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Applications of Riemann-Hilbert problem techniques to electromagnetic coupling through apertures

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The generalized dual series approach, as it is applied to the mixed boundary value problems that characterize the coupling of electromagnetic energy through apertures, is described. This approach is based upon techniques borrowed from the analysis of the Riemann-Hilbert problem of complex variable theory. It allows one to obtain essentially analytical solutions to families of canonical coupling problems in separable geometries. The generalized dual series approach is illustrated with the example of the coupling of an H-polarized plane wave to a perfectly conducting infinite cylinder with an infinite axial slot enclosing a concentric impedance cylinder. This problem encompasses the coupling to an empty cylinder as well as to a cylinder enclosing a conducting wire or cable. The solution explicitly exhibits the correct behavior near the edge of the aperture, and it can handle small to large ratios of cylinder radius to wavelength without additional special considerations. The angle of incidence is arbitrary. Results are shown for a normal incidence case and for a case in which the direction of incidence coincides with the edge of the aperture. A brief comparison with related moment method results is given.

1. INTRODUCTION

Solutions of an analytic nature to the electromagnetic coupling problem as it applies to an enclosed region, an external source, and a coupling aperture would provide insight into the coupling mechanism by which electromagnetic energy penetrates apertures into enclosed regions. Moreover, accurate solutions of this type would provide standards for the evaluation of numerical code results. In addition, it could lead to general engineering analysis and design "rules of thumb" for more generally shaped cavities and apertures and to the development of improved numerical techniques for dealing with aperture coupling in general geometries, especially near the edges of the aperture, where purely numerical techniques may encounter difficulties.

Recent developments in the theory and applications of dual series equations [*Casey*, 1982] and their relationship to the Riemann-Hilbert problem [*Ziolkowski*, 1984] make it possible to obtain essentially analytic solutions to families of canonical problems descriptive of electromagnetic (and, indeed, acoustic) coupling via apertures into enclosed regions. The resultant generalized dual series equations approach will be used in this paper to calculate the azimuthal current induced on an infinite cylinder of radius b with an infinite axial slot which encloses a concentric impedance surface by an H-polarized plane wave with an arbitrary angle of incidence. The associated electric and magnetic fields will also be obtained. This problem encompasses the coupling to an empty cylinder as well as to a cylinder enclosing a conducting wire or cable. The results are valid for small to large kb, for any angle of incidence, and for any aperture size.

Several alternate approaches to the empty cylinder case exist. One is a two-dimensional version of the method of moments solution based on vector and scalar potentials analogous to that developed by *Glisson and Wilton* [1980]. As shown by *Johnson and Ziolkowski* [1984], in contrast to this method of moments solution, the generalized dual series approach explicitly contains the behavior of the solution near the aperture rim and does not incur numerical difficulties in the shadow region for any angle of incidence. Singular integral equation treatments have been given by *Hayashi* [1965] and by *Morris* [1982]. Hayashi's solution procedure resembles the Riemann-Hilbert analysis presented here. However,

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no treatment of the resultant infinite linear system for the solution coefficients was given there. In contrast to Morris' method, which utilizes the Hadamard finite part interpretation of the integrals, the generalized dual series equations approach is straightforward.

It has recently been brought to our attention that work of a similar nature has been reported in the Russian literature by V. P. Shestopalov and his coworkers. Specifically, *Koshparënok and Shestopalov* [1971] have applied the Riemann-Hilbert analysis of the dual series equations that was developed by *Agranovich et al.* [1962] to the empty cylinder case; *Nosich and Shestopalov* [1979] have applied it to the case where the cylinder is filled with a dielectric substrate. In contrast with these works, this paper relies on the novel approach to the solution of the infinite system of equations obtained with the Riemann-Hilbert technique discussed in detail by *Johnson and Ziolkowski* [1984]. Our truncation procedure generates a general solution to that system that is not restricted to any special parameter regime. Furthermore, by using an asymptotic form of the solution coefficients, we are able to demonstrate analytically that our general results for the currents and fields exhibit the correct behavior near an edge of the aperture. Finally, much of the related Russian work on diffraction grating and waveguide iris problems is summarized by *Shestopalov* [1971]. Unfortunately, this book is available only in Russian.

