\[ t = \frac{4 \pi L}{\alpha} \]
4.31. (a) $E = -\frac{\Delta T}{\beta} \frac{1}{h} \frac{\partial}{\partial x}$ from Eq. (3.37) at $J_n = 0$

$$E = \frac{-\Delta T}{\beta} \frac{1}{h} \frac{\partial}{\partial x}$$

(b) $E = 0.022 \frac{\text{mV}}{\text{m}^2}$

(c) $E = \frac{\partial T}{\partial x}$

4.33. $J_{500} = 3 \times 10^{-7} \text{A/m}$ from Eq. (4.20) $= 2 \times 10^{-12} \text{A/m} = \frac{1}{2} \text{mA/m}$

$$J_{500} = 3 \times 10^{-7} \text{A/m}$$

$$V_e = 2 \times 10^{-7} \times 1.15 \times 0.1 \text{V} = 0.22 \text{V}$$

$$V_e = 2 \frac{\Delta T}{\beta} \times 1.15 \times 0.1 = 0.22 \text{V}$$
Solutions to Sample Problems

Table 3.2

Common Single-Species Diffusion Equation Solutions

<table>
<thead>
<tr>
<th>Solution no.</th>
<th>GIVEN:</th>
<th>SIMPLIFIED</th>
<th>DIFF. EQN.</th>
<th>SOLUTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. 1</td>
<td>Steady state, no light.</td>
<td>( \Gamma = \frac{D_0}{\nu} )</td>
<td>( \frac{d\Delta x}{dt} = \frac{d^2\Delta x}{dx^2} )</td>
<td>( \Delta x(x) = Ae^{-x/\nu} + Be^{x/\nu} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \gamma = \frac{\nu}{\sqrt{4\pi t}} )</td>
<td>( \gamma ) and ( A, B ) are solution constants.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution no.</th>
<th>GIVEN:</th>
<th>SIMPLIFIED</th>
<th>DIFF. EQN.</th>
<th>SOLUTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. 2</td>
<td>No concentration gradient, no light.</td>
<td>( \frac{d\Delta x}{dx} = \frac{d^2\Delta x}{dx^2} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Solution no.</th>
<th>GIVEN:</th>
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<th>DIFF. EQN.</th>
<th>SOLUTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. 3</td>
<td>Steady state, no concentration gradient.</td>
<td>( \frac{d\Delta x}{dt} = \frac{d^2\Delta x}{dx^2} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Solution no.</th>
<th>GIVEN:</th>
<th>SIMPLIFIED</th>
<th>DIFF. EQN.</th>
<th>SOLUTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. 4</td>
<td>Steady state, no light.</td>
<td>( \frac{d\Delta x}{dt} = \frac{d^2\Delta x}{dx^2} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
<td>( \Delta x(x) = \frac{d\Delta x}{dx}e^{-x/\nu} )</td>
</tr>
</tbody>
</table>

The single advection is shown, \( \Gamma = 300 \) K, the density of the air is the same everywhere with \( D_0 = \frac{\nu}{\sqrt{4\pi t}} \) and \( \gamma = \frac{\nu}{\sqrt{4\pi t}} \). Also, the statement of the problem implies equilibrium conditions exist for \( r = 0 \).

Steps 2-3: Characterize the system under equilibrium conditions.

1. Assume the system is at room temperature \( \phi = 300 \) K. Thus, \( \phi = \phi_0 = N_0 = 10^{10}m^3 \) and \( p_0 = N_0/\phi_0 = 10^5 \). With the density being uniform, the equilibrium \( \phi_0 \) and \( p_0 \) values are the same everywhere throughout the sample.

Steps 4-5: Analyze the problem qualitatively.

Prior to \( r = 0 \), equilibrium conditions prevail and \( \Delta x = 0 \). Entering at \( r = 0 \) light creates excited electrons and holes, and \( \Delta x \) will begin to increase. The growing excess carrier number, however, in turn lead to an increased indirect thermal recombination rate which is proportional to \( \Delta x \). Consequently, as \( \Delta x \) grows, a result of photogeneration, more and more of the excited holes are eliminated per second by recombination through R-G centers. Eventually, a point is reached where the carriers annihilated per second by indirect thermal recombination balance the carriers created per second by the light, and a steady state condition is attained.

