• Impedance \((Z)\) plays the same role in the frequency domain as resistance, inductance, and capacitance play in the time domain. Specifically, the relationship between phasor current and phasor voltage for resistors, inductors, and capacitors is

\[ V = ZI, \]

where the reference direction for \(I\) obeys the passive sign convention. The reciprocal of impedance is admittance \((Y)\), so another way to express the current-voltage relationship for resistors, inductors, and capacitors in the frequency domain is

\[ V = I/Y. \]

(See pages 345 and 350.)

• All of the circuit analysis techniques developed in Chapters 2–4 for resistive circuits also apply to sinusoidal steady-state circuits in the frequency domain. These techniques include KVL, KCL, series, and parallel combinations of impedances, voltage and current division, node voltage and mesh current methods, source transformations and Thévenin and Norton equivalents.

- The two-winding linear transformer is a coupling device made up of two coils wound on the same nonmagnetic core. Reflected impedance is the impedance of the secondary circuit as seen from the terminals of the primary circuit or vice versa. The reflected impedance of a linear transformer seen from the primary side is the conjugate of the self-impedance of the secondary circuit scaled by the factor \(\left(\frac{\omega M}{|Z_{22}|}\right)^2\). (See pages 361 and 363.)

- The two-winding ideal transformer is a linear transformer with the following special properties: perfect coupling \((k = 1)\), infinite self-inductance in each coil \((L_1 = L_2 = \infty)\), and lossless coils \((R_1 = R_2 = 0)\). The circuit behavior is governed by the turns ratio \(a = N_2/N_1\). In particular, the volts per turn is the same for each winding, or

\[ \frac{V_1}{N_1} = \pm \frac{V_2}{N_2}, \]

and the ampere turns are the same for each winding, or

\[ N_1I_1 = \pm N_2I_2. \]

(See page 365 and 366.)

### Table 9.3 Impedance and Related Values

<table>
<thead>
<tr>
<th>Element</th>
<th>Impedance ((Z))</th>
<th>Reactance</th>
<th>Admittance ((Y))</th>
<th>Susceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>(R) (resistance)</td>
<td></td>
<td>(G) (conductance)</td>
<td></td>
</tr>
<tr>
<td>Capacitor</td>
<td>(j(-1/\omega C))</td>
<td>(-1/\omega C)</td>
<td>(j\omega C)</td>
<td>(\omega C)</td>
</tr>
<tr>
<td>Inductor</td>
<td>(j\omega L)</td>
<td>(\omega L)</td>
<td>(j(-1/\omega L))</td>
<td>(-1/\omega L)</td>
</tr>
</tbody>
</table>

### Problems

#### Section 9.1

9.1 A sinusoidal voltage is given by the expression

\[ v = 100 \cos(240\pi t + 45^\circ) \text{ mV}. \]

Find (a) \(f\) in hertz; (b) \(T\) in milliseconds; (c) \(V_m\); (d) \(v(0)\); (e) \(\phi\) in degrees and radians; (f) the smallest positive value of \(t\) at which \(v = 0\); and (g) the smallest positive value of \(t\) at which \(dv/dt = 0\).

9.2 In a single graph, sketch \(v = 60 \cos(\omega t + \phi)\) versus \(\omega t\) for \(\phi = -60^\circ, -30^\circ, 0^\circ, 30^\circ,\) and \(60^\circ\).

a) State whether the voltage function is shifting to the right or left as \(\phi\) becomes more positive.

b) What is the direction of shift if \(\phi\) changes from 0 to \(-30^\circ\)?

9.3 At \(t = -250/6 \mu s\), a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at \(t = 1250/6 \mu s\). It is also known that the voltage is 75 V at \(t = 0\).

a) What is the frequency of \(v\) in hertz?

b) What is the expression for \(v\)?