Numerical results for the more general impedance surface cases have been obtained. However, because of length constraints and because they adequately illustrate the capabilities of the approach, only characteristic examples of the current and electric field results for the empty cylinder case will be given in this paper. Papers summarizing the more general H-polarized cases and more recent E-polarized case results are currently in preparation.

2. REDUCTION TO THE DUAL SERIES PROBLEM

The problem geometry is shown in Figure 1. A cylindrical coordinate system (ρ, ϕ, z) is erected; the z axis coincides with the cylinder's axis. The impedance surface lies on the coordinate surface $\rho = a$; the cylinder lies on $\rho = b$. The metallic portion of the cylinder lies in the region $-\Theta < \phi < \Theta$; the aperture is the complimentary region $\Theta < \phi < 2\pi - \Theta$. The H-polarized plane wave is assumed normally incident on the cylinder; hence the problem is two-dimensional.

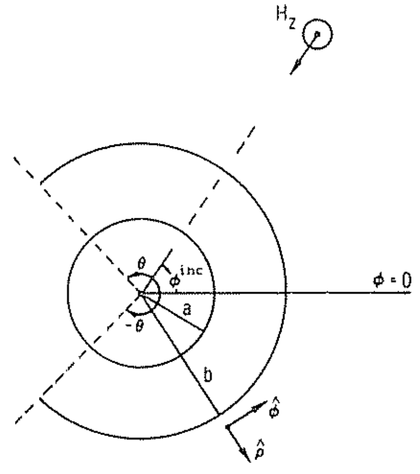

Fig. 1. Configuration of the scattering of an H-polarized plane wave from a cylinder with an infinite axial slot

For the given polarization, Maxwell's equations decouple and only the E_ρ, E_ϕ, and H_z components are excited. The components of the field tangential to the surface of the aperture and the cylinder are of particular importance in the dual series formulation. They are related by $E_\phi = (j/\omega\varepsilon)\partial_\rho H_z$, where, as throughout this paper, the time dependence $e^{j\omega t}$ has been assumed and suppressed.

The incident fields are first decomposed into Fourier modes. For the incident plane wave, the magnetic field

$$H_z^{inc} = \tilde{H}_0 e^{jk\rho\cos(\phi - \phi^{inc})}$$
$$= \tilde{H}_0 \sum_{m=-\infty}^{\infty} j^{|m|} J_{|m|}(k\rho) e^{jm(\phi - \phi^{inc})} \quad (1)$$

hence the electric field component

$$E_\phi^{inc} = jZ_0 \tilde{H}_0 \sum_{m=-\infty}^{\infty} [j^{|m|} J'_{|m|}(k\rho) e^{-jm\phi^{inc}}] e^{jm\phi} \quad (2)$$

where $J'_m(x) = dJ_m/dx$ and $Z_0 = k/\omega\varepsilon$ is the free-space impedance. It then follows that the Fourier expansions of the total fields can be expressed as

$$H_z = \tilde{H}_0 \sum_{m=-\infty}^{\infty} [A_m J_{|m|}(k\rho) + B_m H_{|m|}(k\rho)] e^{jm\phi} \qquad a < \rho < b$$
$$H_z = \tilde{H}_0 \sum_{m=-\infty}^{\infty} C_m H_{|m|}(k\rho) e^{jm\phi} + \sum_{m=-\infty}^{\infty} j^{|m|} J_{|m|}(k\rho) e^{jm(\phi - \phi^{inc})} \qquad \rho > b \quad (3)$$

$$E_\phi = jZ_0 \tilde{H}_0 \sum_{m=-\infty}^{\infty} [A_m J'_{|m|}(k\rho) + B_m H'_{|m|}(k\rho)]e^{jm\phi} \qquad a < \rho < b$$

$$E_\phi = jZ_0 \tilde{H}_0 \sum_{m=-\infty}^{\infty} C_m H'_{|m|}(k\rho)e^{jm\phi} + \sum_{m=-\infty}^{\infty} j^{|m|} J'_{|m|}(k\rho)e^{jm(\phi-\phi^{\rm inc})} \qquad \rho > b \tag{4}$$

where H_m is the Hankel function of second kind and order m, and $H'_m(x) = dH_m/dx$.

