in-

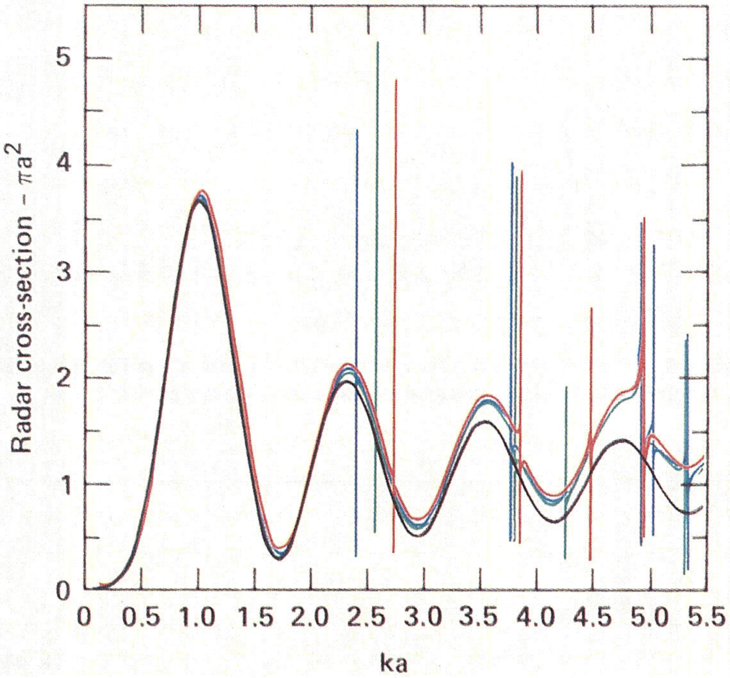

Plate 1. The presence of interior information in the exterior scattering data is demonstrated with *ka* scans of the radar cross sections resulting from a plane wave normally incident upon an open spherical shell enclosing various interior loads. The open shell has a radius $a = 1.0$ m and a circular aperture with an angular extent of $\Theta_{ap} = 10°$. The incident plane wave is directed into the aperture along the axis of symmetry of the open shell. The red curve represents the case where the open shell is empty. The blue curve represents the case where the open shell encloses a concentric, perfectly conducting metallic interior sphere whose radius $b = 0.3$ m. The green curve represents the case where the open shell encloses a concentric, dielectric interior sphere. This dielectric sphere is homogeneous and lossless with a relative permittivity of $\varepsilon_r = 3.0$ and has a radius $b = 0.3$ m. The black curve represents the corresponding closed sphere results and is given for comparison and reference.

