PRACTICE PROBLEM SET II

- DtFt (properties & filter analysis)
- DFT with applications

PART A

Multiple choice questions: You may circle more than one answer. For a given question a point will be awarded only if ALL correct answers (and no incorrect ones) are circled.

1. Which is the correct gain response for the digital filter: \( y(n) = x(n) + x(n-1) \)?

(a) \( \cdots \) \( \cdots \) (b) \( \cdots \) \( \cdots \) (c) \( \cdots \) \( \cdots \)

2. Which of the following is a notch filter?

(a) \( y(n) = y(n-1) + x(n) \)
(b) \( y(n) = x(n) + 0.2x(n-1) \)
(c) \( y(n) = 0.5y(n-1) + x(n) \)
(d) \( y(n) = 0.5x(n) + 0.5x(n-2) \)
(e) None of the above.

3. Which of the following is a minimum-phase filter?

(a) \( H(z) = 1 + 2z^{-1} \)
(b) \( H(z) = \frac{0.5 + 2z^{-1}}{1 - 0.4z^{-1}} \)
(c) \( H(z) = \frac{2 + 4z^{-1}}{1 + 0.5z^{-1}} \)
(d) \( H(z) = 2 + z^{-1} \)
(e) None of these.
4. The following system has an overall transfer function of:

(a) \( H(\omega) \)
(b) \( 1 - H(\omega) \)
(c) \( H(\omega - \pi) \)
(d) \( H(\omega) + H(\omega - \pi) \)
(e) None of these

5. Which filter is linear phase?

(a) \( y(n) = x(n) + y(n-1) \)
(b) \( y(n) = x(n) - 3x(n-1) + x(n-3) \)
(c) \( y(n) = x(n) - 2x(n-2) \)
(d) \( y(n) = -x(n) - x(n-1) + x(n-2) \)
(e) None are linear phase

6. The coefficients of a certain FIR filter are all \( \geq 0 \). Which statement cannot be true?

(a) Filter has linear phase
(b) Filter is stable
(c) Filter is lowpass
(d) Filter introduces finite time delay
(e) All can be true.

7. When doing spectral analysis using the FFT, better frequency resolution is **always** obtained by:

(a) Recording more data
(b) Recording data over a longer time window
(c) Increasing the sampling rate

8. If the DFT of the 4-sample sequence \{a b c d\} yields the sequence \{A B C D\}, then the 8-pt DFT of the sequence \{a 0 b 0 c 0 d 0\} is:

(a) \{A -B C -D E -F G -H\}
(b) \{A 0 C 0 E 0 G 0\}
(c) \{E F G H A B C D\}
(d) \{A B C D A B C D\}
(e) None of these.

9. Zero-padding is used in spectral analysis to:

(a) reduce spectral leakage
(b) reduce the picket-fence effect
(c) reduce computation time
(d) eliminate wraparound error.
PART B

1. Find the impulse response of the total system. Express your answer in terms of $h(n)$.

Assume that each $h(n)$ shown in the diagram is the impulse response of (the same) causal and LTI filter.

2. Find the impulse response of the total system given that:

$$h(n) = \begin{cases} 
1, & 0 \leq n \leq 9 \\
0, & \text{elsewhere} 
\end{cases}$$

($h(n)$ is the impulse response of the LTI system represented by each box.)
3. Given two causal LTI filters $H_1(z)$ and $H_2(z)$ with poles, zeros and gain constant $K$ as shown:\

\[ H_1(z) \quad K = 3 \]
\[ H_2(z) \quad K = 1 \]

Find the ratio $\frac{|H_2(\omega)|}{|H_1(\omega)|}$ and discuss the significance and implications of your answer. (Hint: Lesson 22, page 5.)

4. An analog signal is sampled using a sampling rate of 40 kHz. Design a FIR filter to remove interference at (only) the following specific frequencies:

500Hz, 1.5kHz, 2.5kHz, 3.5kHz, 4.5kHz, … 18.5kHz, 19.5kHz.