The impedance condition at $\rho = a$ is enforced to reduce the number of unknown coefficients. It is

$$E_\phi(a, \phi) = ZH_z(a, \phi) \tag{5}$$

Satisfaction of this condition by (3) and (4) requires

$$\frac{B_m}{A_m} = -\left[\frac{ZJ_{|m|}(ka) - jZ_0 J'_{|m|}(ka)}{ZH_{|m|}(ka) - jZ_0 H'_{|m|}(ka)}\right] \equiv \Omega_{|m|}(ka) \tag{6}$$

Several special cases are encompassed by these results. A concentric electric conductor is described by $Z = 0$, hence by $E_\phi(a, \phi) \equiv 0$ and $\Omega_m(ka) = -J'_m(ka)/H'_m(ka)$. A concentric magnetic conductor is described by $Z = \infty$, hence, by $H_z(a, \phi) \equiv 0$ and $\Omega_m(ka) = -J_m(ka)/H_m(ka)$. An empty cylinder is described by $\Omega_m(ka) \equiv 0$.

The number of unknown coefficients is reduced further by requiring the continuity of E_ϕ across the cylinder $\rho = b$. This condition is satisfied if

$$A_m[J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)] = C_m H'_{|m|}(kb) + j^{|m|} J'_{|m|}(kb)e^{-jm\phi^{\rm inc}}$$

which means

$$C_m = A_m \frac{[J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)]}{H'_{|m|}(kb)} - j^{|m|} \frac{J'_{|m|}(kb)e^{-jm\phi^{\rm inc}}}{H'_{|m|}(kb)} \tag{7}$$

Consequently, the total fields can now be expressed solely in terms of the unknown modal coefficient A_m as

$$H_z = \tilde{H}_0 \sum_{m=-\infty}^{\infty} A_m[J_{|m|}(k\rho) + \Omega_{|m|}(ka)H_{|m|}(k\rho)]e^{jm\phi} \qquad a < \rho < b$$

$$H_z = \tilde{H}_0 \sum_{m=-\infty}^{\infty} A_m \frac{H_{|m|}(k\rho)}{H'_{|m|}(kb)} [J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)]e^{jm\phi} + \sum_{m=-\infty}^{\infty} j^{|m|} J_{|m|}(k\rho) - \frac{J'_{|m|}(kb)}{H'_{|m|}(kb)} H_{|m|}(k\rho)e^{jm(\phi-\phi^{\rm inc})} \qquad \rho > b \tag{8}$$

$$E_\phi = jZ_0 \tilde{H}_0 \sum_{m=-\infty}^{\infty} A_m[J'_{|m|}(k\rho) + \Omega_{|m|}(ka)H'_{|m|}(k\rho)]e^{jm\phi} \qquad a < \rho < b$$

$$E_\phi = jZ_0 \tilde{H}_0 \sum_{m=-\infty}^{\infty} A_m \frac{H'_{|m|}(k\rho)}{H'_{|m|}(kb)} [J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)]e^{jm\phi} + \sum_{m=-\infty}^{\infty} j^{|m|}\left[J'_{|m|}(k\rho) - \frac{J'_{|m|}(kb)}{H'_{|m|}(kb)} H'_{|m|}(k\rho)\right] e^{jm(\phi-\phi^{\rm inc})} \qquad \rho > b \tag{9}$$