Summarizing, we expect \( \Delta x \) to grow from zero at \( r = 0 \) to build up at a decreasing rate, and to ultimately become constant. Since the light generation and thermal recombination rates are in balance under steady state conditions, we can even state \( \phi_0 = \Delta x(x) \) at \( r = 0 \), \( \Delta x(x) < \phi_0 \), \( \phi_0 \) is a slow-mode ignition point.

Steps 6-7: Perform a quantitative analysis.

The minority carrier diffusion equation is the starting point for most first-order quan-
titative analyses. After reevaluating the problem for obvious conditions that would invalidate the use of the diffusion equation, the appropriate minority carrier diffusion equation is written down. The equation is simplified, and a solution is sought subject to boundary con-
titions stated or implied in the problem.
For the problem under consideration a cursory inspection reveals that all simplifying assumptions involved in deriving the diffusion equations are readily satisfied. Specifically, only the minority carrier concentration is of interest; the equilibrium carrier concentrations are not a function of position, indirect thermal generation is nonexistent, and there are no "other processes" except for photogeneration. Because the photogeneration is uniform throughout the semiconductor, the perturbed carrier concentrations are also position-independent and the electric field E must clearly be zero in the perturbed system. Finally, if \( \Delta n_{\text{max}} = G_0 T \tau \), then \( \Delta n_{\text{max}} \approx \Delta n_0 \), in consistent with low-level injection prevailing at all times.

With no obstacles to utilizing the diffusion equation, the desired qualitative solution can now be obtained by solving

\[
\frac{\partial \Delta n}{\partial t} = D \frac{\partial^2 \Delta n}{\partial x^2} - \Delta n + G_0
\]

subject to the boundary condition

\[
\Delta n(0, t) = 0
\]

Since \( \Delta n \) is not a function of position, the diffusion equation becomes an ordinary differential equation and simplifies to

\[
\frac{\partial \Delta n}{\partial t} = \frac{\partial \Delta n}{\partial t} = \frac{1}{\tau} \Delta n + G_0
\]

The general solution of Eq. (3.57) is

\[
\Delta n(x, t) = G_0 \tau + A e^{-x/\tau}
\]

Applying the boundary condition yields

\[
A = -G_0 \tau
\]

and

\[
\Delta n(x, t) = G_0 \tau [1 - e^{-x/\tau}]
\]

\[\text{Solution.}\]

Failing to examine the mathematical solution to a problem is like growing vegetables and then failing to eat the produce. Relative to the Eq. (3.80) result, \( G_0 \tau \) has the dimensions of a concentration (ordinate/m²/s) and the solution is at least dimensionally correct. A plot of the Eq. (3.80) result is shown in Fig. 3.15. Moreover, in agreement with qualitative predications, \( \Delta n(x, t) \) starts from zero at \( t = 0 \) and eventually saturates at \( G_0 \tau \) after a few \( \tau \).

Equilibrium—We would be remiss if we did not point out the connection between the hypothetical problem just completed and the photoconductive decay measurement described in Subsection 3.1.4. Light output from the photoconductor used in the measurement can be modeled to first order by the pulse train picture in Fig. 3.26(a). The Eq. (3.80) solution can be approximately described the carrier build-up during a light pulse. It should be noted, however, that the photomultiplier light pulse have a duration of \( \tau_{\text{p}} = 1 \mu s \), which compared the minority carrier lifetime \( \tau_c = 150 \mu s \). With \( \tau_{\text{p}} \tau_c \approx 1 \), the seen only the very

![Figure 3.15](image_url)

**Figure 3.15** Solution to Sample Problem No. 1. Photogeneration-induced increase in the excess hole concentration as a function of time.
Figure 3.35 (a) Approximate model for the light source from the specimen is the photoconductively deep measurement, (b) sketch of the observed light-off-light-on transition for the sodium ion-conducting membrane.