The FIR filter should have a maximum gain of 1.

Specify the linear difference equation of the filter.

\[ H(z) = K \prod_{m} \left( \frac{1}{z - \text{zero}_m} \right) \prod_{m} \left( \frac{1}{z - \text{pole}_m} \right) \]
5. A FIR filter with N coefficients was designed and its gain response is shown below (highlighted black trace):

Describe how to convert this filter to a new FIR filter that can be used to remove interference at specific frequencies 5 kHz and 15 kHz. Assume that the analog signal to be processed was sampled at a sampling rate of 40 kHz.

6. Sketch $Y(\omega)$ for $-\pi \leq \omega \leq \pi$ in the following system.

where $x(n)$ has the spectrum:
7. Sketch $Y(\omega)$ for $-\pi \leq \omega \leq \pi$ in the following system.

```
x(n) \uparrow 2 \quad \text{H(\omega)} \quad \uparrow 2 \quad \text{H(\omega)} \quad y(n)
```

where $x(n)$ has the spectrum:

```
X(\omega)
```

8. (a) Fill in the missing two elements of the array $X(k)$. Explain how you did this.

```
x(n) \quad X(k)
[5 2 3 2] \quad [___ 2 ___ 2]
```

(b) From the DFT pair in (a), use DFT properties to find the 8pt DFT of the array $y(n)$. Explain how you found $Y(k)$.

```
y(n) \quad Y(k)
[3 0 2 0 5 0 2 0] \quad [___ ___ ___ ___ ___ ___ ___]
```

(c) Let $x_1(n) = [3 1 0 -2]$ and $x_2(n) = [2 0 1 3]$. Let $X_1(k)$ and $X_2(k)$ be the 4-pt DFT’s of $x_1(n)$ and $x_2(n)$, respectively. Find $x_3(n)$, where $X_3(k) = X_1(k)X_2(k)$. Show work.
9. (The sampling rate for this problem was 2048Hz.)

The following graph was generated from the 64-pt DFT of the data y(n), where:

- \( y(n) = x(n) w(n) \)
- \( w(n) = 64\text{-pt tapered window} \)
- \( x(n) = 3\cos\left(\frac{7\pi}{12} n\right) + 1.2\cos\left(\frac{31\pi}{62} n\right) \) for \( 0 \leq n \leq 63 \)

The next graph shows part of the DFT of the window:

From the information given, find:

(a) \( P_1, P_2 \) (heights of the peaks in the first graph)

(b) \( f_1, f_2 \) (frequencies of the two sinusoids in Hz)
10. A continuous-time signal $x(t)$ is corrupted by interference $v(t)$ as shown. Design a digital filter to remove this interference. *Hint: See attached trig identities.*

![Diagram](image)

11. Use the inverse discrete-time Fourier transform to find an expression for the impulse response $h(n)$ of the ideal lowpass filter shown.

![Diagram](image)
12. The spectrum shown was computed from 64 points of a sequence of the form

\[ x(n) = \sum_{k=1}^{M} A_k \cos[2\pi f_k nT], \]

where \( T^{-1} = 128 \) Hz, the \( A_k \) are measured in volts, and the \( f_k \) in Hz.

The code used to compute and plot the spectrum was:

```matlab
(x was created in previous code)
xw=x.*hanning(64)';
XW=fft(xw);
semilogy(k,abs(XW))
axis([0 63 -inf inf])
grid
xlabel('k')
max(abs(fft(hanning(64))))
ans = 32.5000
(See plot)
```