Finally, the boundary conditions for the tangential electric and magnetic fields at the surface $\rho = b$ are enforced to obtain the dual series equations. The continuity of H_z in the aperture and the Wronskian

$$J'_{|m|}(kb)H_{|m|}(kb) - J_{|m|}(kb)H'_{|m|}(kb) = \frac{2j}{\pi kb} \tag{10}$$

yield the relation

$$\sum_{m=-\infty}^{\infty} \left[\frac{A_m}{H'_{|m|}(kb)}\right]e^{jm\phi} = \sum_{m=-\infty}^{\infty} \left[\frac{j^{|m|}e^{-jm\phi^{\rm inc}}}{H'_{|m|}(kb)}\right]e^{jm\phi} \qquad |\phi| > \Theta \tag{11}$$

The condition that the total tangential electric field be zero on the metal yields

$$\sum_{m=-\infty}^{\infty} A_m[J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)]e^{jm\phi} = 0 \qquad |\phi| < \Theta \tag{12}$$

Introducing the coefficients

$$b_m = \frac{A_m - j^{|m|}e^{-jm\phi^{\rm inc}}}{H'_{|m|}(kb)} \tag{13}$$

$$\tau_m = j\pi(kb)^2\{J'_{|m|}(kb)H'_{|m|}(kb) + \Omega_{|m|}(ka)[H'_{|m|}(kb)]^2\} \tag{14}$$

and the expression

$$f(\phi) = \sum_{m=-\infty}^{\infty} f_m e^{jm\phi} \tag{15a}$$

where

$$f_m = -j^{|m|+1}\pi(kb)^2 e^{-jm\phi^{\rm inc}}[J'_{|m|}(kb) + \Omega_{|m|}(ka)H'_{|m|}(kb)] \tag{15b}$$

Equations (11) and (12) can be rewritten as the dual series system

$$\sum_{m=-\infty}^{\infty} b_m e^{jm\phi} = 0 \qquad |\phi| > \Theta$$
$$\sum_{m=-\infty}^{\infty} b_m \tau_m e^{jm\phi} = f(\phi) \qquad |\phi| < \Theta \tag{16}$$

The dc components of the fields (their small kb, hence small ka behavior) can be extracted by introducing the functions χ_m so that

$$\tau_m = -\xi \qquad m = 0$$
$$\tau_m = |m|(1 + \chi_m) \qquad m \neq 0 \tag{17}$$

where $\chi_m(0) = -(a/b)^{2m}$ for $m \neq 0$, and χ_m rapidly approaches zero as $m \to \infty$. The resultant dual series equations are

$$\sum_{m=-\infty}^{\infty} b_m e^{jm\phi} = 0 \qquad |\phi| > \Theta \tag{18a}$$

$$\sum_{m=-\infty}^{\infty} b_m |m|(1 + \chi_m) e^{jm\phi} = \xi b_0 + f(\phi) \qquad |\phi| < \Theta \tag{18b}$$

3 EQUIVALENT RIEMANN-HILBERT PROBLEM

The Riemann-Hilbert problem is a classical problem in complex variable theory. It concerns the construction of the analytic function x whose limits x_+ and x_- from the inside and outside of a closed curve satisfy a transition condition $x_- = gx_+ + h$ on an open segment of that curve. The problem and its solution are described thoroughly, for instance, by *Gakhov*, [1966]. Its relationship to the aperture coupling problem is discussed in detail by *Ziolkowski* [1984].

The form of the dual series system (18) is identical to the one obtained by *Johnson and Ziolkowski* [1984] for the empty cylinder case. Consequently, its solution follows immediately from those results. Note that this conclusion is highly desirable from a practical point of view. It was the primary motivation for introducing the auxiliary coefficients b_m to generate (18) rather than dealing directly with the dual series system formed by (11) and (12). Moreover, since the terms in the dual series system (18) reduce to those derived by *Johnson and Ziolkowski* [1984] when $\Omega_{|m|}(ka) = 0$, comparisons between the empty and nonempty cylinder configurations are readily acquired.

