1. \( \phi(b, \phi, z) - \phi(a, \phi, z) = \nabla \cdot \mathbf{V} > 0 \)

Because outer cylinder is grounded,
\( \phi(a, \phi, z) = 0 \Rightarrow \phi(b, \phi, z) = V_0 \)

a. \( \nabla^2 \phi = 0 \) for \( b \leq r \leq a \)

Solution is independent of \( \phi \) and \( z \) \( \Rightarrow \)
\( \phi(r, \phi, z) = A \ln r + B \)

Thus
\( \phi(r = b, \phi, z) = A \ln b + B = V_0 \)
\( \phi(r = a, \phi, z) = A \ln a + B = 0 \)

\( \Rightarrow B = -A \ln a \Rightarrow V_0 = A \ln b - A \ln a = A \frac{\ln b}{\ln a} \)

\( A = \frac{V_0}{\ln b/a} \)

\( \Rightarrow \phi(r, \phi, z) = \frac{V_0}{\ln b/a} \ln r - \frac{V_0}{\ln b/a} \ln a \)

\( = \frac{V_0}{\ln b/a} \ln r/a \)

\( 
\phi(r, \phi, z) = \frac{V_0}{\ln (a/b)} \ln \left(\frac{a}{r}\right) 
\)

b. \( \mathbf{E} = -\nabla \phi = -\frac{\partial \phi}{\partial r} \mathbf{r} \)

\( \Rightarrow \mathbf{E}(r, \phi, z) = -\frac{V_0}{\ln (a/b)} \frac{1}{a/r} \left( -\frac{a}{r^2} \right) \mathbf{r} \)

\( = \frac{V_0}{\ln (a/b)} \frac{1}{r} \mathbf{r} \)
c. \( \hat{\mathbf{e}} \cdot \nabla \mathbf{E} = D_{2n} - D_{1n} = \rho_s \) but \( \mathbf{E} = 0 \) inside the conductor.

Thus

\[
\rho_s = \frac{\varepsilon V_0}{\ln(a/b)} \frac{1}{b}
\]

Note: Total charge on inside cylinder of length \( L \):

\[
q_{in} = \rho_s \frac{2\pi}{2} b L = \frac{2\pi \varepsilon e V_0}{\ln(a/b)} L
\]

d. \( \mathbf{E} \neq 0 \) \( \hat{n} \cdot \mathbf{E} = 0 \), \( D_{2n} - D_{1n} = \rho_s \) \( \implies \rho_s = -D_{1n} \)

Thus

\[
\rho_s = -\varepsilon V_0 \frac{1}{\ln(a/b)} \frac{1}{a}
\]

Note: Total charge on outside cylinder of length \( L \):

\[
q_{out} = \rho_s \frac{2\pi}{2} a L = \frac{2\pi \varepsilon e V_0}{\ln(a/b)} L = -q_{in}
\]

The densities have to be different because the surface areas are different. This allows \( q_{out} = -q_{in} \) in the end.

e. \( C = \frac{Q}{V} = \frac{2\pi \varepsilon V_0}{\ln(a/b)} \frac{L}{V_0} = \frac{2\pi \varepsilon L}{\ln(a/b)} \)

capacitance per unit length \( C_0 = \frac{2\pi \varepsilon}{\ln(a/b)} \)
\[
\vec{B}(0, y, 0) = -\mu I y \hat{x} \int_{-a-L}^{-a} \frac{dz'}{4\pi} \frac{\left(\hat{y} - \hat{z'}\right)}{(y^2 + z'^2)^{3/2}}
\]

