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Nonradiating sources, the Aharonov-Bohm effect, and the question of measurability of electromagnetic potentials

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[1] A new characterization of nonradiating (NR) sources is derived that is based on electromagnetic potentials. In the new description a hierarchy of NR sources is systematically created that includes certain nonlocalized NR sources having the property that their curl is localized. The important class of spatially localized NR sources, whose fields vanish everywhere in the exterior of the source, corresponds to a special case of the general theory. The new NR source developments are discussed in connection with the question of measurability of electromagnetic potentials as enabled by the Aharonov-Bohm (A-B) effect, whereby quantum mechanical effects of the potentials can be observed in regions of vanishing electromagnetic fields but nonvanishing electromagnetic potentials. A necessary condition is derived for an electrodynamic A-B effect in the exterior of a spatially localized NR source. By exploring this condition, it is concluded that for time-varying, information-carrying fields (as required, e.g., in communications and remote sensing applications) the required A-B conditions of vanishing fields and nonvanishing potentials are not possible in the exterior of a NR source; i.e., electrodynamically, if the fields vanish everywhere outside the source, then the potentials also vanish there. This does not necessarily hold under static conditions in which nontrivial potentials with physically observable quantum effects can exist in the exterior of a source having zero external fields. *INDEX TERMS:* 0619 Electromagnetics: Electromagnetic theory; 0634 Electromagnetics: Measurement and standards; *KEYWORDS:* Aharonov-Bohm effect, nonradiating sources, electromagnetic potentials

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1. Introduction

[2] In classical electromagnetic theory the electromagnetic potentials ($\mathbf{A}$, Φ) are introduced mainly as useful mathematical constructs to compute the true force fields ($\mathbf{E}$, $\mathbf{B}$). Only the fields are thought to be physically real. In contrast, in quantum physics the potentials can play, under certain circumstances, even a more fundamental role than the corresponding fields. In fact, it is known that an electron can be influenced in a physically measurable way, e.g., in the form of a quantum-mechanical wavefunction phase shift, in regions of vanishing electromagnetic fields ($\mathbf{E} = 0$, $\mathbf{B} = 0$) but nonvanishing electromagnetic potentials ($\mathbf{A} \neq 0$, $\Phi \neq 0$). For example, quantum mechanics tells us that if an electron in an electromagnetic field region is split into two alternative trajectories, say, $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$, then an ($\mathbf{A}$, Φ)-dependent relative phase shift [*Lee et al.*, 1992]

$$\Delta\phi(t) = \phi_2(t) - \phi_1(t) = \left(\frac{e}{\hbar c}\right)\left[\int_{\mathbf{x}_2(t)} d\mathbf{x}_2(t)\cdot\mathbf{A}(\mathbf{x}_2(t), t) - \frac{c}{e}\int_0^t dt_2\Phi(\mathbf{x}_2(t_2), t_2) - \int_{\mathbf{x}_1(t)} d\mathbf{x}_1(t) \cdot\mathbf{A}(\mathbf{x}_1(t), t) + \frac{c}{e}\int_0^t dt_1\Phi(\mathbf{x}_1(t_1), t_1)\right] \quad (1)$$

appears between the Schrödinger wavefunctions $\Psi_1(\mathbf{r}, t) = \Psi_1^{(0)}(\mathbf{r}, t)e^{i\phi_1}(t)$ and $\Psi_2(\mathbf{r}, t) = \Psi_2^{(0)}(\mathbf{r}, t)e^{i\phi_2}(t)$ associated with each path; $\Psi_1^{(0)}(\mathbf{r}, t)$ and $\Psi_2^{(0)}(\mathbf{r}, t)$ are reference electron wavefunctions associated with the first and second trajectories, respectively, in the absence of