9.4 A sinusoidal current is zero at \(t = 150 \mu s\) and increasing at a rate of \(2 \times 10^4\pi\) A/s. The maximum amplitude of the voltage is 10 A.

a) What is the frequency of \(v\) in radians per second?

b) What is the expression for \(v\)?
5. Consider the sinusoidal voltage
\[ v(t) = 170 \cos(120\pi t - 60^\circ) \text{ V}. \]

a) What is the maximum amplitude of the voltage?
b) What is the frequency in hertz?
c) What is the frequency in radians per second?
d) What is the phase angle in radians?
e) What is the phase angle in degrees?
f) What is the period in milliseconds?
g) What is the first time after \( t = 0 \) that \( v = 170 \text{ V} \)?
h) The sinusoidal function is shifted \( 125/18 \text{ ms} \) to the right along the time axis. What is the expression for \( v(t) \)?
i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for \( v(t) \) is \( 170 \sin(120\pi t) \text{ V} \)?
j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for \( v(t) \) is \( 170 \cos(120\pi t) \text{ V} \)?

6. Show that
\[ \int_0^{t_0 + T} \frac{V_m^2 \cos^2(\omega t + \phi)}{2} dt = \frac{V_m^2 T}{2} \]

7. The rms value of the sinusoidal voltage supplied to the convenience outlet of a U.S. home is 120 V. What is the maximum value of the voltage at the outlet?

8. Find the rms value of the half-wave rectified sinusoidal voltage shown.

Figure P9.8

\[ v = V_m \sin \frac{2\pi}{T} t, \quad 0 \leq t \leq T/2 \]

Section 9.2

9.9 The voltage applied to the circuit shown in Fig. 9.5 at \( t = 0 \) is \( 100 \cos(400t + 60^\circ) \text{ V}. \) The circuit resistance is 40 \( \Omega \) and the initial current in the 75 mH inductor is zero.

a) Find \( i(t) \) for \( t \geq 0 \).
b) Write the expressions for the transient and steady-state components of \( i(t) \).
c) Find the numerical value of \( i \) after the switch has been closed for 1.875 ms.
d) What are the maximum amplitude, frequency (in radians per second), and phase angle of the steady-state current?
e) By how many degrees are the voltage and the steady-state current out of phase?

9.10 a) Verify that Eq. 9.9 is the solution of Eq. 9.8. This can be done by substituting Eq. 9.9 into the left-hand side of Eq. 9.8 and then noting that it equals the right-hand side for all values of \( t > 0 \). At \( t = 0 \), Eq. 9.9 should reduce to the initial value of the current.
b) Because the transient component vanishes as time elapses and because our solution must satisfy the differential equation for all values of \( t \), the steady-state component, by itself, must also satisfy the differential equation. Verify this observation by showing that the steady-state component of Eq. 9.9 satisfies Eq. 9.8.

9.11 Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:

a) \( y = 100 \cos(300t + 45^\circ) + 500 \cos(300t - 60^\circ) \),
b) \( y = 250 \cos(377t + 30^\circ) - 150 \sin(377t + 140^\circ) \),
c) \( y = 60 \cos(100t + 60^\circ) - 120 \sin(100t - 125^\circ) + 100 \cos(100t + 90^\circ) \), and
d) \( y = 100 \cos(\omega t + 40^\circ) + 100 \cos(\omega t + 160^\circ) + 100 \cos(\omega t - 80^\circ) \).

9.12 A 50 Hz sinusoidal voltage with a maximum amplitude of 340 V at \( t = 0 \) is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 8.5 A.

a) What is the frequency of the inductor current?
b) If the phase angle of the voltage is zero, what is the phase angle of the current?
c) What is the inductive reactance of the inductor?
d) What is the inductance of the inductor in millihenrys?
e) What is the impedance of the inductor?
9.13 A 40 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 2.5 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 125.67 μA.

a) What is the frequency of the current in radians per second?

b) What is the phase angle of the current?

c) What is the capacitive reactance of the capacitor?

d) What is the capacitance of the capacitor in microfarads?

e) What is the impedance of the capacitor?