Since \( \hat{z} \times \hat{z} = 0 \) and \( \hat{z} \times \hat{y} = -\hat{x} \),

\[
\vec{B}(0, y, 0) = -\mu I y \hat{x} \int_{-a-L}^{-a} \frac{dz'}{(y^2 + z'^2)^{3/2}}
\]

\[
= -\mu I \frac{y}{4\pi} \hat{x} \left[ \frac{1}{y^2} \left( \frac{z'}{\sqrt{y^2 + z'^2}} \right) \right]_{z'=-a}^{z'=-a-L}
\]

\[
= \frac{\mu I}{4\pi} \frac{1}{y} \hat{x} \left[ \frac{a + L}{\left[ y^2 + (a + L)^2 \right]^{1/2}} - \frac{a}{\left[ y^2 + a^2 \right]^{1/2}} \right]
\]

**Note:** with

\[
\cos \alpha_1 = \frac{a + L}{\left[ y^2 + (a + L)^2 \right]^{1/2}}
\]

\[
\cos \alpha_2 = \frac{a}{\left[ y^2 + a^2 \right]^{1/2}}
\]
Given 
\[ \mathbf{B}(0, y, 0) = \frac{\mu}{4\pi} \frac{I}{y} \left[ \cos \alpha_1 - \cos \alpha_2 \right] \hat{x} \]

Note: \( \mathbf{B} \) is orthogonal to current and line connecting source to observation pt.

Note: for infinite wire \( \alpha_1 = 0 \), \( \alpha_2 = \pi \) giving
\[ \mathbf{B}(0, y, 0) = \frac{\mu I}{2\pi y} \hat{x} = \frac{\mu I}{2\pi} \hat{\phi} \text{ for z-plane} \]

b. \( \mathbf{B} = \mu \mathbf{H} \)
\[ \mathbf{H}(0, y, 0) = \frac{I}{4\pi y} \hat{x} \left[ \frac{a+L}{[y^2+(a+L)^2]^{1/2}} - \frac{a}{[y^2+a^2]^{1/2}} \right] \]

c. \( \mathbf{E}(0, y, 0) = -\mathbf{V}(0, y, 0) \times \mathbf{B}(0, y, 0) \)
\[ \hat{y} \times \hat{x} = -\hat{z} \]
\[ \mathbf{E}(0, y, 0) = -\frac{\mu}{4\pi} \frac{I}{y} \hat{z} \left[ \frac{a+L}{[y^2+(a+L)^2]^{1/2}} - \frac{a}{[y^2+a^2]^{1/2}} \right] \]
**Problem 4.48** With reference to Fig. 4-19, find $E_1$ if $E_2 = \hat{x}3 - \hat{y}2 + \hat{z}2$ (V/m), $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 18\varepsilon_0$, and the boundary has a surface charge density $\rho_s = 3.54 \times 10^{-11}$ (C/m$^2$). What angle does $E_2$ make with the $z$-axis?

**Solution:** We know that $E_{1t} = E_{2t}$ for any 2 media. Hence, $E_{1t} = E_{2t} = \hat{x}3 - \hat{y}2$. Also, $(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{n} = \rho_s$ (from Table 4.3). Hence, $\varepsilon_1 (E_{1z}) - \varepsilon_2 (E_{2z}) = \rho_s$, which gives

$$E_{1z} = \frac{\rho_s + \varepsilon_2 E_{2z}}{\varepsilon_1} = \frac{3.54 \times 10^{-11}}{2\varepsilon_0} + \frac{18(2)}{2} = \frac{3.54 \times 10^{-11}}{2 \times 8.85 \times 10^{-12}} + 18 = 20 \text{ (V/m)}.$$