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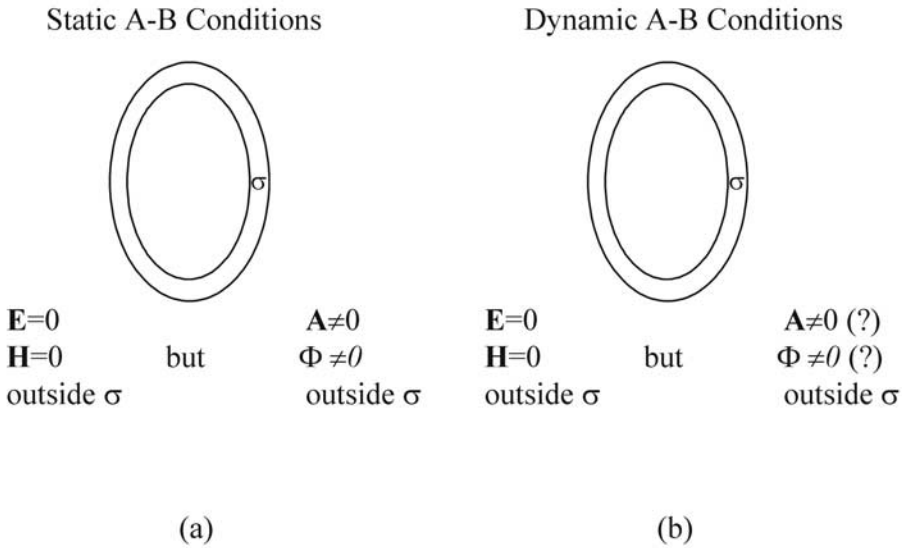

Figure 1. (a) Conceptual illustration of the postulated A-B conditions in the static case. A source contained within the multiply connected source region σ produces vanishing static fields but nonvanishing static potentials outside its support σ. (b) Corresponding electrodynamic picture. Are the postulated A-B conditions (i.e., vanishing fields, nonvanishing potentials, outside σ) possible (or not) under general electrodynamic conditions?

electromagnetic potentials ($\mathbf{A} = 0$, $\Phi = 0$); e is the electron charge, h is Planck's constant and c is the speed of light. The phase shift in Equation (1) is physically measurable, e.g., it can be observed by carrying out electron interference experiments [*Peshkin and Tonomura*, 1989]. More importantly, this form of phase shift can occur even in the extreme situation where the electron traveling paths $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are entirely inside regions of vanishing electromagnetic fields and nonvanishing electromagnetic potentials. This situation is known to occur, e.g., in the exterior of certain static toroidal solenoids having vanishing external magnetostatic fields but nonvanishing external magnetostatic potentials [*Carron*, 1995]. Under such circumstances that were first postulated by *Aharonov and Bohm* [1959] (see Figure 1a) and later verified experimentally in electron interference experiments [*Peshkin and Tonomura*, 1989], it appears that the potentials (as opposed to the fields) are the physically relevant electromagnetic entities. This effect is known as the Aharonov-Bohm (A-B) effect and applies in different forms to other fields, e.g., the gravitational field [*Harris*, 1996].

[3] The A-B effect has been the subject of more than 1400 papers to date, and in recent years has been proposed as the basis of novel devices that claim to measure the electromagnetic vector potential **A** directly [*Lee et al.*, 1992; *Gelinas*, 1984]. *Lee et al.* [1992] proposes the measurement of the potentials associated with a light beam by letting the light beam suffer total internal reflection from a crystal surface; the resulting evanescent electromagnetic potentials that emanate from the crystal are deduced via Equation (1) with electron interference experiments. *Gelinas* [1984] proposes the use of a superconducting Josephson junction, the tunneling current of which can be shown to depend (again, via Equation (1)) on the value of the vector potential around the junction.

[4] Although a few papers have addressed the A-B phase shift under time-varying, electrodynamic conditions [see, e.g., *Lee et al.*, 1992], most have dealt with static versions of the effect [see, e.g., *Aharonov and Bohm*, 1959]. The main focus has been on showing that, even in the extreme case of vanishing electromagnetic fields, one can measure quantum-mechanical effects of nonvanishing electromagnetic potentials such as those produced in the exterior of magnetostatic toroidal solenoids, infinite solenoids, and similar field-confining structures. Thus, the emphasis has been on showing how in certain static situations, the electromagnetic interaction can be mediated locally only by means of the potentials, and not by the corresponding zero fields. It is the enforcing of locality in the electron-electromagnetic field interactions that automatically forces one to attach a special physical significance to the potentials

whenever the A-B conditions ($\mathbf{E} = 0$, $\mathbf{B} = 0$, $\mathbf{A} \neq 0$, $\Phi \neq 0$) are met.

