where $v_1$ and $i_1$ are the voltage and current in circuit 1, and $v_2$ and $i_2$ are the voltage and current in circuit 2. For coils wound on nonmagnetic cores, $M_{12} = M_{21} = M$ (see page 210.)

- The **dot convention** establishes the polarity of mutually induced voltages:

  When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

  Or, alternatively,

  When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

(See page 204.)

- The relationship between the self-inductance of each winding and the mutual inductance between windings is

$$M = k \sqrt{L_1 L_2}.$$  

The **coefficient of coupling**, $k$, is a measure of the degree of magnetic coupling. By definition, $0 \leq k \leq 1$. (See page 211.)

- The energy stored in magnetically coupled coils in a linear medium is related to the coil currents and inductances by the relationship

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M_{12} i_1 i_2.$$

(See page 213.)

## Problems

### Section 6.1

6.1 The triangular current pulse shown in Fig. P6.1 is applied to a 375 mH inductor.

- a) Write the expressions that describe $i(t)$ in the four intervals $t < 0$, $0 \leq t \leq 25$ ms, $25$ ms $\leq t \leq 50$ ms, and $t > 50$ ms.

- b) Derive the expressions for the inductor voltage, power, and energy. Use the passive sign convention.

Figure P6.1

![Figure P6.1](image)

6.2 The voltage at the terminals of the 300 $\mu$H inductor in Fig. P6.2(a) is shown in Fig. P6.2(b). The inductor current $i$ is known to be zero for $t \leq 0$.

- a) Derive the expressions for $i$ for $t \geq 0$.

- b) Sketch $i$ versus $t$ for $0 \leq t \leq \infty$.

Figure P6.2

![Figure P6.2](image)

6.3 The current in the 4 mH inductor in Fig. P6.3 is known to be 2.5 A for $t < 0$. The inductor voltage for $t \geq 0$ is given by the expression

$$v_L(t) = 30e^{-3t} \text{ mV}, \quad 0^+ \leq t < \infty$$

Sketch $v_L(t)$ and $i_L(t)$ for $0 \leq t < \infty$.

Figure P6.3

![Figure P6.3](image)
6.4 The current in a 100 $\mu$H inductor is known to be
\[ i_L = 20te^{-5t} \quad \text{A for } t \geq 0. \]
a) Find the voltage across the inductor for $t > 0$. (Assume the passive sign convention.)
b) Find the power (in microwatts) at the terminals of the inductor when $t = 100$ ms.
c) Is the inductor absorbing or delivering power at 100 ms?
d) Find the energy (in microjoules) stored in the inductor at 100 ms.
e) Find the maximum energy (in microjoules) stored in the inductor and the time (in microseconds) when it occurs.

6.5 The current in and the voltage across a 2.5 H inductor are known to be zero for $t \leq 0$. The voltage across the inductor is given by the graph in Fig. P6.5 for $t \geq 0$.
a) Derive the expression for the current as a function of time in the intervals $0 \leq t \leq 2$ s, $2 \leq t \leq 6$ s, $6 \leq t \leq 10$ s, $10 \leq t \leq 12$ s, and $12 \leq t < \infty$.
b) For $t > 0$, what is the current in the inductor when the voltage is zero?
c) Sketch $i$ versus $t$ for $0 \leq t < \infty$.

Figure P6.5

![Figure P6.5](image)

6.6 The current in a 20 mH inductor is known to be
\[ i = 7 + (15 \sin 140t - 35 \cos 140t)e^{-20t} \quad \text{mA for } t \geq 0. \]
Assume the passive sign convention.
a) At what instant of time is the voltage across the inductor maximum?
b) What is the maximum voltage?

6.7 a) Find the inductor current in the circuit in Fig. P6.7 if $v = 250 \sin 1000t$ V, $L = 50$ mH, and $i(0) = -5$ A.
b) Sketch $v$, $i$, $p$, and $w$ versus $t$. In making these sketches, use the format used in Fig. 6.8. Plot over one complete cycle of the voltage waveform.
c) Describe the subintervals in the time interval between 0 and 2$\pi$ ms when power is being absorbed by the inductor. Repeat for the subintervals when power is being delivered by the inductor.

Figure P6.7

![Figure P6.7](image)

6.8 The current in a 15 mH inductor is known to be
\[ i = 1A, \quad t \leq 0; \]
\[ i = A_1e^{-2000t} + A_2e^{-8000t}, \quad t \geq 0. \]
The voltage across the inductor (passive sign convention) is 60 V at $t = 0$.
a) Find the expression for the voltage across the inductor for $t > 0$.
b) Find the time, greater than zero, when the power at the terminals of the inductor is zero.