9.14 The expressions for the steady-state voltage and current at the terminals of the circuit seen in Fig. P9.14 are

\[ v_g = 150 \cos (8000\pi t + 20') \text{ V}, \]

\[ i_g = 30 \sin (8000\pi t + 38') \text{ A} \]

\[ R_1 = \frac{R_2}{1 + \omega^2 R_2 C_2^2} \]

\[ C_1 = \frac{1 + \omega^2 R_2 C_2^2}{\omega^2 R_2 C_2}. \]

a) What is the impedance seen by the source?

b) By how many microseconds is the current out of phase with the voltage?

9.15 A 20 Ω resistor and a 1 μF capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of a 1 Ω resistor and a 40 μH inductor. These three parallel branches are driven by a sinusoidal current source whose current is 20 cos(50,000t − 20') A.

a) Draw the frequency-domain equivalent circuit.

b) Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.

c) Find the steady-state expression for v(t).

9.16 A 400 Ω resistor, a 87.5 mH inductor, and a 312.5 nF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is 500 cos (8000t + 60') V.

a) Draw the frequency-domain equivalent circuit.

b) Reference the current in the direction of the voltage rise across the source, and find the phasor current.

c) Find the steady-state expression for i(t).

9.17 a) Show that at a given frequency \( \omega \), the circuits in Fig. P9.17(a) and (b) will have the same impedance between the terminals a,b if

\[ R_1 = \frac{R_2}{1 + \omega^2 R_2 C_2^2} \]

\[ C_1 = \frac{1 + \omega^2 R_2 C_2^2}{\omega^2 R_2 C_2}. \]

b) Find the values of resistance and capacitance that when connected in series will have the same impedance at 80 krad/s as that of a 500 Ω resistor connected in parallel with a 25 nF capacitor.

9.18 a) Show that at a given frequency \( \omega \), the circuits in Fig. 9.17(a) and (b) will have the same impedance between the terminals a,b if

\[ R_2 = \frac{1 + \omega^2 R_1 C_1^2}{\omega^2 R_1 C_1^2}, \]

\[ C_2 = \frac{C_1}{1 + \omega^2 R_1 C_1^2}. \]

(Hint: The two circuits will have the same impedance if they have the same admittance.)
b) Find the values of resistance and capacitance that when connected in parallel will give the same impedance at 20 krad/s as that of a 2 kΩ resistor connected in series with a capacitance of 50 nF.

19 a) Show that, at a given frequency \( \omega \), the circuits in Fig. P9.19(a) and (b) will have the same impedance between the terminals a,b if

\[
R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2}, \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}.
\]

b) Find the values of resistance and inductance that when connected in series will have the same impedance at 20 krad/s as that of a 50 kΩ resistor connected in parallel with a 2.5 H inductor.

Figure P9.19

![Figure P9.19](image)

20 a) Show that at a given frequency \( \omega \), the circuits in Fig. P9.19(a) and (b) will have the same impedance between the terminals a,b if

\[
R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1}, \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}.
\]

(Hint: The two circuits will have the same impedance if they have the same admittance.)

b) Find the values of resistance and inductance that when connected in parallel will have the same impedance at 10 krad/s as a 5 kΩ resistor connected in series with a 500 mH inductor.

9.22 a) For the circuit shown in Fig. P9.22, find the frequency (in radians per second) at which the impedance \( Z_{ab} \) is purely resistive.

b) Find the value of \( Z_{ab} \) at the frequency of (a).

Figure P9.22

![Figure P9.22](image)

9.23 Find the impedance \( Z_{ab} \) in the circuit seen in Fig. P9.23. Express \( Z_{ab} \) in both polar and rectangular form.

Figure P9.23

![Figure P9.23](image)

9.24 Find the admittance \( Y_{ab} \) in the circuit seen in Fig. P9.24. Express \( Y_{ab} \) in both polar and rectangular form. Give the value of \( Y_{ab} \) in millisiemens.

Figure P9.24

![Figure P9.24](image)
9.25 Find the steady-state expression for \( i_o(t) \) in the circuit in Fig. P9.25 if \( v_o = 750 \cos 5000t \) mV.

