SOLUTIONS
ECE 352 - DEVICE ELECTRONICS
Fall Semester 2004

Examination No.3

The time for examination is 1 hr 15 mins. Students are allowed to use 3 sheets of notes. Please show your work, partial credit may be given. At each problem state explicitly all assumptions that you make in the solution process (please make explicit references, if needed, to formulas or concepts/definitions from the class notes or textbook useful in your solution). Show your work with units, where applicable. Please write on the paper provided. Use additional sheets only if necessary.

Data: 
- Source charge: \( q = 1.5 \times 10^{-16} [\text{Coul}] = 1.6 \times 10^{-19} [\text{esu}] \) 
- \( \varepsilon_0 = 11.8 \), \( \varepsilon_\infty = 3.9 \), \( \varepsilon_s = 8.85 \times 10^{-14} [\text{F/cm}] \)
- Bolzmann const.: \( k = 8.616 \times 10^{-5} [\text{eV/K}] \) 
- \( t = 1.8 \times 10^{-12} [\text{esu}] = 1.6 \times 10^{-10} [\text{Coul}] \) 
- \( n_s = n_0 e^{E_s} \cdot E_s \)
- Room temp. dependent data: \( n_s = 1.0 \times 10^{10} [\text{Coul/cm}^3] \) 
- \( kT = 0.0259 [\text{eV}] \) 
- \( E_s = 1.12 [\text{eV}] \)

1. (20 pts) A current-voltage (\( I_D - V_{DS} \)) characteristic of an ideal MOSFET is shown in the figure below. Assume "square law" and use the information provided in the figure to answer questions that follow.

\[ I_D = \begin{cases} 
0 & \text{for } V_{DS} < 5 \\
1 & \text{for } V_{DS} = 5 
\end{cases} \]

\[ V_{GS} \]

\[ V_{DD} = 5 \]

a) (5 pts) Given a threshold voltage \( V_{th} = 1[V] \), what is the gate voltage that should be applied to the MOSFET gate to obtain the pictured characteristic?

In the "square law" theory we have \( V_{GS} = V_{DD} - V_{th} \). Therefore, we obtain \( V_G = V_{DD} + V_{th} = 6[V] \)

b) (10 pts) Using the graph below sketch the inversion layer and depletion region inside the silicon for the biasing corresponding to point 1 on the characteristic.

\[ S \quad G \quad D \quad B \]

Inversion layer

Depletion region

c) (5 pts) What is the inversion layer charge, \( Q_s \), at the drain end of the channel when the MOSFET is biased in such a way that it is in the state determined by the point 2 on the characteristic.

The point 2 corresponds to the pinch-off. At the pinch-off, according to the square law theory, the charge in the inversion channel goes to zero, thus \( Q_s(L) = 0 \).
2. (5 pt) The energy band diagram of an ideal MOS capacitor structure with oxide thickness, \( t_{ox} = 9.2[\mu m] \), operating at the temperature \( T = 300[K] \) is sketched below. Note that the applied voltage caused band bending such that \( E_F = E_i \) at the oxide-silicon interface.

\[
\begin{align*}
E_F & - - - - - - - - \quad 0.29 [eV] \\
E_i & \quad - - - - - - - - \quad 0.56 [eV]
\end{align*}
\]

a) (3 pts) What is the surface potential, \( \psi_s \)?

\[
\psi_s = \frac{1}{q} \left[ E_{i0} - E_i (0) \right] = -0.29 [V]
\]

b) (7 pts) Sketch the electrostatic potential, \( \psi \), in the semiconductor as a function of the position, \( x \).

c) (5 pts) Sketch the electric field, in the semiconductor as a function of the position, \( x \).

d) (3 pts) What is the electron concentration at the oxide-silicon interface?