6.9 Assume in Problem 6.8 that the value of the voltage across the inductor at $t = 0$ is $-300$ V instead of 60 V.
a) Find the numerical expressions for $i$ and $v$ for $t \leq 0$.
b) Specify the time intervals when the inductor is storing energy and the time intervals when the inductor is delivering energy.
c) Show that the total energy extracted from the inductor is equal to the total energy stored.

6.10 The current in a 2 H inductor is
\[ i = 25 A, \quad t \leq 0; \]
\[ i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t} A, \quad t \geq 0. \]
The voltage across the inductor (passive sign convention) is 100 V at $t = 0$. Calculate the power at the terminals of the inductor at $t = 0.5$ s. State whether the inductor is absorbing or delivering power.
6.11 Initially there was no energy stored in the 20 H inductor in the circuit in Fig. P6.11 when it was placed across the terminals of the voltmeter. At $t = 0$ the inductor was switched instantaneously to position b where it remained for 1.2 s before returning instantaneously to position a. The d’Arsonval voltmeter has a full-scale reading of 25 V and a sensitivity of 1000 Ω/V. What will the reading of the voltmeter be at the instant the switch returns to position a if the inertia of the d’Arsonval movement is negligible?

Figure P6.11

![Circuit Diagram]

14 mV

20 H

Voltmeter

6.12 Evaluate the integral

$$\int_0^\infty p \, dt$$

for Example 6.2. Comment on the significance of the result.

6.13 The expressions for voltage, power, and energy derived in Example 6.5 involved both integration and manipulation of algebraic expressions. As an engineer, you cannot accept such results on faith alone. That is, you should develop the habit of asking yourself, “Do these results make sense in terms of the known behavior of the circuit they purport to describe?” With these thoughts in mind, test the expressions of Example 6.5 by performing the following checks:

a) Check the expressions to see whether the voltage is continuous in passing from one time interval to the next.

b) Check the power expression in each interval by selecting a time within the interval and seeing whether it gives the same result as the corresponding product of $v$ and $i$. For example, test at 10 and 30 μs.

c) Check the energy expression within each interval by selecting a time within the interval and seeing whether the energy equation gives the same result as $\frac{1}{2}Cv^2$. Use 10 and 30 μs as test points.

Section 6.2

6.14 A 0.5 μF capacitor is subjected to a voltage pulse having a duration of 2 s. The pulse is described by the following equations:

$$v_c(t) = \begin{cases} 
40t^3 \text{ V}, & 0 \leq t \leq 1 \text{ s}; \\
40(2 - t)^3 \text{ V}, & 1 \text{ s} \leq t \leq 2 \text{ s}; \\
0 & \text{elsewhere}.
\end{cases}$$

Sketch the current pulse that exists in the capacitor during the 2 s interval.

6.15 The rectangular-shaped current pulse shown in Fig. P6.11 is applied to a 0.2 μF capacitor. The initial voltage on the capacitor is a 40 V drop in the reference direction of the current. Derive the expression for the capacitor voltage for the time intervals in (a)–(c).

a) $0 \leq t \leq 100 \mu s$;

b) $100 \mu s \leq t \leq 300 \mu s$;

c) $300 \mu s \leq t < \infty$;

d) Sketch $v(t)$ over the interval $-100 \mu s \leq t \leq 500 \mu s$.

Figure P6.15

<table>
<thead>
<tr>
<th>$i$ (mA)</th>
<th>$t$ (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>-40</td>
<td>200</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
</tr>
</tbody>
</table>

6.16 The voltage at the terminals of the capacitor in Fig. 6.10 is known to be

$$v = \begin{cases} 
-30 \text{ V}, & t \leq 0; \\
10 - 10e^{-1000t}(4 \cos 3000t + \sin 3000t) \text{ V} & t \geq 0.
\end{cases}$$

Assume $C = 0.5 \mu F$.

a) Find the current in the capacitor for $t < 0$.

b) Find the current in the capacitor for $t > 0$. 
c) Is there an instantaneous change in the voltage across the capacitor at \( t = 0 \)?

6.17 The current shown in Fig. P6.17 is applied to a 0.25 \( \mu \text{F} \) capacitor. The initial voltage on the capacitor is zero.

a) Find the charge on the capacitor at \( t = 30 \text{ \mu s} \).

b) Find the voltage on the capacitor at \( t = 50 \text{ \mu s} \).

c) How much energy is stored in the capacitor by this current?