**Figure P9.25**

\[
\begin{array}{c}
\text{400 \Omega} \\
\text{40 mH} \\
\text{0.4 \mu F}
\end{array}
\]

\[ v_o(t) \]

\[ i_o(t) \]

9.26 The circuit shown in Fig. P9.26 is operating in the sinusoidal steady state. Find the value of \( \omega \) if

\[ i_o = 100 \sin (\omega t + 81.87^\circ) \] mA,

\[ v_o = 50 \cos (\omega t - 45^\circ) \] V.

**Figure P9.26**

\[
\begin{array}{c}
\text{400 \Omega} \\
\text{40 mH} \\
\text{400 nF}
\end{array}
\]

9.27 Find the steady-state expression for \( v_o \) in the circuit of Fig. P9.27 if \( i_g = 200 \cos 5000t \) mA.

**Figure P9.27**

\[
\begin{array}{c}
\text{240 \Omega} \\
\text{v_o} \\
\text{80 \Omega} \\
\text{2.5 \mu F} \\
\text{48 mH}
\end{array}
\]

9.28 The circuit in Fig. P9.28 is operating in the sinusoidal steady state. Find the steady-state expression for \( v_o(t) \) if \( v_g = 64 \cos 8000t \) V.

**Figure P9.28**

\[
\begin{array}{c}
31.25 \text{ nF} \\
2 \text{ k\Omega} \\
500 \text{ mH}
\end{array}
\]

9.29 The phasor current \( I_a \) in the circuit shown in Fig. P9.29 is 40 \( \angle 0^\circ \) mA.

a) Find \( I_b, I_c, \) and \( V_g \).

b) If \( \omega = 800 \) rad/s, write the expressions for \( i_a(t) \), \( i_b(t) \), and \( v_g(t) \).

**Figure P9.29**

\[
\begin{array}{c}
\text{25 \Omega} \\
\text{120 \Omega} \\
\text{160 \Omega} \\
\text{40 + j80 mA}
\end{array}
\]

9.30 a) For the circuit shown in Fig. P9.30, find the steady-state expression for \( v_o \) if \( i_g = 5 \cos (8 \times 10^2 t) \) A.

b) By how many nanoseconds does \( v_o \) lag \( i_g \)?

**Figure P9.30**

\[
\begin{array}{c}
\text{12 \Omega} \\
\text{125 nF} \\
\text{20 \Omega} \\
\text{25 \mu H} \\
\text{v_o}
\end{array}
\]

9.31 The circuit in Fig. P9.31 is operating in the sinusoidal steady state. Find \( v_o(t) \) if \( i_a(t) = 15 \cos 8000t \) mA.

**Figure P9.31**

\[
\begin{array}{c}
\text{5 k\Omega} \\
\text{10 k\Omega} \\
\text{50 nF} \\
\text{1.25 H} \\
\text{30 k\Omega} \\
\text{v_o(t)}
\end{array}
\]

9.32 Find \( I_b \) and \( Z \) in the circuit shown in Fig. P9.32 if \( V_g = 60 \angle 0^\circ \) V and \( I_a = 5 \angle -90^\circ \) A.

**Figure P9.32**

\[
\begin{array}{c}
-j5 \Omega \\
\text{j5 \Omega} \\
\text{j2 \Omega} \\
\text{I_b} \\
\text{Z} \\
\text{I_b} \\
\text{-j8 \Omega}
\end{array}
\]
33 Find the value of Z in the circuit seen in Fig. P9.33 if \( V_g = 100 - j50 \, \text{V}, \) \( I_g = 20 + j30 \, \text{A}, \) and \( V_I = 40 + j30 \, \text{V}. \)

Figure P9.33

34 Find \( Z_{ab} \) for the circuit shown in Fig P9.34.

Figure P9.34

35 The frequency of the sinusoidal voltage source in the circuit in Fig. P9.35 is adjusted until the current \( i_g \) is in phase with \( v_g. \)

a) Find the frequency in hertz.

b) Find the steady-state expression for \( i_g \) (at the frequency found in [a]) if \( v_g = 10 \cos \omega t \) \( \text{V}. \)