Hence, $E_1 = \hat{x}3 - \hat{y}2 + \hat{z}20$ (V/m). Finding the angle $E_2$ makes with the $z$-axis:

$$E_2 \cdot \hat{z} = |E_2| \cos \theta, \quad 2 = \sqrt{9 + 4 + 4 \cos \theta}, \quad \theta = \cos^{-1} \left( \frac{2}{\sqrt{17}} \right) = 61^\circ.$$
Problem 4.56  Figure P4.56(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance $d$. The space between the plates contains two adjacent dielectrics, one with permittivity $\varepsilon_1$ and surface area $A_1$ and another with $\varepsilon_2$ and $A_2$. The objective of this problem is to show that the capacitance $C$ of the configuration shown in Fig. P4.56(a) is equivalent to two capacitances in parallel, as illustrated in Fig. P4.56(b), with

$$C = C_1 + C_2$$  \hspace{1cm} (19)$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d}$$ \hspace{1cm} (20)$$
$$C_2 = \frac{\varepsilon_2 A_2}{d}$$ \hspace{1cm} (21)$$

To this end, proceed as follows:

(a) Find the electric fields $E_1$ and $E_2$ in the two dielectric layers.

(b) Calculate the energy stored in each section and use the result to calculate $C_1$ and $C_2$.

(c) Use the total energy stored in the capacitor to obtain an expression for $C$. Show that (19) is indeed a valid result.

![Figure P4.56](image-url)

**Figure P4.56:** (a) Capacitor with parallel dielectric section, and (b) equivalent circuit.
Solution:

\[
\begin{align*}
E_1 &= E_2 = \frac{V}{d}, \\
W_{e1} &= \frac{1}{2} \varepsilon_1 E_1^2 \cdot d = \frac{1}{2} \varepsilon_1 \frac{V^2}{d} \cdot A_1 \cdot d = \frac{1}{2} \varepsilon_1 V^2 \frac{A_1}{d}, \\
W_{e1} &= \frac{1}{2} C_1 V^2.
\end{align*}
\]

Hence, \( C_1 = \varepsilon_1 \frac{A_1}{d} \). Similarly, \( C_2 = \varepsilon_2 \frac{A_2}{d} \).

(c) Total energy is

\[
W_e = W_{e1} + W_{e2} = \frac{1}{2} \frac{V^2}{d} (\varepsilon_1 A_1 + \varepsilon_2 A_2) = \frac{1}{2} CV^2.
\]

Hence,

\[
C = \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} = C_1 + C_2.
\]
**Problem 5.12** Two infinitely long, parallel wires are carrying 6-A currents in opposite directions. Determine the magnetic flux density at point $P$ in Fig. P5.12.

![Diagram of two parallel wires with currents](image)

Figure P5.12: Arrangement for Problem 5.12.

**Solution:**

$$
B = \hat{\phi} \frac{\mu_0 I_1}{2\pi (0.5)} + \hat{\phi} \frac{\mu_0 I_2}{2\pi (1.5)} = \hat{\phi} \frac{\mu_0}{\pi} (6 + 2) = \hat{\phi} \frac{8\mu_0}{\pi} \text{ (T)}. 
$$
Problem 5.14  Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the $x$–$y$ plane with its center at the origin, and the second loop’s center is at $z = 2$ m. If the two loops have the same radius $a = 3$ m, determine the magnetic field at:

(a) $z = 0$
(b) $z = 1$ m
(c) $z = 2$ m

\[ z = 2 \text{ m} \]

\[ z = 0 \]

\[ z = 1 \text{ m} \]

\[ z = 2 \text{ m} \]

**Figure P5.14:** Parallel circular loops of Problem 5.14.

**Solution:** The magnetic field due to a circular loop is given by (5.34) for a loop in the $x$–$y$ plane carrying a current $I$ in the $+\hat{\phi}$-direction. Considering that the bottom loop in Fig. is in the $x$–$y$ plane, but the current direction is along $-\hat{\phi}$,

\[ H_1 = -\hat{z} \frac{Ia^2}{2(a^2 + z^2)^{3/2}}, \]

where $z$ is the observation point along the $z$-axis. For the second loop, which is at a height of 2 m, we can use the same expression but $z$ should be replaced with $(z - 2)$. Hence,

\[ H_2 = -\hat{z} \frac{Ia^2}{2(a^2 + (z - 2)^2)^{3/2}}. \]