The following is a listing of the spectrum data:

```plaintext
abs(XW)
an
Columns 1 through 9
  0.0629   0.0791   0.1725   15.8789   32.4929   15.8752   0.0639   0.0352
Columns 10 through 18
  0.0227   0.0160   0.0120   0.0094   0.0077   0.0065   0.0057   0.0052   0.0051
Columns 19 through 27
  0.0055   0.0067   0.0094   0.0162   0.0409   3.9683   8.1241   3.9682   0.0407
Columns 28 through 36
  0.0159   0.0089   0.0059   0.0045   0.0038   0.0035   0.0038   0.0045   0.0059
Columns 37 through 45
  0.0089   0.0159   0.0407   3.9682   8.1241   3.9683   0.0409   0.0162   0.0094
Columns 46 through 54
  0.0067   0.0055   0.0051   0.0052   0.0057   0.0065   0.0077   0.0094   0.0120
Columns 55 through 63
  0.0160   0.0227   0.0352   0.0639   0.1642   15.8752   32.4929   15.8789   0.1725
Column 64
  0.0791
```

From the plot and data shown, your task is to estimate:

- \( M \)
- \( (A_k, f_k) \) for \( 1 \leq k \leq M \)
13. Consider the length-12 sequence, defined for $0 \leq n \leq 11$,

$$x[n] = \{3 \ -1 \ 2 \ 4 \ -3 \ -2 \ 0 \ 1 \ -4 \ 6 \ 2 \ 5\}$$

with a 12-point DFT given by $X(k)$, for $0 \leq k \leq 11$. Evaluate the following functions of $X(k)$ without explicitly computing the DFT.

(a) $X(0)$

(b) $X(6)$

(c) $\sum_{k=0}^{11} X(k)$

(d) $\sum_{k=0}^{11} (-1)^k X(k)$

(e) $\sum_{k=0}^{11} |X(k)|^2$

14. Determine and sketch the gain $|H|$ and phase response $\angle H$ for each of the following systems:

(a) $x(n) \rightarrow z^{-1} \rightarrow y(n)$

(b) $x(n) \rightarrow z^{-1} \rightarrow + \rightarrow y(n)$

15. Find the impulse response of the following system. (Show all work.)
16. (a) Find the DtFt of the following sequence:

\[ x(n) = \begin{cases} 
1; & 0 \leq n \leq N - 1 \\
0; & \text{elsewhere} 
\end{cases} \]

You must give your answer in closed form, i.e. no power series, or summations.

(b) Hence, determine \( X(e^{j\omega}) \)

17. The sequence:

\[ x(n) = 4 \sin \left( n \frac{\pi}{2} - \frac{\pi}{8} \right) \forall n \]

is input to a system whose frequency response is shown below. Determine the output \( y(n) \).

18. (a) Given \( x(n) \leftrightarrow X(z) \), derive an expression for the z-transform of \( z_0^n x(n) \), where \( z_0 \) is a complex constant.

(b) Let \( x(n) \) be the sequence whose z-transform has the pole-zero plot shown below. Using your result in part (a), sketch the pole-zero plot for the sequence:

\[ y(n) = \cos \left( \frac{\pi}{2} n \right) x(n) \]
19. A signal is passed through a causal system with zeros at $1.5e^{j3\pi/4}$, -2 and 1/3, and four poles at 0. Design an inverse filter $H_{\text{inv}}(z)$ to compensate for the magnitude distortion caused by the system. The gain of the inverse filter should be positive at $\omega=0$. Obtain the overall system function $H_{\text{overall}}(z)$ after compensation.

20. A continuous-time signal $x_s(t)$ that occupies the frequency band $2\pi 1000 \leq |\Omega| \leq 2\pi 1250$ rad/sec is corrupted by additive 60-Hz sinusoidal noise. The combined time signal can be represented as $x(t) = x_s(t) + x_n(t)$, where $x_s$ is the desired signal and $x_n$ is the noise. It is desired to design a simple digital filter to reject the 60-Hz noise while passing the signal component. Assume that $x(t)$ is sampled at intervals of $T=0.25x10^{-3}$ sec. Design an appropriate filter and specify its system function $H(z)$. The gain of the filter in the center of the frequency band of $x_s$ should be 1.0.

21. You are given this pole-zero diagram for a causal LTI system.

![Pole-Zero Diagram](image)

You are also told that the dc gain of the system is 1.