Figure P9.35

36 a) The frequency of the source voltage in the circuit in Fig. P9.36 is adjusted until \( i_g \) is in phase with \( v_g. \) What is the value of \( \omega \) in radians per second?

b) If \( v_g = 45 \cos \omega t \) \( \text{V} \) (where \( \omega \) is the frequency found in [a]), what is the steady-state expression for \( v_o? \)

Figure P9.36

9.37 The frequency of the sinusoidal current source in the circuit in Fig. P9.37 is adjusted until \( v_o \) is in phase with \( i_g. \)

a) What is the value of \( \omega \) in radians per second?

b) If \( i_g = 2.5 \cos \omega t \) \( \text{mA} \) (where \( \omega \) is the frequency found in [a]), what is the steady-state expression for \( v_o? \)

Figure P9.37

9.38 The circuit shown in Fig. P9.38 is operating in the sinusoidal steady state. The capacitor is adjusted until the current \( i_g \) is in phase with the sinusoidal voltage \( v_g. \)

a) Specify the capacitance in microfarads if \( v_g = 250 \cos 1000t \) \( \text{V}. \)

b) Give the steady-state expression for \( i_g \) when \( C \) has the value found in (a).

Figure P9.38
9.39 a) The source voltage in the circuit in Fig. P9.39 is $v_s = 96 \cos 10,000 \pi t$ V. Find the values of $L$ such that $i_g$ is in phase with $v_g$ when the circuit is operating in the steady state.

b) For the values of $L$ found in (a), find the steady-state expressions for $i_g$.

Section 9.7

9.40 Use source transformations to find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.40.

9.41 Use source transformations to find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.41.

9.42 The sinusoidal voltage source in the circuit in Fig. P9.42 is developing a voltage equal to $22.36 \cos (5000t + 26.565^\circ)$ V.

a) Find the Thévenin voltage with respect to the terminals a,b.

b) Find the Thévenin impedance with respect to the terminals a,b.

c) Draw the Thévenin equivalent.

9.43 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.43.

9.44 The device in Fig. P9.44 is represented in the frequency domain by a Norton equivalent. When an inductor having an impedance of $j100 \Omega$ is connected across the device, the value of $V_0$ is $100/120^\circ$ mV. When a capacitor having an impedance of $-j100 \Omega$ is connected across the device, the value of $I_0$ is $-3/210^\circ$ mA. Find the Norton current $I_N$ and the Norton impedance $Z_N$.

9.45 Find $Z_{ab}$ in the circuit shown in Fig. P9.45 when the circuit is operating at a frequency of 1.6 Mrad/s.
9.46 Find the Thévenin impedance seen looking into the terminals a,b of the circuit in Fig. P9.46 if the frequency of operation is 25 kHz/s.

Figure P9.46

9.47 Find the Norton equivalent with respect to terminals a,b in the circuit of Fig. P9.47.

Figure P9.47

9.48 Find the Thévenin equivalent circuit with respect to the terminals a,b of the circuit shown in Fig. P9.48.

Figure P9.48

9.49 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.49 when $V_g = 25 \angle 0^\circ$ V.

Figure P9.49

9.50 The circuit shown in Fig. P9.50 is operating at a frequency of 10 kHz/s. Assume $\alpha$ is real and lies between $-50$ and $+50$, that is, $-50 \leq \alpha \leq 50$.

a) Find the value of $\alpha$ so that the Thévenin impedance looking into the terminals a,b is purely resistive.

b) What is the value of the Thévenin impedance for the $\alpha$ found in (a)?

c) Can $\alpha$ be adjusted so that the Thévenin impedance equals $5 + j5$? If so, what is the value of $\alpha$?

d) For what values of $\alpha$ will the Thévenin impedance be inductive?