The total field is

\[ H = H_1 + H_2 = -\hat{z} \frac{Ia^2}{2} \left[ \frac{1}{(a^2 + z^2)^{3/2}} + \frac{1}{[a^2 + (z - 2)^2]^{3/2}} \right] \text{ A/m}. \]
(a) At $z = 0$, and with $a = 3$ m and $I = 40$ A,

$$H = -\hat{z} \frac{40 \times 9}{2} \left[ \frac{1}{3^3} + \frac{1}{(9+4)^{3/2}} \right] = -\hat{z} 10.5 \text{ A/m}.$$ 

(b) At $z = 1$ m (midway between the loops):

$$H = -\hat{z} \frac{40 \times 9}{2} \left[ \frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}} \right] = -\hat{z} 11.38 \text{ A/m}.$$ 

(c) At $z = 2$ m, $H$ should be the same as at $z = 0$. Thus,

$$H = -\hat{z} 10.5 \text{ A/m}.$$
Problem 4.49 An infinitely long conducting cylinder of radius $a$ has a surface charge density $\rho_s$. The cylinder is surrounded by a dielectric medium with $\varepsilon_r = 4$ and contains no free charges. The tangential component of the electric field in the region $r \geq a$ is given by $\mathbf{E}_t = -\hat{\phi}\cos\phi/r^2$. Since a static conductor cannot have any tangential field, this must be cancelled by an externally applied electric field. Find the surface charge density on the conductor.

Solution: Let the conducting cylinder be medium 1 and the surrounding dielectric medium be medium 2. In medium 2,

$$\mathbf{E}_2 = \hat{r}E_t - \hat{\phi}\frac{1}{r^2}\cos\phi,$$

with $E_t$, the normal component of $\mathbf{E}_2$, unknown. The surface charge density is related to $E_t$. To find $E_t$, we invoke Gauss’s law in medium 2:

$$\nabla \cdot \mathbf{D}_2 = 0,$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} (rE_t) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( -\frac{1}{r^2}\cos\phi \right) = 0,$$

which leads to

$$\frac{\partial}{\partial r} (rE_t) = \frac{\partial}{\partial \phi} \left( \frac{1}{r^2}\cos\phi \right) = -\frac{1}{r^2}\sin\phi.$$

Integrating both sides with respect to $r$,

$$\int \frac{\partial}{\partial r} (rE_t) \, dr = -\sin\phi \int \frac{1}{r} \, dr$$

$$rE_t = \frac{1}{r}\sin\phi,$$

or

$$E_t = \frac{1}{r^2}\sin\phi.$$

Hence,

$$\mathbf{E}_2 = \hat{r} \frac{1}{r^2}\sin\phi.$$

According to Eq. (4.93),

$$\hat{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s,$$
where \( \hat{n}_2 \) is the normal to the boundary and points away from medium 1. Hence, \( \hat{n}_2 = \hat{r} \). Also, \( \mathbf{D}_1 = 0 \) because the cylinder is a conductor. Consequently,

\[
\rho_s = -\hat{r} \cdot \mathbf{D}_2 |_{r=a} \\
= -\hat{r} \cdot \varepsilon_2 \mathbf{E}_2 |_{r=a} \\
= -\hat{r} \cdot \varepsilon_2 \varepsilon_0 \left[ \hat{r} \frac{1}{r^2} \sin \phi \right] |_{r=a} \\
= -\frac{4\varepsilon_0}{a^2} \sin \phi \quad \text{(C/m²)}.
\]
Problem 4.50  If \( \mathbf{E} = 150 \) (V/m) at the surface of a 5-cm conducting sphere centered at the origin, what is the total charge \( Q \) on the sphere’s surface?