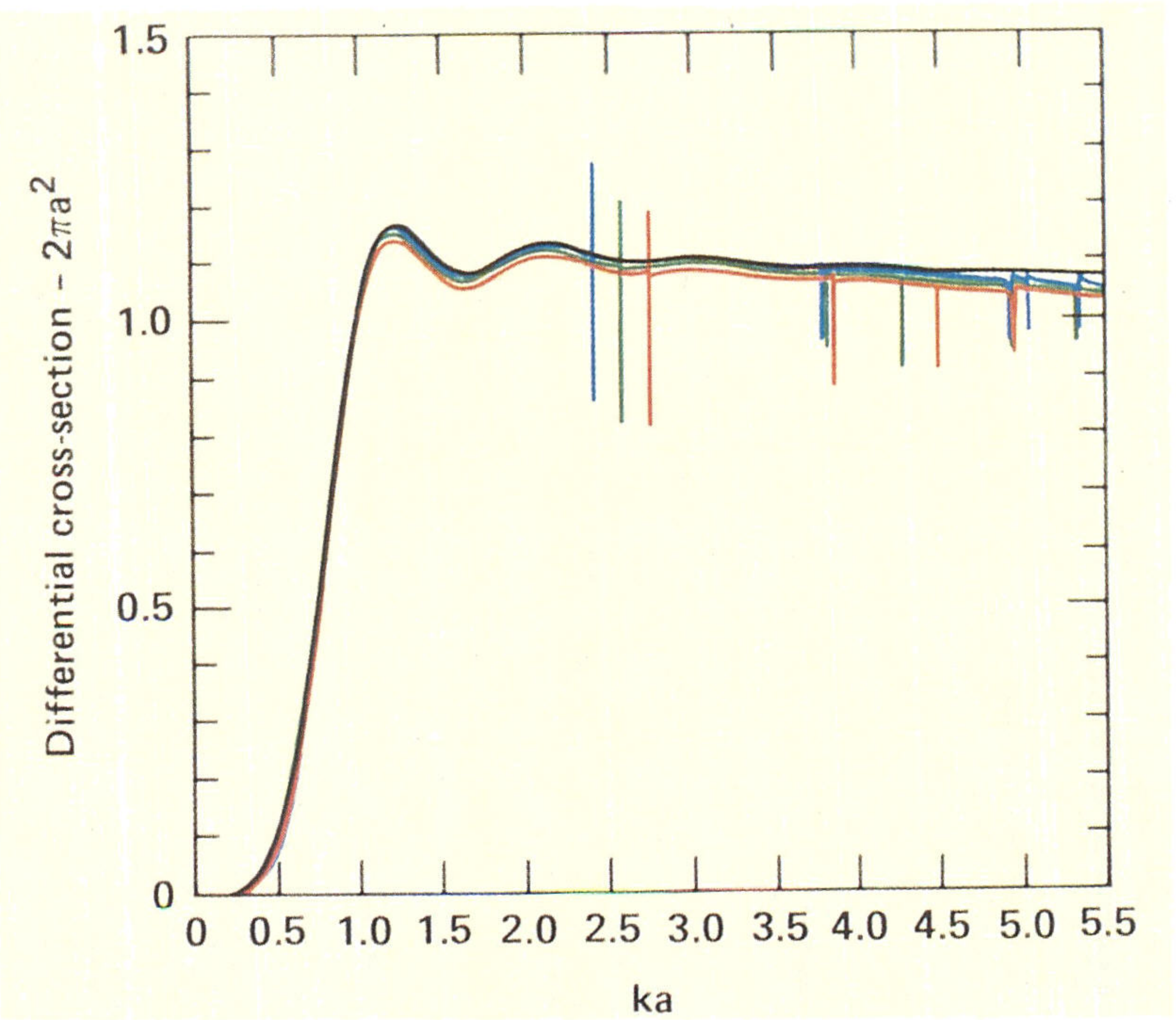

Plate 2. The presence of interior information in the exterior scattering data is also demonstrated with *ka* scans of the total differential (scattering) cross sections for the cases given in Plate 1.