Figure P9.50

Section 9.8

9.51 Use the node-voltage method to find $V_o$ in the circuit in Fig. P9.51.

Figure P9.51

9.52 Use the node-voltage method to find the steady-state expression for $v_e(t)$ in the circuit in Fig. P9.52 if

$v_{g1} = 10 \cos (5000t + 53.13^\circ) \text{ V},$

$v_{g2} = 8 \sin 5000t \text{ V}.$

Figure P9.52
Section 9.9

9.57 Use the mesh-current method to find the steady-state expression for \( i_a(t) \) in the circuit in Fig. P9.57 if
\[
\begin{align*}
v_a &= 60 \cos 40,000t \, \text{V}, \\
v_b &= 90 \sin (40,000t + 180^\circ) \, \text{V}.
\end{align*}
\]

Figure P9.57

9.58 Use the mesh-current method to find the steady-state expression for \( v_o(t) \) in the circuit in Fig. P9.52.

9.59 Use the mesh-current method to find the phasor current \( I_a \) in the circuit in Fig. P9.54.

9.60 Use the mesh-current method to find the branch currents \( I_a, I_b, I_c, \) and \( I_d \) in the circuit shown in Fig. P9.60.

Figure P9.60

9.61 Use the mesh-current method to find the steady-state expression for \( v_o \) in the circuit seen in Fig. P9.61 if \( v_g \) equals \( 72 \cos 5000t \, \text{V} \).

Figure P9.61
9.62 Use the concept of voltage division to find the steady-state expression for \( v_o(t) \) in the circuit in Fig. P9.62 if \( v_g = 75 \cos 5000t \) V.

![Figure P9.62](image)

9.63 Use the concept of current division to find the steady-state expression for \( i_g \) in the circuit in Fig. P9.63 if \( i_g = 125 \cos 500t \) mA.

![Figure P9.63](image)

9.64 The sinusoidal voltage source in the circuit shown in Fig. P9.64 is generating the voltage \( v_g = 1.2 \cos 100t \) V. If the op amp is ideal, what is the steady-state expression for \( v_o(t) \)?

![Figure P9.64](image)

9.65 The 1 \( \mu \)F capacitor in the circuit seen in Fig. P9.64 is replaced with a variable capacitor. The capacitor is adjusted until the output voltage leads the input voltage by 120°.

a) Find the values of \( C \) in microfarads.

b) Write the steady-state expression for \( v_o(t) \) when \( C \) has the value found in (a).

9.66 The op amp in the circuit in Fig. P9.66 is ideal.

a) Find the steady-state expression for \( v_o(t) \).

b) How large can the amplitude of \( v_g \) be before the amplifier saturates?

![Figure P9.66](image)

9.67 The op amp in the circuit seen in Fig. P9.67 is ideal. Find the steady-state expression for \( v_o(t) \) when \( v_g = 20 \cos 10^5t \) V.

![Figure P9.67](image)

9.68 The operational amplifier in the circuit shown in Fig. P9.68 is ideal. The voltage of the ideal sinusoidal source is \( v_g = 10 \cos 2 \times 10^2t \) V.

a) How small can \( C_o \) be before the steady-state output voltage no longer has a pure sinusoidal waveform?

b) For the value of \( C_o \) found in (a), write the steady-state expression for \( v_o \).

![Figure P9.68](image)
9.69  a) Find the input impedance $Z_{ab}$ for the circuit in Fig. P9.69. Express $Z_{ab}$ as a function of $Z$ and $K$ where $K = (R_2/R_1)$.

b) If $Z$ is a pure capacitive element, what is the capacitance seen looking into the terminals a,b?

Figure P9.69

---

9.70  For the circuit in Fig. P9.57, suppose

$$v_a = 5 \cos 80,000t \text{ V}$$

$$v_b = -2.5 \cos 320,000t \text{ V}.$$  

a) What circuit analysis technique must be used to find the steady-state expression for $i_a(t)$?

b) Find the steady-state expression for $i_a(t)$.

---

9.71  For the circuit in Fig. P9.71 suppose

$$v_1 = 20 \cos(2000t - 36.87°) \text{ V}$$

$$v_2 = 10 \cos(5000t + 16.26°) \text{ V}.$$  

a) What circuit analysis technique must be used to find the steady-state expression for $v_a(t)$?

b) Find the steady-state expression for $v_a(t)$.