SOLUTION TO SAMPLE PROBLEM NO. 2

b) The semiconductor is again silicon uniformly doped with an \( N_i = 10^{17} \text{cm}^{-3} \). Steady state conditions are inferred from the statement of the problem, since we are asked for \( \Delta n_s \) and \( \Delta n_{sc} \). Moreover, \( x = 0 \), \( \Delta n_i(0) = \Delta n_{sc}(0) = 10^{17} \text{cm}^{-3} \), and \( \Delta n_s = 0 \) as \( x \to \infty \). The former boundary condition follows from the semi-infinite nature of the bar. The penetration into \( \Delta n_s \) due to the transgranular light cannot possibly extend out to \( x = \infty \). The nondegenerating nature of the light allows us to set \( \Delta n_s = 0 \) for \( x > 0 \). Finally, note that the problem statement fails to mention the temperature of operation. When this happens, it is reasonable to assume an intended \( T = 300 \text{ K} \).

If the light were removed, the silicon bar in Sample Problem 2 maintained at 300 K would revert to an equilibrium condition described as that described in Sample Problem 1. Under equilibrium conditions, then, \( \Delta n_i = 10^{17} \text{cm}^{-3} \), \( \Delta n_{sc} = 10^{17} \text{cm}^{-3} \), and the carrier concentrations remain throughout the semiconductor bar.

Qualitatively it is a simple matter to predict the expected effect of the nonmeasuring light on the silicon bar. The light first causes excess carriers \( \Delta n_{sc} \) in the illumination region. In addition, excess holes are generated in the illuminated region itself. Thus, the diffusion holes move into the bar from their existence are reduced by recombination. In addition, since the majority carrier holes live for only a limited period, a time \( t_p \), on the average, fewer excess holes will survive into the depth of penetration into the light becomes larger and larger. Under steady state conditions it is reasonable to expect an equal depletion of both near \( x = 0 \) with \( \Delta n_{sc}(x) \) monotonically decreasing from \( \Delta n_{sc} \) at \( x = 0 \) to \( \Delta n_s = 0 \) as \( x \to \infty \).
In preparation for obtaining a quantitative solution, we observe that the system under consideration is one-dimensional, the analysis is restricted to the minority carrier holes, the equilibrium carrier concentrations are position independent, incident thermal $N$–$O$ donors, there are no "other processes" for $x > 0$, and low-level injection conditions clearly prevail ($\Delta p_{\text{drift}} = \Delta p_{\text{diff}} = 10^{17} \text{cm}^{-3}$). The only question that might be raised concerning the use of the diffusion equation as the starting point for the quantitative analysis is whether $N = 0$. With the lights on, a nonuniform distribution of holes and associated electric field will appear near the $x = 0$ surface. The excess hole pile-up, however, is very small ($\Delta p_{\text{drift}} = \Delta p_{\text{diff}}$) and the associated electric field is therefore expected to be correspondingly small. Moreover, problems of this type it is found that the majority carriers, negatively charged, decrease in the given problem, redissolve in such a way as to partly cancel the minority carrier charge. Thus experience indicates the $N = 0$ assumption to be reasonable, and use of the minority carrier diffusion equation as justified.

Under steady-state conditions with $Q_i = 0$ for $x > 0$ the hole diffusion equation reduces to the form

$$\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{\tau_p} = 0 \quad \text{for} \quad x > 0$$

(3.62)

which is to be solved subject to the boundary conditions

$$\Delta p_{\text{drift}} = \Delta p_{\text{diff}} = \Delta p_{\text{in}}$$

(3.63)

and

$$\Delta p_{\text{drift}} = 0$$

(3.64)

Equation (3.62) should be recognized as one of the simplified diffusion equations cited in Table 3.2, with the general solution

$$\Delta p(x) = A e^{-\lambda x} + B e^{\lambda x}$$

(3.65)

where

$$\lambda = \sqrt{D_p \tau_p}$$

(3.66)