Since \( E_F = E_i \) at the interface, we have \( n = n_i = 10^{20} [1/cm^3] \).

i) (4 pts) Roughly sketch the electron concentration versus position, \( x \), in the semiconductor.

\[
\begin{align*}
n & \quad \uparrow \\
0 & \quad \downarrow \\
x_d & \quad \downarrow \\
\end{align*}
\]

e) (3 pts) What is the doping level of the bulk?

\[
N_d^+ = n_i e^{(E_F - E_i)/kT} = \left(10^{20}\right) e^{29.3/0.0259} = 7.29 \times 10^9 [1/cm^3]
\]
3. (20 pts) A set of points a through e, is indicated on the enclosed plot of C-V characteristic of the MOS capacitor. A specific bias condition is associated with each point on the characteristic.

The sketches of energy band diagrams (numbered 1 through 5) in this ideal MOS capacitance structure under specific bias conditions are given below.

Identify each bias condition listed in with the suitable charge sketch and a point (or points) on the C-V characteristic and complete the Table:

<table>
<thead>
<tr>
<th>Bias Condition</th>
<th>Point on C-V Characteristic (indicate suitable letter)</th>
<th>Charge Density Diagram (indicate suitable number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accumulation</td>
<td>a</td>
<td>3</td>
</tr>
<tr>
<td>Depletion</td>
<td>c</td>
<td>5</td>
</tr>
<tr>
<td>Inversion</td>
<td>e</td>
<td>4</td>
</tr>
<tr>
<td>Flat Band</td>
<td>b</td>
<td>1</td>
</tr>
<tr>
<td>$V_G = V_A$</td>
<td>d</td>
<td>2</td>
</tr>
</tbody>
</table>
4. (20 pts) The DC state of an ideal MOS capacitor has charges depicted by the simplified (approximated) block diagram given below.

![Diagram of MOS capacitor with charges](image)

a) (3 pts) Is the bulk semiconductor p- or n-type?

The material is p-type, because there is an inversion layer of negative charges (electrons) and also a depletion layer of negative charges.

b) (6 pts) Is the device in (circle one): accumulation depletion inversion

The structure is in inversion, because there is a n inversion layer with electrons and as it is shown in the sketch of charges $n_e > N_x$.

c) (11 pts) Sketch the energy band diagram corresponding to the described DC state of the MOS capacitor structure.

![Energy band diagram](image)
The onset of strong inversion is defined by a bias that creates a layer of mobile charges and the bias level is such that at this level the rate of increase of mobile charges, with the bias (or with the surface potential), begins to exceed the rate of increase of fixed charges.

Specifying this condition mathematically, we can say that at the onset of strong inversion we have

\[
\frac{d\left(\frac{Q}{L}\right)}{d\psi} = \frac{d\left(\frac{Q}{L}\right)}{d\psi},
\]

and at higher bias we have

\[
\frac{d\left(\frac{Q}{L}\right)}{d\psi} > \frac{d\left(\frac{Q}{L}\right)}{d\psi}.
\]

b) (5 pts) What are 2 major assumptions in derivation of condition \(\psi_s = -2\psi_b\) associated with the onset of strong inversion.

1) We assume existence of layer of mobile charges, but such that the mobile charge is negligible with respect to the fixed charge.

2) We assume that the potential inside the silicon and near the silicon surface changes linearly with the distance, \(x\), from the surface.

c) (4 pts) What is the major assumption in derivation of the formula

\[
I_D = \frac{W}{L} \mu_a \int_{V_a}^{V_s} \left(\psi - V_a\right) dV.
\]

The major assumption in this derivation is existence of a uniform layer of mobile charges with the mobility, \(\mu_a\), independent of position.

d) (5 pts) What is the major assumption in derivation of the formula

\[
I_D = C_s \frac{W}{L} \mu_a \left[ V_a - V_a - \frac{1}{2} V_a \right] V_a.
\]

The major assumption in this derivation is that a bias is just above that which creates the onset of strong inversion (but not much higher) so that for the surface potential, \(\psi_s\), we can write \(\psi_s \equiv -2\psi_b\).

With this assumption we can introduce the threshold voltage \(V_a = \frac{Q_s}{C_s} + \psi_s + V_{th}\) in the expression for

\[
Q_a \sim C_s \left[ V_a - V_a - V(x) \right].
\]