---

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Section 9.10

9.72  a) Find the steady-state expressions for the currents $i_a$ and $i_L$ in the circuit in Fig. P9.72 when $v_g = 200 \cos 10,000t \text{ V}$.

b) Find the coefficient of coupling.

c) Find the energy stored in the magnetically coupled coils at $t = 50\pi \mu s$ and $t = 100\pi \mu s$.

Figure P9.72

---

9.73  The sinusoidal voltage source in the circuit seen in Fig. P9.73 is operating at a frequency of 50 krad/s. The coefficient of coupling is adjusted until the peak amplitude of $i_1$ is maximum.

a) What is the value of $k$?

b) What is the peak amplitude of $i_1$ if $v_g = 369 \cos(5 \times 10^4 t) \text{ V}$?

Figure P9.73

---

9.74  For the circuit in Fig. P9.74, find the Thévenin equivalent with respect to the terminals c,d.

Figure P9.74

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9.75  The value of $k$ in the circuit in Fig. P9.75 is adjusted so that $Z_{ab}$ is purely resistive when $\omega = 25 \text{ krad/s}$. Find $Z_{ab}$.
A series combination of a 150 Ω resistor and a 20 nF capacitor is connected to a sinusoidal voltage source by a linear transformer. The source is operating at a frequency of 500 kHz/s. At this frequency, the internal impedance of the source is 5 + j16 Ω. The rms voltage at the terminals of the source is 125 V when it is not loaded. The parameters of the linear transformer are $R_1 = 12 \, \Omega$, $L_1 = 80 \, \mu H$, $R_2 = 50 \, \Omega$, $L_2 = 500 \, \mu H$, and $M = 100 \, \mu H$.

a) What is the value of the impedance reflected into the primary?

b) What is the value of the impedance seen from the terminals of the practical source?

Section 9.11

9.77 Find the impedance $Z_{ab}$ in the circuit in Fig. P9.77 if $Z_L = 200 + j150 \, \Omega$.

Figure P9.77

9.78 At first glance, it may appear from Eq. 9.69 that an inductive load could make the reactance seen looking into the primary terminals (i.e., $X_{ab}$) look capacitive. Intuitively, we know this is impossible. Show that $X_{ab}$ can never be negative if $X_L$ is an inductive reactance.

9.79 a) Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.79 is given by the expression

$$Z_{ab} = \frac{Z_L}{\left(1 + \frac{N_1}{N_2}\right)^2}$$

b) Show that if the polarity terminal of either one of the coils is reversed that

$$Z_{ab} = \frac{Z_L}{\left(1 - \frac{N_1}{N_2}\right)^2}.$$

Figure P9.79

9.80 a) Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.80 is given by the expression

$$Z_{ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L.$$ 

b) Show that if the polarity terminals of either one of the coils is reversed, 

$$Z_{ab} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_L.$$ 

Figure P9.80
Section 9.12

9.81 The parameters in the circuit shown in Fig. 9.53 are $R_1 = 0.1\ \Omega$, $\omega L_1 = 0.8\ \Omega$, $R_2 = 24\ \Omega$, $\omega L_2 = 32\ \Omega$, and $V_L = 240 + j0\ \text{V}$. 

a) Calculate the phasor voltage $V_s$.

b) Connect a capacitor in parallel with the inductor, hold $V_L$ constant, and adjust the capacitor until the magnitude of $I$ is a minimum. What is the capacitive reactance? What is the value of $V_s$?

c) Find the value of the capacitive reactance that keeps the magnitude of $I$ as small as possible and that at the same time makes $|V_s| = |V_L| = 240\ \text{V}$.

9.82 Show by using a phasor diagram what happens to the magnitude and phase angle of the voltage $v_s$ in the circuit in Fig. P9.82 as $R_s$ is varied from zero to infinity. The amplitude and phase angle of the source voltage are held constant as $R_s$ varies.