(a) Find the impulse response of the system. Check the value of $h(0)$ using the initial-value theorem.

(b) Sketch $|H(e^{j\omega})|$ for $-\pi \leq \omega \leq \pi$.

22. Design a digital filter to give the following gain response when operated at a sampling rate of 700 samples/sec.

![Gain Response](image)

Give the linear difference equation and draw the signal flow graph of your digital filter.
23. The signal $x(n)$ is distorted by the causal LTI system shown, giving distorted signal $d(n)$.

\[ x(n) \xrightarrow{\text{z}^{-1}} x(n) \xrightarrow{\text{z}^{-1}} x(n) \xrightarrow{\text{z}^{-1}} d(n) \xrightarrow{H_c(z)} y(n) \]

Find the linear difference equation of the filter $H_c(z)$ such that
\[ \frac{|Y(e^{j\omega})|}{|X(e^{j\omega})|} = 1 \; \forall \; \omega. \]

24. Parts (b) and (c) should be answered using DFT properties, without explicit computation of the DFT.

(a) Find the 2-pt DFT of the sequence $x_1(n) = [2 \; 1]$.

(b) Hence, find the 4-pt DFT of the sequence $x_2(n) = [2 \; 1 \; 2 \; 1]$. Hint: Check your answer at $k = 0$.
(Explain how you arrived at the answer.)

(c) From the results in (a) and (b), find the 4-pt DFT of $x_3(n) = [4 \; 1 \; 3 \; 1]$.
(Explain how you arrived at the answer.)

25. Is $\sin(2\omega)$ for $0 \leq |\omega| \leq \pi$ a valid steady-state frequency response $H(\omega)$ for a discrete-time LTI system with real coefficients? If so, plot the gain and phase responses.

26. (a) Which of the following systems A, B, C are BIBO stable, given they are causal?
(b) Which of the following systems A, B, C are BIBO stable, given they are noncausal?
(c) A second order filter has poles at $0.6e^{\pm j\pi/6}$. The impulse response includes the term: $h(n) = K(\alpha)^n \cos(\theta n + \beta)u(n)$. Identify the values of $\alpha$ and $\theta$.
(d) What type (lowpass, bandpass, highpass, etc.) do the following filters belong to?
\[ y(n) = x(n) + 0.81x(n-2) \]
\[ y(n) = x(n) - 0.81x(n-2) \]
27. A discrete time signal $x(n)$ has the spectrum shown:

$x(n)$ is processed as follows:

Sketch the spectrum at points (i), (ii) ... (v) using the axes below.

Let $x(n)$ be a speech signal. Would you expect a playback of $y(n)$ to be intelligible? *Explain.*
28. (a) Show that \( x^* (n) \Leftrightarrow X^* (z^*) \) and that \( x(-n) \Leftrightarrow X(1/z) \)

(b) It is desired to filter a signal \( x(n) \). A method for compensating for the nonlinear phase of the filter \( H(z) \) is shown below. The method can only be applied “off-line” (not real-time) since the action of time reversal requires a data buffer.

\[
\begin{align*}
  x(n) & \rightarrow H(z) \rightarrow w(n) \rightarrow \text{Reverse time} \rightarrow w(-n) \rightarrow H(z) \rightarrow \text{Reverse time} \rightarrow y(n)
\end{align*}
\]

Derive \( Y(\omega) \) in terms of \( X(\omega) \) and \( H(\omega) \). How is the phase of \( H(z) \) nullified? What are the overall gain and phase responses of the filter shown outlined?

29. An ideal BPF (bandpass filter) has gain 1 over \( 0.125\pi \leq |\omega| \leq 0.375\pi \) and zero gain at all other frequencies. Show that the filter’s impulse response can be expressed as the product of \( \cos(n\pi/4) \) and the impulse response of an ideal LPF (lowpass filter).

30. Sketch roughly the gain of the LTI filters that have the following pole-zero patterns.