Retracing the analysis by *Ziolkowski* [1984], differentiate (18a) with respect to ϕ and introduce the coefficients $x_m = b_m m$ and the functions

$$x_+(z) = \sum_{m>0} x_m z^m \tag{19}$$

$$x_-(z) = -\sum_{m<0} x_m z^m \tag{20}$$

$$F(e^{j\phi}) = \xi b_0 + f(\phi) - \sum_{m\neq 0} x_m \frac{|m|}{m} \chi_m e^{jm\phi} \tag{21}$$

The dual series equations (18) can then be written simply as

$$x_+(e^{j\phi}) - x_-(e^{j\phi}) = 0 \qquad |\phi| > \Theta \tag{22a}$$

$$x_+(e^{j\phi}) + x_-(e^{j\phi}) = F(e^{j\phi}) \qquad |\phi| < \Theta \tag{22b}$$

This system represents the equivalent Riemann-Hilbert problem. Equation (22a) expresses the continuity of x across the aperture; equation (22b) represents the transition condition x must satisfy across the metal. As shown by *Johnson and Ziolkowski* [1984] and *Ziolkowski* [1984], its solution defines the Fredholm equation of the second kind

$$x_m + \sum_{n=-\infty}^{\infty} \Lambda_{mn} x_n = \sum_{n=-\infty}^{\infty} \Gamma_{mn} f_n \tag{23}$$

for the modal coefficients. The coefficients Λ_{mn} and the forcing terms f_n and Γ_{mn} are known and defined in terms of readily calculable functions (Legendre polynomials, Bessel and Hankel functions). The system (23) is completed with the auxiliary condition

$$b_0 = -\sum_{m\neq 0} \frac{(-)^m}{m} x_m \tag{24}$$

which is obtained from (18a) with $\phi = \pi$; it is introduced to account for the constant eliminated in the differentiation that led from (18a) to (22a). Equations

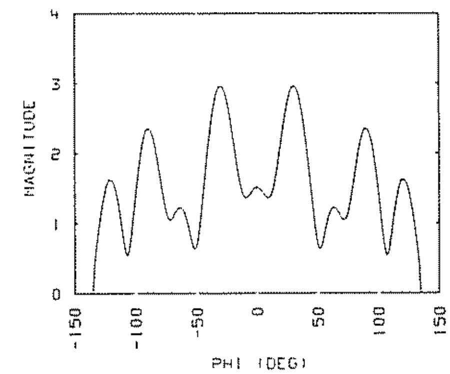

Fig. 2 Currents calculated by the dual series (solid line) and the method of moments (dotted line) for an H-polarized plane wave incident at $\phi^{inc} = 180°$ on an empty cylinder of radius 1.0 λ with an aperture angle $\Theta_{ap} = \pi - \Theta = 45°$.

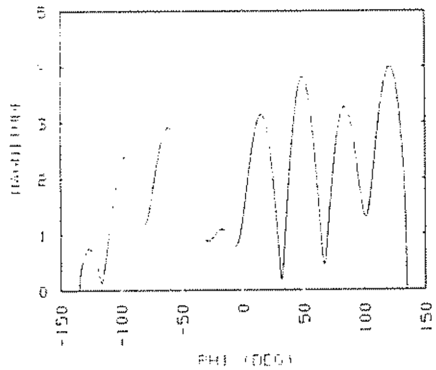

Fig. 3. Currents calculated by the dual series (solid line) and the method of moments (dotted line) for an H-polarized plane wave incident at $\phi^{inc} = 135°$ on an empty cylinder of radius 1.0 λ with an aperture angle $\Theta_{ap} = \pi - \Theta = 45°$.

(23) and (24) uniquely define the desired modal coefficients.

4. NUMERICAL IMPLEMENTATION

The infinite system (23) and (24) can be treated in several ways. It was found by *Johnson and Ziolkowski* [1984] that because f_n and Λ_{mn} rapidly approach zero for large values of n, truncating f_n and Λ_{mn} for $|n|$ greater than some value N and using Gauss elimination to solve the remaining finite square system

$$x_m + \sum_{n=-N}^{N} \Lambda_{mn} x_n = \sum_{n=-N}^{N} \Gamma_{mn} f_n \tag{25a}$$

$$b_0 = \sum_{n=-N}^{N} \sum_{m \neq 0} \frac{(-)^m}{m} \Lambda_{mn} x_n - \sum_{n=-N}^{N} \sum_{m \neq 0} \frac{(-)^m}{m} \Gamma_{mn} f_n \tag{25b}$$

where $m = -N, -N+1, \cdots, +N$ yields good numerical approximations for the coefficients b_0, $x_{\pm 1}$, $\cdots$, $x_{\pm N}$. The remaining coefficients, x_m, for $|m| > N$ are given by (25a). As N approaches infinity, this approximate solution scheme becomes exact.