![Figure P9.82](image)

$v_s = V_m \cos \omega t$

9.83 a) For the circuit shown in Fig. P9.83, compute $V_s$ and $V_i$.

b) Construct a phasor diagram showing the relationship between $V_s$, $V_i$, and the load voltage of $440 / 0^\circ\ \text{V}$.

c) Repeat parts (a) and (b), given that the load voltage remains constant at $440 / 0^\circ\ \text{V}$, when a capacitive reactance of $-22\ \Omega$ is connected across the load terminals.

![Figure P9.83](image)

Sections 9.1–9.12

9.84 You may have the opportunity as an engineering graduate to serve as an expert witness in lawsuits involving either personal injury or property damage. As an example of the type of problem on which you may be asked to give an opinion, consider the following event. At the end of a day of fieldwork, a farmer returns to his farmstead, checks his hog confinement building, and finds to his dismay that the hogs are dead. The problem is traced to a blown fuse that caused a 240 V fan motor to stop. The loss of ventilation led to the suffocation of the livestock. The interrupted fuse is located in the main switch that connects the farmstead to the electrical service. Before the insurance company settles the claim, it wants to know if the electric circuit supplying the farmstead functioned properly. The lawyers for the insurance company are puzzled because the farmer’s wife, who was in the house on the day of the accident convalescing from minor surgery, was able to watch TV during the afternoon. Furthermore, when she went to the kitchen to start preparing the evening meal, the electric clock indicated the correct time. The lawyers have hired you to explain (1) why the electric clock in the kitchen and the television set in the living room continued to operate after the fuse in the main switch blew and (2) why the second fuse in the main switch didn’t blow after the fan motor stalled. After ascertaining the loads on the three-wire distribution circuit prior to the interruption of fuse $A$, you are able to construct the circuit model shown in Fig. P9.84 on page 389. The impedances of the line conductors and the neutral conductor are assumed negligible.

a) Calculate the branch currents $I_1$, $I_2$, $I_3$, $I_4$, $I_5$, and $I_6$ prior to the interruption of fuse $A$.

b) Calculate the branch currents after the interruption of fuse $A$. Assume the stalled fan motor behaves as a short circuit.

c) Explain why the clock and television set were not affected by the momentary short circuit that interrupted fuse $A$.

d) Assume the fan motor is equipped with a thermal cutout designed to interrupt the motor circuit if the motor current becomes excessive. Would you expect the thermal cutout to operate? Explain.

e) Explain why fuse $B$ is not interrupted when the fan motor stalls.
9.85 a) Calculate the branch currents $I_1 - I_6$ in the circuit in Fig. P9.58.

b) Find the primary current $I_p$.

9.86 Suppose the 40 $\Omega$ resistance in the distribution circuit in Fig. P9.58 is replaced by a 20 $\Omega$ resistance.

a) Recalculate the branch current in the 2 $\Omega$ resistor, $I_2$.

b) Recalculate the primary current, $I_p$.

c) On the basis of your answers, is it desirable to have the resistance of the two 120 V loads be equal?

9.87 A residential wiring circuit is shown in Fig. P9.87. In this model, the resistor $R_3$ is used to model a 240 V appliance (such as an electric range), and the resistors $R_1$ and $R_2$ are used to model 120 V appliances (such as a lamp, toaster, and iron). The branches carrying $I_1$ and $I_2$ are modeling what electricians refer to as the hot conductors in the circuit, and the branch carrying $I_n$ is modeling the neutral conductor. Our purpose in analyzing the circuit is to show the importance of the neutral conductor in the satisfactory operation of the circuit. You are to choose the method for analyzing the circuit.

a) Show that $I_n$ is zero if $R_1 = R_2$.

b) Show that $V_1 = V_2$ if $R_1 = R_2$.

c) Open the neutral branch and calculate $V_1$ and $V_2$ if $R_1 = 60 \Omega$, $R_2 = 600 \Omega$, and $R_3 = 10 \Omega$.

d) Close the neutral branch and repeat (c).

e) On the basis of your calculations, explain why the neutral conductor is never fused in such a manner that it could open while the hot conductors are energized.

Figure P9.87

9.88 a) Find the primary current $I_p$ for (c) and (d) in Problem 9.87.

b) Do your answers make sense in terms of known circuit behavior?