Solution: From Table 4-3, \( \mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \). \( \mathbf{E}_2 \) inside the sphere is zero, since we assume it is a perfect conductor. Hence, for a sphere with surface area \( S = 4\pi a^2 \),

\[
D_{1R} = \rho_s, \quad E_{1R} = \frac{\rho_s}{\varepsilon_0} = \frac{Q}{S\varepsilon_0},
\]

\[
Q = E_R S\varepsilon_0 = (150)4\pi(0.05)^2\varepsilon_0 = \frac{3\pi\varepsilon_0}{2} \quad \text{(C)}.
\]
**Problem 4.58** The capacitor shown in Fig. P4.58 consists of two parallel dielectric layers. Use energy considerations to show that the equivalent capacitance of the overall capacitor, $C$, is equal to the series combination of the capacitances of the individual layers, $C_1$ and $C_2$, namely

$$C = \frac{C_1 C_2}{C_1 + C_2} \quad (22)$$

where

$$C_1 = \varepsilon_1 \frac{A}{d_1}, \quad C_2 = \varepsilon_2 \frac{A}{d_2}$$

(a) Let $V_1$ and $V_2$ be the electric potentials across the upper and lower dielectrics, respectively. What are the corresponding electric fields $E_1$ and $E_2$? By applying the appropriate boundary condition at the interface between the two dielectrics, obtain explicit expressions for $E_1$ and $E_2$ in terms of $\varepsilon_1$, $\varepsilon_2$, $V$, and the indicated dimensions of the capacitor.

(b) Calculate the energy stored in each of the dielectric layers and then use the sum to obtain an expression for $C$.

(c) Show that $C$ is given by Eq. (22).

\[ \text{Figure P4.58: (a) Capacitor with parallel dielectric layers, and (b) equivalent circuit (Problem 4.58).} \]
Solution:

\[ \begin{align*}
V &= V_1 + V_2, \\
E_1 &= \frac{V_1}{d_1}, \quad E_2 = \frac{V_2}{d_2}.
\end{align*} \]

According to boundary conditions, the normal component of \( \mathbf{D} \) is continuous across the boundary (in the absence of surface charge). This means that at the interface between the two dielectric layers,

\[ D_{1n} = D_{2n} \]

or

\[ \varepsilon_1 E_1 = \varepsilon_2 E_2. \]

Hence,

\[ V = E_1 d_1 + E_2 d_2 = E_1 d_1 + \frac{\varepsilon_1 E_1}{\varepsilon_2} d_2, \]

which can be solved for \( E_1 \):

\[ E_1 = \frac{V}{d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2}. \]

Similarly,

\[ E_2 = \frac{V}{d_2 + \frac{\varepsilon_2}{\varepsilon_1} d_1}. \]
(b)

\[ W_{e_1} = \frac{1}{2} \varepsilon_1 E_1^2 \cdot \gamma_1 = \frac{1}{2} \varepsilon_1 \left( \frac{V}{d_1 + \varepsilon_1 d_2} \right)^2 \cdot A d_1 = \frac{1}{2} V^2 \left[ \frac{\varepsilon_1 \varepsilon_2^2 A d_1}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2} \right], \]

\[ W_{e_2} = \frac{1}{2} \varepsilon_2 E_2^2 \cdot \gamma_2 = \frac{1}{2} \varepsilon_2 \left( \frac{V}{d_2 + \varepsilon_2 d_1} \right)^2 \cdot A d_2 = \frac{1}{2} V^2 \left[ \frac{\varepsilon_1 \varepsilon_2^2 A d_2}{(\varepsilon_1 d_2 + \varepsilon_2 d_1)^2} \right], \]

\[ W_e = W_{e_1} + W_{e_2} = \frac{1}{2} V^2 \left[ \frac{\varepsilon_1 \varepsilon_2^2 A d_1 + \varepsilon_2^2 \varepsilon_1 A d_2}{(\varepsilon_1 d_2 + \varepsilon_2 d_1)^2} \right]. \]

