Results of Oblique Incidence Analysis:

Snell's Law: \( k_1 \sin \theta_i = k_2 \sin \theta_t \)

Since in general: \( k = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} \sqrt{\frac{\mu}{\varepsilon}} \)

Index of Refraction: \( N = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}} \)

Snell's Law takes its common form:

\( N_1 \sin \theta_i = N_2 \sin \theta_t \)

or

\( \sin \theta_t = \frac{N_1}{N_2} \sin \theta_i \)

Case: \( N_1, N_2 > 0, \quad N_2 > N_1 \)

then \( \theta_t = \sin^{-1} \left( \frac{N_1}{N_2} \sin \theta_i \right) \)

Transmitted wave has a smaller angle to the normal.

Case: \( N_1 > 0, N_2 < 0 \) (Research: double negative materials with \( \varepsilon < 0, \mu < 0 \))

\( |N_2| > |N_1| \):

\( \theta_t = -\sin^{-1} \left( \frac{N_1}{|N_2|} \sin \theta_i \right) \)

Case: \( N_1, N_2 > 0 \quad N_1 > N_2 \) \( \Rightarrow \theta_t = \sin^{-1} \left( \frac{N_1}{N_2} \sin \theta_i \right) \)

has solutions only for a restricted set of incidence angles.
the critical angle is when \( \Theta_t = \frac{\pi}{2} \)

\[
N_1 \sin \Theta_{CR} = N_2 \sin (\Theta_t = \frac{\pi}{2})
\]

or

\[
\Theta_{CR} = \sin^{-1} \left( \frac{N_2}{N_1} \right)
\]

At the critical angle the transmitted wave becomes a surface wave.

For angles \( \Theta_i > \Theta_{CR} \) have reflected wave + surface wave.

The surface wave is "bound" to the interface.

Consider Perp. Pol. case:

\[
E_{wave} = i E_0 e^{-i(k_{2x}x + k_{2z}z)}
\]

where

\[
k_{2z} = \sqrt{k_{zz}^2 - k_{xz}^2} = \sqrt{\omega^2 \varepsilon_{zz} \mu_z - k_{xz}^2}
\]

But \( k_{2x} = k_2 \sin \Theta_t = k_1 \sin \Theta_i \)

So that

\[
k_{2z} = \sqrt{\omega^2 \varepsilon_z \mu_z - k_1^2 \sin^2 \Theta_i}
\]

\[
= \omega \sqrt{\varepsilon_z \mu_z - \varepsilon_1 \mu_1 \sin^2 \Theta_i}
\]
\[
\begin{align*}
R_{zz} &= \frac{\omega}{c} \sqrt{N_2^2 - N_1^2 \sin^2 \theta_i} \\
&= N_1 \frac{\omega}{c} \sqrt{\sin^2 \theta_i \cos \theta_i - \sin^2 \theta_i} \\
\text{If } \theta_i > \theta_{cr}, \text{ then square argument is negative} \\
\Rightarrow R_{zz} &= -j \alpha \text{ for } \theta_i > \theta_{cr} \\
\alpha &= N_1 \frac{\omega}{c} \sqrt{\sin^2 \theta_i \cos \theta_i - \sin^2 \theta_i} \\
\text{and} \\
E_{\omega}^{\text{trans}} &= \frac{\lambda}{j} E_0 \text{ e}^{-\alpha z} \text{ e}^{-j k_{2x} x} \\
\text{boundwave = wave decays exponentially away from interface} \\
\text{Picture: } \theta_r = \theta_i \quad \text{Boundwave } \theta_r = \pi/2 \\
\text{Note: Bound Wave is necessary to maintain power conservation} \\
\text{Process called Total Internal Reflection} \\
\text{Remember: need } N_1 > N_2. \\
\text{Application: optical fiber}
\end{align*}
\]
Optical Fibers are thin fibers of dielectric material typically very pure glass. They are important for broadband, low loss guided wave transmission systems.

Consider the optical fiber in air: \( N_f > N_0 \). The fiber will guide incident light (will capture light at one end and guide it to the other end). This occurs if \( \theta_c \) is such that \( \theta > \theta_c \), i.e.

\[
\sin \theta > \sin \theta_c = \frac{1}{N_f}
\]

But

\[
\sin \theta_i = N_f \sin \theta_t \quad \text{and} \quad \theta_t = \frac{\pi}{2} - \theta
\]

\[
= N_f \sin (\frac{\pi}{2} - \theta) = N_f \cos \theta
\]

Hence

\[
\cos^2 \theta = \left( \frac{1}{N_f} \right)^2 \sin^2 \theta_i
\]

or

\[
\sin^2 \theta = 1 - \left( \frac{1}{N_f} \right)^2 \sin^2 \theta_i
\]

But

\[
\sin \theta > \frac{1}{N_f} \quad \text{so that}
\]

\[
1 - \left( \frac{1}{N_f} \right)^2 \sin^2 \theta_i \geq \left( \frac{1}{N_f} \right)^2
\]

or

\[
N_f^2 \geq 1 + \sin^2 \theta_i
\]
Thus, all waves with $\theta_i$ satisfying

$$\theta_i \leq \sin^{-1}\left(\sqrt{\frac{N_f^2}{N_i^2} - 1}\right)$$

will be trapped in the fiber.

Note: the max $\theta_i$ can be as $\frac{\pi}{2}$. So if

$$N_f > \sqrt{2} = 1.414$$

all waves incident on the fiber will be trapped and guided. Glass has $N=1.5$, so this works nicely.

Brewster's Angle: Note that both $\Gamma_i$ and $\Gamma_f$ have a difference of terms in their numerators. Thus, in principle, both could be zero. The angle of incidence at which there is no reflected wave and only a transmitted wave is called Brewster's Angle.

Paralleled Pol: \[\eta_1 \cos\theta_{B_{ii}} = \eta_2 \cos\theta_{T}\] which means

$$\eta_1 \cos\theta_{B_{ii}} = \eta_2 \sqrt{1 - \sin^2\theta_{T}} = \eta_2 \sqrt{1 - \left(\frac{N_f}{N_i} \sin\theta_i\right)^2}$$

or

$$\sin\theta_{B_{ii}} = \sqrt{1 - \left(\frac{\mu_2 \epsilon_1 \mu_1 \epsilon_2}{\mu_2 \epsilon_2 \mu_1 \epsilon_1}\right)^2}$$
Likewise,
\[ n_z \cos \theta_{B\perp} = n_1 \cos \theta \]
yielding \[ \sin \theta_{B\perp} = \sqrt{\frac{1 - \left( \frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1} \right)^2}{1 - \left( \frac{\mu_1}{\mu_2} \right)^2}} \]

Non-Magnetic Materials: \[ \mu_1 = \mu_2 = \mu_0 \]
\[ \sin \theta_{B\parallel} = \sqrt{\frac{1 - \frac{\varepsilon_1}{\varepsilon_2}}{1 - \left( \frac{\varepsilon_1}{\varepsilon_2} \right)^2}} = \frac{1}{\sqrt{1 + \frac{\varepsilon_1}{\varepsilon_2}}} \]
\[ = \frac{\varepsilon_2}{\sqrt{\varepsilon_1 + \varepsilon_2}} \]
\[ \sin \theta_{B\perp} = \infty \Rightarrow \text{no } \theta_{B\perp}! \]

This allows for using a dielectric material to sort out parallel and perpendicular polarized waves.

\[ \Gamma \] can make a general wave into a parallel pol. wave.

Often "Brewster Windows" are used in lasers to achieve linear pol.

Note: \[ \theta_{B\perp} \] exists for magnetic materials.