Because $\Delta p(\infty) \to 0$ as $x \to \infty$, the only way that the Eq. (3.64) boundary condition can be satisfied is for $B = 0$ to be identically zero. With $B = 0$, application of the Eq. (3.63) boundary condition yields

$$A = \Delta p_{\infty}$$

(3.67)

and

$$\Delta p(x) = \Delta p_{\infty} e^{-\lambda x} \quad \square \text{solution}$$

(3.68)

The Eq. (3.68) result is plotted in Fig. 3.2.1. In agreement with qualitative arguments, the concentrating light merely gives rise to a monotonically decreasing $\Delta p(x)$ starting from $\Delta p_{\infty}$ at $x = 0$ and decreasing to $\Delta p_0 = 0$ as $x \to \infty$. Note that the precise functional form of the falloff in the excess carrier concentration is exponential with a characteristic decay length equal to $\lambda^{-1}$.

![Figure 3.21: Solution to Sample Problem](image)

No. 7 showing the excess hole concentration inside the Si bar as a function of position.
SOLUTION TO EXERCISE 3.5

5(a) Since \( E = E_0 \), it follows from Eq. (3.75a) that \( F_x = E_0 \). Likewise, substituting \( 0 \) preceding \( p \)-expression into Eq. (3.75b), we conclude

\[ F_x = E_0 - KT \ln(p_0/n_0) = E_0 - KT \ln(p_0/n_0) + (\Delta p_0(n_0)e^{-\Delta p_0}) \]

(b) If \( \Delta p_0(x) = p_0 \), then \( \Delta p_0(n_0) = n_0 \) and

\[ F_x = E_0 - KT \ln(p_0/n_0)e^{-\Delta p_0(x)} \]

or

\[ F_x = E_0 - KT \ln(p_0/n_0) + KT \Delta p_0(x) \]

(c) We know from Simplex Problem No. 3 that \( \Delta p_0 = 10^6 \) cm\(^{-2} \), \( n_0 = 10^6 \) cm\(^{-2} \), and \( p_0 = n_0/10^4 \) cm\(^{-2} \). Thus

(i) Near \( x = 0 \), \( \Delta p_0 \approx p_0 \) and \( F_x \) is a linear function of \( x \)

(ii) At \( x = 0 \), \( \Delta p_0 = n_0 \) and we deduce from the part (b) result that \( p_0 = E_0 \).

(iii) For large \( x \), \( F_x \) eventually approaches \( F_x = E_0 \).

(d) \( F_x = E_0 - E_0 = E_0 - KT \Delta p_0(n_0) = 0.20 \) eV

Utilizing the preceding information, one concludes

\[ \begin{array}{c|c}
\text{Region} & \text{Energy State Configuration} \\
\hline
\text{specified} & \text{below} \\
\hline
\end{array} \]

6. Assuming \( \rho = 0 \), it follows from Eq. (3.76a) that there will be a hole current whenever \( dp/\rho dx \neq 0 \). There is obviously a hole current to be illuminated near \( x = 0 \).

\[ \begin{array}{c}
\text{Negative} \\
\text{Charge} \\
\hline
\text{Hole} \\
\end{array} \]

6. Apparent surface recombination is denoted \( J_o = 0 \) if \( x = 0 \). One might conclude from the part (c) result that \( dp/\rho dx = 0 \) and therefore \( J_o = 0 \). Under usual input conditions, however, one must have \( J_o = \text{constant} \) at all points in the band; hence no carriers leaves the spot at \( x = 0 \). However, there exists at \( x = 0 \). Thus \( J_o(x) \approx J_o(x) \approx J_o(x) \) and we know \( J_o \neq 0 \) near \( x = 0 \). The apparent discrepancy here stems from the fact that \( J_o \) is proportional to both \( n \) and \( p \), because the majority carrier electron concentration is much lower than the minority carrier hole concentration. \( dp/\rho dx \) must be correspondingly smaller than \( dp/\rho dx \). The slope to \( F_x \) simply cannot be obtained by inspecting the energy band diagram.