But \( W_e = \frac{1}{2} CV^2 \), hence,

\[ C = \frac{\varepsilon_1 \varepsilon_2^2 A d_1 + \varepsilon_2^2 \varepsilon_1 A d_2}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2} = \frac{\varepsilon_1 \varepsilon_2 A}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2} = \frac{\varepsilon_1 \varepsilon_2 A}{\varepsilon_2 d_1 + \varepsilon_1 d_2}. \]

(c) Multiplying numerator and denominator of the expression for \( C \) by \( A/d_1 d_2 \), we have

\[ C = \frac{\varepsilon_1 A}{d_1} \cdot \frac{\varepsilon_2 A}{d_2} = \frac{C_1 C_2}{C_1 + C_2}, \]

where

\[ C_1 = \frac{\varepsilon_1 A}{d_1}, \quad C_2 = \frac{\varepsilon_2 A}{d_2}. \]
**Problem 5.9** The loop shown in Fig. P5.9 consists of radial lines and segments of circles whose centers are at point $P$. Determine the magnetic field $\mathbf{H}$ at $P$ in terms of $a$, $b$, $\theta$, and $I$.

![Figure P5.9: Configuration of Problem 5.9.](image)

**Solution:** From the solution to Example 5-3, if we denote the $z$-axis as passing out of the page through point $P$, the magnetic field pointing out of the page at $P$ due to the current flowing in the outer arc is $\mathbf{H}_{\text{outer}} = -\frac{2I\theta}{4\pi b}$ and the field pointing out of the page at $P$ due to the current flowing in the inner arc is $\mathbf{H}_{\text{inner}} = \frac{2I\theta}{4\pi a}$. The other wire segments do not contribute to the magnetic field at $P$. Therefore, the total field flowing directly out of the page at $P$ is

$$\mathbf{H} = \mathbf{H}_{\text{outer}} + \mathbf{H}_{\text{inner}} = 2\frac{I\theta}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right) = 2\frac{I\theta(b-a)}{4\pi ab}.$$
**Problem 5.10** An infinitely long, thin conducting sheet defined over the space \(0 \leq x \leq w\) and \(-\infty \leq y \leq \infty\) is carrying a current with a uniform surface current density \(J_s = \$5\, \text{(A/m)}\). Obtain an expression for the magnetic field at point \(P = (0, 0, z)\) in Cartesian coordinates.

**Solution:**

![Diagram of conducting sheet](image)

\[|\mathbf{R}| = \sqrt{x^2 + z^2} \]

\(P = (0, 0, z)\)

Equation (5.30) provides an expression for the magnetic field due to an infinitely long wire carrying a current \(I\) as

\[\mathbf{H} = \frac{B}{\mu_0} = \frac{\hat{l} I}{2\pi r}.\]

We now need to adapt this expression to the present situation by replacing \(I\) with \(I_x = J_s\, dx\), replacing \(r\) with \(R = (x^2 + z^2)^{1/2}\), as shown in Fig. P5.10, and by assigning the proper direction for the magnetic field. From the Biot–Savart law, the direction of \(\mathbf{H}\) is governed by \(\mathbf{l} \times \mathbf{R}\), where \(\mathbf{l}\) is the direction of current flow. In the present case, \(\mathbf{l}\) is in the \(\hat{y}\) direction. Hence, the direction of the field is