6.18 The initial voltage on the 0.2 \( \mu \text{F} \) capacitor shown in Fig. P6.18(a) is -60.6 V. The capacitor current has the waveform shown in Fig. P6.18(b).

a) How much energy, in microjoules, is stored in the capacitor at \( t = 250 \text{ \mu s} \)?

b) Repeat (a) for \( t = \infty \).

6.19 The voltage across the terminals of a 0.4 \( \mu \text{F} \) capacitor is

\[
    v = \begin{cases} 
    25 \text{ V}, & t \leq 0; \\
    A_1 e^{-1500t} + A_2 e^{-1500t} \text{V}, & t \geq 0. 
    \end{cases}
\]

The initial current in the capacitor is 90 mA. Assume the passive sign convention.

a) What is the initial energy stored in the capacitor?

b) Evaluate the coefficients \( A_1 \) and \( A_2 \).

c) What is the expression for the capacitor current?

Section 6.3

6.20 Assume that the initial energy stored in the inductors of Fig. P6.20 is zero. Find the equivalent inductance with respect to the terminals a,b.

6.21 Assume that the initial energy stored in the inductors of Fig. P6.21 is zero. Find the equivalent inductance with respect to the terminals a,b.
6.22 The two parallel inductors in Fig. P6.22 are connected across the terminals of a black box at \( t = 0 \). The resulting voltage \( v \) for \( t > 0 \) is known to be \(-1800e^{-20t} \) V. It is also known that \( i_1(0) = 4 \) A and \( i_2(0) = -16 \) A.

a) Replace the original inductors with an equivalent inductor and find \( i(t) \) for \( t \geq 0 \).

b) Find \( i_1(t) \) for \( t \geq 0 \).

c) Find \( i_2(t) \) for \( t \geq 0 \).

d) How much energy is delivered to the black box in the time interval \( 0 \leq t < \infty \)?

e) How much energy was initially stored in the parallel inductors?

f) How much energy is trapped in the ideal inductors?

g) Show that your solutions for \( i_1 \) and \( i_2 \) agree with the answer obtained in (f).

6.23 The three inductors in the circuit in Fig. P6.23 are connected across the terminals of a black box at \( t = 0 \). The resulting voltage for \( t > 0 \) is known to be \( v_0 = 1250e^{-25t} \) V.

If \( i_1(0) = 10 \) A and \( i_2(0) = -5 \) A, find

a) \( i_3(0) \);

b) \( i_1(t), t \geq 0 \);

c) \( i_2(t), t \geq 0 \);

d) \( i_3(t), t \geq 0 \);

e) the initial energy stored in the three inductors;

f) the total energy delivered to the black box; and

g) the energy trapped in the ideal inductors.

6.24 For the circuit shown in Fig. P6.23, how many milliseconds after the switch is opened is the energy delivered to the black box 80% of the total energy delivered?

6.25 Find the equivalent capacitance with respect to the terminals \( a, b \) for the circuit shown in Fig. P6.25.

6.26 Find the equivalent capacitance with respect to the terminals \( a, b \) for the circuit shown in Fig. P6.26.

6.27 The two series-connected capacitors in Fig. P6.27 are connected to the terminals of a black box at \( t = 0 \). The resulting current \( i(t) \) for \( t > 0 \) is known to be \( 900e^{-2500t} \) \( \mu A \).

a) Replace the original capacitors with an equivalent capacitor and find \( v_0(t) \) for \( t \geq 0 \).

b) Find \( v_1(t) \) for \( t \geq 0 \).
c) Find \( v_2(t) \) for \( t \geq 0 \).
d) How much energy is delivered to the black box in the time interval \( 0 \leq t < \infty \)?
e) How much energy was initially stored in the series capacitors?
f) How much energy is trapped in the ideal capacitors?
g) Show that the solutions for \( v_1 \) and \( v_2 \) agree with the answer obtained in (f).

Figure P6.27

6.28 The four capacitors in the circuit in Fig. P6.28 are connected across the terminals of a black box at \( t = 0 \). The resulting current \( i_b \) for \( t > 0 \) is known to be
\[ i_b = 50e^{-250t} \mu A. \]
If \( v_a(0) = 15 \text{ V}, \ v_b(0) = -45 \text{ V}, \) and \( v_c(0) = 40 \text{ V}, \) find the following for \( t \geq 0 \); (a) \( v_b(t) \), (b) \( v_a(t) \), (c) \( v_c(t) \), (d) \( v_d(t) \), (e) \( i_1(t) \), and (f) \( i_2(t) \).