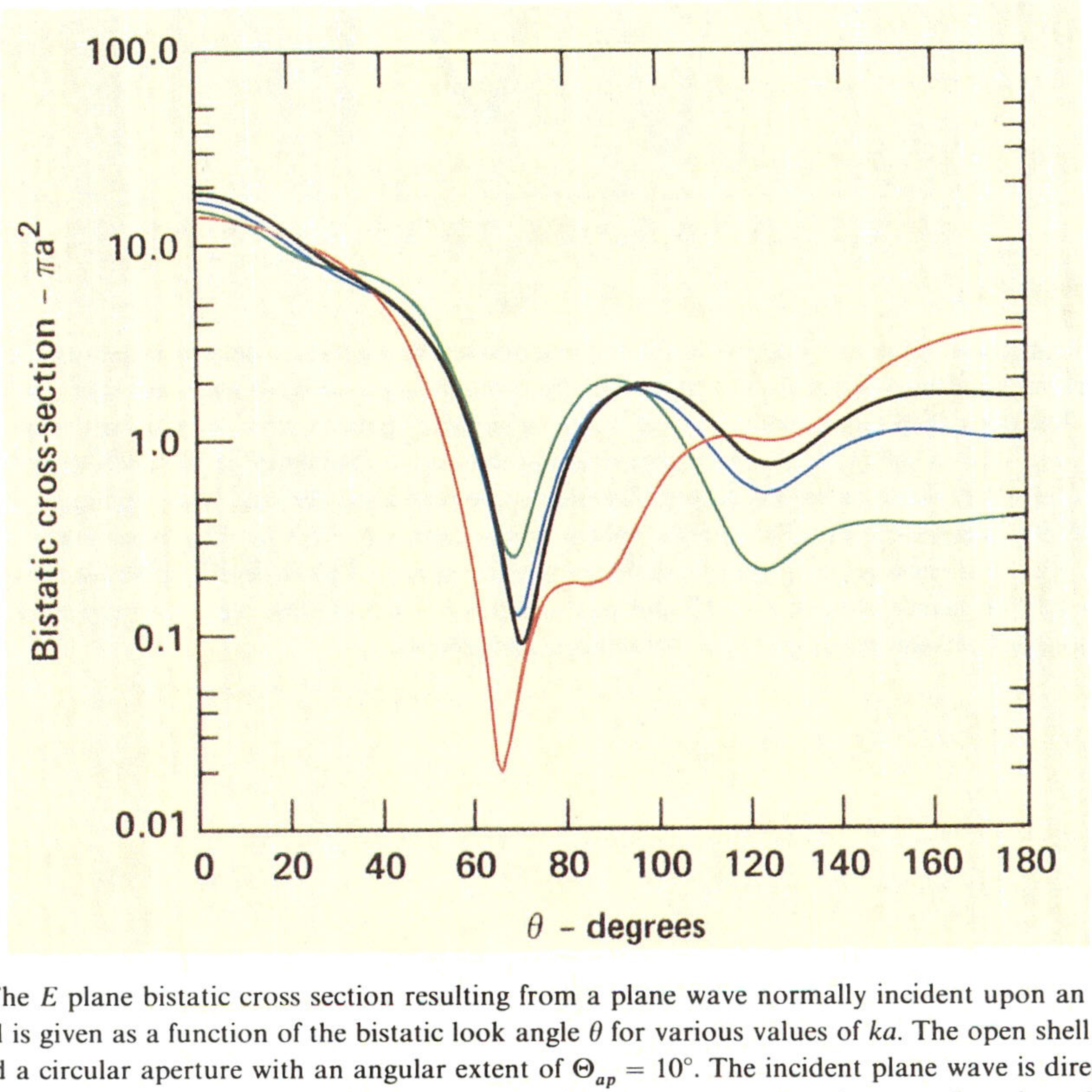

Plate 3. The E plane bistatic cross section resulting from a plane wave normally incident upon an empty, open spherical shell is given as a function of the bistatic look angle θ for various values of ka. The open shell has a radius $a = 1.0$ m and a circular aperture with an angular extent of $\Theta_{ap} = 10°$. The incident plane wave is directed into the aperture along the axis of symmetry of the open shell. The blue curve represents the case where $ka = 3.857$, the red curve for $ka = 3.859$, the green curve for $ka = 3.861$, and the black curve for $ka = 3.863$.

dicative of a reradiation phenomena that is associated with the cavity-backed nature of the aperture. As inferred from Plate 2 and indicated above, the energy contained within the open spherical shell dramatically increases at a CBA resonance. Concurrently the currents induced on the sphere and consequently the scattered fields experience a π-phase shift as ka passes through one of these CBA resonances. Thus a scattered field, whose amplitude is enhanced by the energy resonantly captured in the cavity, is created that at different look angles either constructively or destructively interferes with the incident field. This results in the distinctive antiresonance features present in the radar cross sections. For small apertures they are very narrow which is indicative of the extremely high Q nature of the cavity. Increasing the aperture size broadens them and their locations are translated to lower ka values corresponding to an increased detuning of the cavity. These features also broaden as the wavelength of the incident field decreases to a point where it can begin to resolve the structure of the open spherical shell. The slight dilatation apparent in Plates 1 and 2 as the wavelength nears the radius of the open shell is due to this effect. The antiresonance peaks in the empty open spherical shell radar cross sections have actually been observed experimentally [*Chang and Senior*, 1969; L. F. Libelo, personal communication, 1985] although, for a lack of theoretical proof of their existence, they have been generally attributed to errors in the measurement apparatus (L. F. Libelo, personal communication, 1985).

These resonance features are also found in the bistatic cross sections at the same relative positions for all look angles. This is true even for nonnormal incidence. As one might expect, the shapes and sizes of the individual resonance peaks vary with the angle of incidence and the bistatic look angles. This is illustrated with Plate 3. The E plane bistatic cross section for the empty open sphere case in the previous figures is given as a function of the bistatic look angle θ for various values of ka. The blue curve represents the case where $ka = 3.857$, the red curve for $ka = 3.859$, the green curve for $ka = 3.861$, and the black curve for $ka = 3.863$. The red and green curves correspond, respectively, to the locations of the peak and the minimum of the associated CBA antiresonance feature present on the red curve in Plate 1. A comparison of these curves indicates that the enhanced response in the back-scattered direction at the peak of the resonance occurs at the expense of the signals scattered into the other look angles, particularly in the broadside direction. Similarly, the broadside response is enhanced at the expense of the back-scattered signal at the anti-resonance minimum.