\[\frac{\mathbf{l} \times \mathbf{R}}{|\mathbf{l} \times \mathbf{R}|} = \frac{\hat{y} \times (-\hat{x}x + \hat{z}z)}{|\hat{y} \times (-\hat{x}x + \hat{z}z)|} = \frac{\hat{x}z + \hat{z}x}{(x^2 + z^2)^{1/2}}.\]
Therefore, the field \( dH \) due to the current \( I_x \) is
\[
dH = \frac{\hat{x}z + \hat{z}x}{(x^2 + z^2)^{1/2}} I_x \frac{dx}{2\pi R} = \frac{(\hat{x}z + \hat{z}x)J_s}{2\pi(x^2 + z^2)},
\]
and the total field is
\[
H(0, 0, z) = \int_{x=0}^{w} \left( \frac{\hat{x}z + \hat{z}x}{2\pi(x^2 + z^2)} \right) \frac{J_s dx}{2\pi(x^2 + z^2)}
= \frac{J_s}{2\pi} \left( \hat{x}z \int_{x=0}^{w} \frac{dx}{x^2 + z^2} + \hat{z} \int_{x=0}^{w} \frac{x dx}{x^2 + z^2} \right)
= \frac{J_s}{2\pi} \left( \hat{x}z \left[ \frac{1}{z} \tan^{-1} \left( \frac{x}{z} \right) \right]_{x=0}^{w} + \hat{z} \left( \frac{1}{2} \ln(x^2 + z^2) \right)_{x=0}^{w} \right)
= \frac{5}{2\pi} \left[ 2\pi \tan^{-1} \left( \frac{w}{z} \right) + \hat{z} \frac{1}{2} \left( \ln(w^2 + z^2) - \ln(0^2 + z^2) \right) \right] \text{ (A/m) for } z \neq 0,
= \frac{5}{2\pi} \left[ \hat{x}z \tan^{-1} \left( \frac{w}{z} \right) + \hat{z} \frac{1}{2} \ln \left( \frac{w^2 + z^2}{z^2} \right) \right] \text{ (A/m) for } z \neq 0.
\]
An alternative approach is to employ Eq. (5.24a) directly.
Problem 5.11 An infinitely long wire carrying a 25-A current in the positive $x$-direction is placed along the $x$-axis in the vicinity of a 20-turn circular loop located in the $x$–$y$ plane (Fig. P5.11). If the magnetic field at the center of the loop is zero, what is the direction and magnitude of the current flowing in the loop?

![Figure P5.11: Circular loop next to a linear current (Problem 5.11).](image)

Solution: From Eq. (5.30), the magnetic flux density at the center of the loop due to the wire is

$$ B_1 = \hat{z} \frac{\mu_0}{2\pi d} I_1 $$

where $\hat{z}$ is out of the page. Since the net field is zero at the center of the loop, $I_2$ must be clockwise, as seen from above, in order to oppose $I_1$. The field due to $I_2$ is, from Eq. (5.35),

$$ B = \mu_0 H = -\hat{z} \frac{\mu_0 N I_2}{2a}. $$

Equating the magnitudes of the two fields, we obtain the result

$$ \frac{N I_2}{2a} = \frac{I_1}{2\pi d}, $$

or

$$ I_2 = \frac{2a I_1}{2\pi N d} = \frac{1 \times 25}{\pi \times 20 \times 2} = 0.2 \text{ A}. $$
Problem 5.13  A long, East-West–oriented power cable carrying an unknown current \( I \) is at a height of 8 m above the Earth’s surface. If the magnetic flux density recorded by a magnetic-field meter placed at the surface is 15 \( \mu \)T when the current is flowing through the cable and 20 \( \mu \)T when the current is zero, what is the magnitude of \( I \)?

Solution: The power cable is producing a magnetic flux density that opposes Earth’s, own magnetic field. An East–West cable would produce a field whose direction at the surface is along North–South. The flux density due to the cable is

\[
B = (20 - 15) \, \mu \text{T} = 5 \, \mu \text{T}.
\]

As a magnet, the Earth’s field lines are directed from the South Pole to the North Pole inside the Earth and the opposite on the surface. Thus the lines at the surface are from North to South, which means that the field created by the cable is from South to North. Hence, by the right-hand rule, the current direction is toward the East. Its magnitude is obtained from

\[
5 \, \mu \text{T} = 5 \times 10^{-6} = \frac{\mu_0 I}{2\pi d} = \frac{4\pi \times 10^{-7} I}{2\pi \times 8},
\]

which gives \( I = 200 \, \text{A} \).