5. SAMPLE NUMERICAL RESULTS

Although the fields and the current induced on the cylinder's surface may be readily computed via (9) and (10) once the modal coefficients b_0 and $b_m = x_m/m$ ($m \neq 0$) are known, alternate expressions that

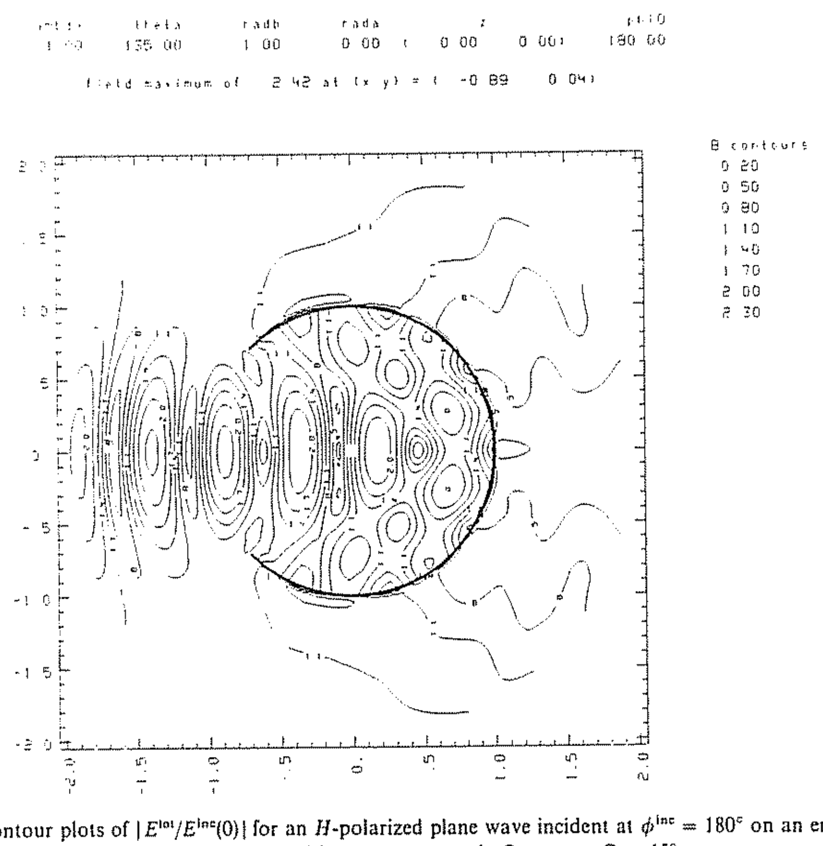

Fig. 4. Contour plots of $|E^{tot}/E^{inc}(0)|$ for an H-polarized plane wave incident at $\phi^{inc} = 180°$ on an empty cylinder of radius 1.0 λ with an aperture angle $\Theta_{ap} = \pi - \Theta = 45°$.