The locations of the CBA resonances are also dependent on the characteristics of the interior load. As observed in Plates 1 and 2, there are clear distinctions between the metallic and the dielectric interior sphere cases. Some of the resonance locations are nearly coincident while others are not. This depends intimately on the particular modal pattern that is established in the interior of the cavity. If the mode "interacts" with the interior object, large shifts in the positions of the resonance features may be produced. Comparing the empty and the loaded cavity cases, one finds that as the size of the interior load is increased, very large translations of particular CBA resonance locations occur. As a result, one might expect to and does observe the sequence in which resonances appear to be altered or even a disappearance of some or the appearance of additional resonances in a fixed ka interval. As the relative permittivity of the interior dielectric sphere is increased, the number of resonances found at lower ka values is dramatically increased. Of course, the number of available cavity modes, hence, CBA resonance features becomes quite large as ka (frequency) increases. Mode splitting, as well as mode translation, occurs when the interior object is moved off-axis. This effect has been observed with an off-set wire in the interior of a slit cylinder.

3. CONCLUSIONS

Parameter studies of the generalized dual series solutions of several cavity-backed aperture canonical problems are enhancing our understanding of the aperture coupling and scattering processes. The presence of the resonance features in the scattering cross sections are extremely interesting. The cross section resonances indicate that for cavity-backed apertures there is interior information contained in the exterior scattering data. The dependence of the location of these peaks on the interior structure and their presence at all look angles may have very important ramifications for diagnostic and object identification applications.

Acknowledgments. This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract W-7405-ENG-48.

REFERENCES

Chang, S., and T. B. A. Senior, Scattering by a spherical shell with a circular aperture, *Air Force Weapons Lab. Interaction Notes*, *141*, 1–144, 1969.

Johnson, W. A., and R. W. Ziolkowski, The scattering of an *H*-polarized plane wave from an axially slotted infinite cylinder: A dual series approach, *Radio Sci.*, *19*(1), 275–291, 1984.

Ziolkowski, R. W., *n*-Series problems and the coupling of electromagnetic waves to apertures: A Riemann-Hilbert approach, *SIAM J. Math. Anal.*, *16*(2), 358–378, 1985*a*.

Ziolkowski, R. W., Scattering from a spherical shell with a circular aperture: cross-sections and energy storage for a normally incident plane wave, paper presented at National Radio Science Meeting, URSI IRC-6, Vancouver, B. C., June 1985*b*.

Ziolkowski, R. W., and J. B. Grant, Scattering from cavity-backed apertures: The generalized dual series solution of the concentrically loaded *E*-pol slit cylinder problem, *IEEE Trans. Antennas Propag.*, in press, 1987.

Ziolkowski, R. W., and W. A. Johnson, Electromagnetic scattering of an arbitrary plane wave from a spherical shell with a circular aperture, *J. Math. Phys.*, in press, 1987.

Ziolkowski, R. W., and R. F. Schmucker, Scattering from a slit cylinder enclosing an off-set impedance surface, paper presented at National Radio Science Meeting, URSI B8-8, Boulder, Colo., Jan. 1986.

Ziolkowski, R. W., W. A. Johnson, and K. F. Casey, Applications of Riemann-Hilbert problem techniques to electromagnetic coupling through apertures, *Radio Sci.*, *19*(11), 1425–1431, 1984.

Ziolkowski, R. W., D. P. Marsland, and L. F. Libelo, Scattering from an open spherical shell having a circular aperture and enclosing a metallic or dielectric sphere, paper presented at National Radio Science Meeting, URSI B8-9, Boulder, Colo., Jan. 1986.

Ziolkowski, R. W., D. P. Marsland, L. F. Libelo, and G. Pisane, Scattering from an open spherical shell having a circular aperture and enclosing a concentric dielectric sphere, *Rep. UCRL-960704*, Lawrence Livermore Natl. Lab., Livermore, Calif., 1987.

R. W. Ziolkowski, Engineering Research Division, Lawrence Livermore National Laboratory, P.O. Box 5504, L-156, Livermore, CA 94550.