Figure P6.28

6.29 For the circuit in Fig. P6.28, calculate
a) the initial energy stored in the capacitors;
b) the final energy stored in the capacitors;
c) the total energy delivered to the black box;
d) the percentage of the initial energy stored that is delivered to the black box; and
e) the time, in milliseconds, it takes to deliver 5 \( \mu J \) to the black box.

6.30 Derive the equivalent circuit for a series connection of ideal capacitors. Assume that each capacitor has its own initial voltage. Denote these initial voltages as \( v_1(t_0), v_2(t_0), \) and so on. (Hint: Sum the voltages across the string of capacitors, recognizing that the series connection forces the current in each capacitor to be the same.)

6.31 Derive the equivalent circuit for a parallel connection of ideal capacitors. Assume that the initial voltage across the paralleled capacitors is \( v(t_0) \). (Hint: Sum the currents into the string of capacitors, recognizing that the parallel connection forces the voltage across each capacitor to be the same.)

Sections 6.1–6.3

6.32 The current in the circuit in Fig. P6.32 is known to be
\[ i_o = 50e^{-800t}(\cos 6000t + 2 \sin 6000t) \text{ mA} \]
for \( t \geq 0 \). Find \( v_1(0^+) \) and \( v_2(0^+) \).

Figure P6.32

6.33 At \( t = 0 \), a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Fig. P6.33. For \( t > 0 \), it is known that
\[ i_o = -e^{-80t} \sin 60t \text{ A}. \]
If \( v_0(0) = -300 \text{ V} \) find \( v_o \) for \( t \geq 0 \).

Figure P6.33
Section 6.4

6.34 There is no energy stored in the circuit in Fig. P6.34 at the time the switch is opened.
   a) Derive the differential equation that governs the behavior of $i_2$ if $L_1 = 10 \text{ H}$, $L_2 = 40 \text{ H}$, $M = 5 \text{ H}$, and $R_s = 90 \Omega$.
   b) Show that when $i_s = 10e^{-t} - 10 \text{ A}$, $t \geq 0$, the differential equation derived in (a) is satisfied when $i_2 = e^{-t} - 5e^{-2.25t} \text{ A}$, $t \geq 0$.
   c) Find the expression for the voltage $v_1$ across the current source.
   d) What is the initial value of $v_1$? Does this make sense in terms of known circuit behavior?

Figure P6.34

6.35 Let $v_o$ represent the voltage across the 16 H inductor in the circuit in Fig. 6.25. Assume $v_o$ is positive at the dot. As in Example 6.6, $i_g = 16 - 16e^{-4t} \text{ A}$.
   a) Can you find $v_o$ without having to differentiate the expressions for the currents? Explain.
   b) Derive the expression for $v_o$.
   c) Check your answer in (b) using the appropriate current derivatives and inductances.

6.36 Let $v_s$ represent the voltage across the current source in the circuit in Fig. 6.25. The reference for $v_s$ is positive at the upper terminal of the current source.
   a) Find $v_s$ as a function of time when $i_g = 16 - 16e^{-4t} \text{ A}$.
   b) What is the initial value of $v_s$?
   c) Find the expression for the power developed by the current source.
   d) How much power is the current source developing when $t$ is infinite?
   e) Calculate the power dissipated in each resistor when $t$ is infinite.

6.37 a) Show that the differential equations derived in (a) of Example 6.6 can be rearranged as follows:
   
   \[
   4 \frac{di_1}{dt} + 25i_1 - 8 \frac{di_2}{dt} - 20i_2 = 5i_s - 8 \frac{di_s}{dt}.
   \]
   
   \[
   -8 \frac{di_1}{dt} - 20i_1 + 16 \frac{di_2}{dt} + 80i_2 = 16 \frac{di_s}{dt}.
   \]
   
   b) Show that the solutions for $i_1$ and $i_2$ given in (b) of Example 6.6 satisfy the differential equations given in part (a) of this problem.

6.38 The physical construction of four pairs of magnetically coupled coils is shown in Fig. P6.38. (See page 225.) Assume that the magnetic flux is confined to the core material in each structure. Show two possible locations for the dot markings on each pair of coils.

6.39 The polarity markings on two coils are to be determined experimentally. The experimental setup is shown in Fig. P6.39. Assume that the terminal connected to the negative terminal of the battery has been given a polarity mark as shown. When the switch is closed, the dc voltmeter kicks up scale. Where should the polarity mark be placed on the coil connected to the voltmeter?

Figure P6.39

6.40 a) Show that the two coupled coils in Fig. P6.40 can be replaced by a single coil having an inductance of $L_{ab} = L_1 + L_2 + 2M$. (Hint: Express $v_{ab}$ as a function of $i_{ab}$.)
   b) Show that if the connections to the terminals of the coil labeled $L_2$ are reversed, $L_{ab} = L_1 + L_2 - 2M$.