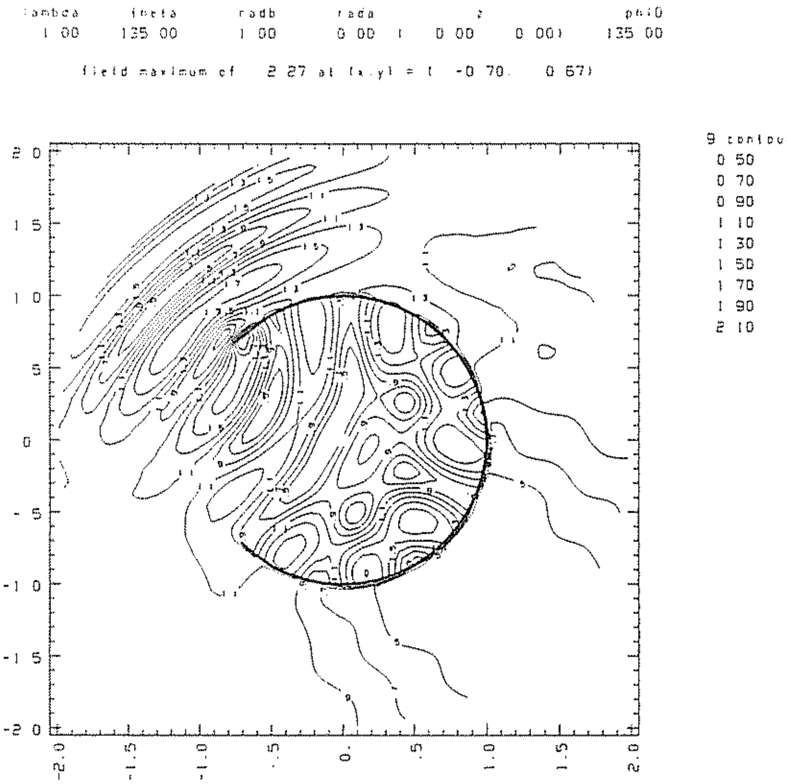

Fig. 5. Contour plots of $|E^{tot}/E^{inc}(0)|$ for an H-polarized plane wave incident at $\phi^{inc} = 135°$ on an empty cylinder of radius 1.0 λ with an aperture angle $\Theta_{ap} = \pi - \Theta = 45°$

are particularly useful in studying the behavior of those quantities near an edge of the aperture speed up the summation process. For instance, the current induced on the cylinder may be expressed as

$$J_\phi(b, \phi) = H_{z<}(\phi) - H_{z>}(\phi)$$
$$= \frac{2\tilde{H}_0}{j\pi kb}\left(b_0 + \sum_{m \neq 0} b_m e^{jm\phi}\right) \tag{26}$$

The rate of convergence of this infinite sum can be accelerated by expressing it as

$$\sum_{m \neq 0} b_m e^{jm\phi} = \sum_{m \neq 0} \tilde{b}_m e^{jm\phi} + \sum_{m \neq 0} (b_m - \tilde{b}_m) e^{jm\phi} \tag{27}$$

where $\tilde{b}_m$ is a large m approximation of b_m. The first sum can be treated analytically; it contains the singular component of the current near an edge of the aperture. The second sum is rapidly converging and is truncated for $|m| > M$.

The magnitudes of the currents generated with this generalized dual series scheme (solid lines) and with a two-dimensional method of moments code (dotted lines) for the empty cylinder ($\Omega_{|m|} \equiv 0$) case are shown in Figures 2 and 3. In Figure 2, the angle of incidence $\phi^{inc} = 180°$; in Figure 3, $\phi^{inc} = 135°$. The radius of the cylinder in terms of wavelength $(b/\lambda) = 1.0$ and the aperture angle $\Theta_{ap} = \pi - \Theta = 45°$ in both cases. The truncation numbers were chosen to be large: $N = 25$ and $M = 190$, to guarantee the convergence of the generalized dual series results. Note that both figures clearly demonstrate that the generalized dual series solution readily models the correct behavior of the currents near the edges of the aperture. Furthermore, as discussed by *Johnson and Ziolkowski* [1984], the generalized dual series solution has revealed that the moment method solution will properly describe the current (especially in the shadow region) only if a sufficiently fine gridding near the aperture edges is employed. The slight inaccuracy of the moment method solution present in both figures disappears when finer gridding is utilized.

Contour plots of the corresponding total E-field magnitudes are given, respectively, in Figures 4 and 5. They were obtained by calculating E_ρ and E_ϕ for 0.033-λ increments in radius and 5.625° increments in

ϕ The magnitude of the total electric field is normalized to unity at the center of the cylinder Both figures indicate an enhancement of the field amplitudes in the interior of the cylinder

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