Figure P6.40
6.41 a) Show that the two magnetically coupled coils in Fig. P6.41 can be replaced by a single coil having an inductance of

\[ L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}. \]

*(Hint: Let \( i_1 \) and \( i_2 \) be clockwise mesh currents in the left and right “windows” of Fig. P6.41, respectively. Sum the voltages around the two meshes. In mesh 1 let \( v_{ab} \) be the unspecified applied voltage. Solve for \( di_1/dt \) as a function of \( v_{ab} \)).

b) Show that if the magnetic polarity of coil 2 is reversed, then

\[ L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}. \]
Section 6.5

6.42 Two magnetically coupled coils are wound on a nonmagnetic core. The self-inductance of coil 1 is 250 mH, the mutual inductance is 100 mH, the coefficient of coupling is 0.5, and the physical structure of the coils is such that $\Phi_{11} = \Phi_{22}$.

a) Find $L_2$ and the turns ratio $N_1/N_2$.

b) If $N_1 = 1000$, what is the value of $\Phi_1$ and $\Phi_2$?

6.43 The self-inductances of two magnetically coupled coils are $L_1 = 400 \mu$H and $L_2 = 900 \mu$H. The coupling medium is nonmagnetic. If coil 1 has 250 turns and coil 2 has 500 turns, find $\Phi_{11}$ and $\Phi_{21}$ (in nanowebers per ampere) when the coefficient of coupling is 0.75.

6.44 Two magnetically coupled coils have self-inductances of 52 mH and 13 mH, respectively. The mutual inductance between the coils is 19.5 mH.

a) What is the coefficient of coupling?

b) For these two coils, what is the largest value that $M$ can have?

c) Assume that the physical structure of these coupled coils is such that $\Phi_1 = \Phi_2$. What is the turns ratio $N_1/N_2$ if $N_1$ is the number of turns on the 52 mH coil?

6.45 The self-inductances of two magnetically coupled coils are 288 mH and 162 mH, respectively. The 288 mH coil has 1000 turns, and the coefficient of coupling between the coils is ½. The coupling medium is nonmagnetic. When coil 1 is excited with coil 2 open, the flux linking only coil 1 is 0.5 as large as the flux linking coil 2.

a) How many turns does coil 2 have?

b) What is the value of $\Phi_2$ in nanowebers per ampere?

c) What is the value of $\Phi_{11}$ in nanowebers per ampere?

d) What is the ratio ($\phi_{22}/\phi_{12}$)?

6.46 a) Starting with Eq. 6.59, show that the coefficient of coupling can also be expressed as

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}.$$

b) On the basis of the fractions $\phi_{21}/\phi_1$ and $\phi_{12}/\phi_2$, explain why $k$ is less than 1.0.

6.47 The self-inductances of the coils in Fig. 6.30 are $L_1 = 25$ mH and $L_2 = 100$ mH. If the coefficient of coupling is 0.8, calculate the energy stored in the system in millijoules when (a) $i_1 = 10$ A, $i_2 = 15$ A; (b) $i_1 = -10$ A, $i_2 = -15$ A; (c) $i_1 = -10$ A, $i_2 = 15$ A; and (d) $i_1 = 10$ A, $i_2 = -15$ A.

6.48 The coefficient of coupling in Problem 6.47 is increased to 1.0.

a) If $i_1$ equals 10 A, what value of $i_2$ results in zero stored energy?

b) Is there any physically realizable value of $i_2$ that can make the stored energy negative?

Sections 6.1–6.5

6.49 Rework the Practical Perspective example, except that this time, put the button on the bottom of the divider circuit, as shown in Fig. P6.49. Calculate the output voltage $v(t)$ when a finger is present.

Figure P6.49

![Figure P6.49]

6.50 Some lamps are made to turn on or off when the base is touched. These use a one-terminal variation of the capacitive switch circuit discussed in the Practical Perspective. Figure P6.50 shows a circuit model of such a lamp. Calculate the change in the voltage $v(t)$ when a person touches the lamp. Assume all capacitors are initially discharged.

Figure P6.50

![Figure P6.50]
In the Practical Perspective example, we calculated the output voltage when the elevator button is the upper capacitor in a voltage divider. In Problem 6.49, we calculated the voltage when the button is the bottom capacitor in the divider, and we got the same result! You may wonder if this will be true for all such voltage dividers. Calculate the voltage difference (finger versus no finger) for the circuits in Figs. P6.51(a) and (b), which use two identical voltage sources.