[5] However, the ideas for measuring the electromagnetic potentials contained in works by *Lee et al.* [1992] and *Gelinas* [1984] suggest the possibility of carrying out such measurements by means of the A-B effect also under time-varying conditions. From an engineering standpoint, this immediately leads to a number of fundamental and practical questions. For example, from the points of view of communications and remote sensing, one naturally wonders whether anything fundamentally new can be obtained by using devices that measure the potentials, e.g., by means of the A-B effect, as opposed to more conventional devices (antennas) that sense the electric and magnetic field vectors. The central goal of this paper is thus to elucidate the role of time-dependent potentials in more general electrodynamic versions of the A-B effect. The question raised in the electrodynamic case is schematically illustrated in Figure 1b. Ultimately we wish to clarify whether the potentials and the A-B effect can (or cannot) yield anything fundamentally new in communications and remote sensing applications with time-varying, information-carrying electromagnetic fields. Methodologically, we use a new nonradiating (NR) source [*Devaney and Wolf*, 1973] presentation from the point of view of electromagnetic potentials to examine the realizability of the postulated A-B conditions.

2. Nonradiating Sources and the Aharonov-Bohm Conditions

[6] To observe the A-B effect one must create first a source that generates vanishing fields and nonvanishing potentials in a region of interest where the A-B experiments will be carried out. Spatially localized NR sources are essential for this purpose, particularly in the context of communications and remote sensing applications. These sources generate vanishing fields everywhere outside their region of support [*Devaney and Wolf*, 1973], whether the support is simply or multiply connected. Of particular interest for communications and remote sensing applications is the possibility of observing electrodynamic A-B effects in the exterior of a NR source. If this were possible, then information could be transmitted remotely from a transmitter (or an imaging object) to a receiver by means of the potentials even in the absence of radiation fields.

[7] To motivate the matter further, we note that it was Bohm himself (with Weinstein) [*Bohm and Weinstein*, 1948] who gave renewed impetus in 1948 to the old idea of using NR sources to model stable particles and atoms. Thus, Bohm knew of and contributed to the theory of NR sources in electrodynamics. However, throughout the A-B literature, little reference is made to NR sources or electrodynamic conditions; and particular attention is given to magnetostatic situations.

[8] Here we use a general NR source formalism to show that quantum-mechanical effects of the potentials in the exterior of a spatially localized NR source can arise only if a certain static condition is met. Only then may a special physical significance be attributed to the potentials in the sense originally intended by *Aharonov and Bohm* [1959]. In contrast, time-varying effects of NR potentials in the exterior of a NR source are simply not possible. Thus, the possibility is ruled out of using the A-B effect for new, secure communications or object interrogation with vanishing fields and nonvanishing potentials. For example, A-B experiments with time-varying fields of the type described by *Lee et al.* [1992] and *Gelinas* [1984] simply do not convey electrodynamically additional information that is not already available from the corresponding field measurements.

3. Nonradiating Sources and Their Potentials

[9] Current distributions that do not radiate (NR sources) [*Devaney and Wolf*, 1973] have received attention since the early days of electromagnetic theory, particularly in connection with models of atoms and electrons and with questions of the electromagnetic self-force and radiation-reaction [see *Goedecke*, 1964, and references therein]. These NR sources have also received attention in the inversion disciplines where they arise naturally as members of the null space of the mapping from the source (scatterer) to the field [*Bleistein and Cohen*, 1977]. Various tools such as multipole expansions, Fourier and Radon transforms, and Green function techniques [*Devaney and Wolf*, 1973; *Bleistein and Cohen*, 1977; *Marengo et al.*, 1999; *Marengo and Ziolkowski*, 1999] have been employed to characterize NR sources. Here we derive a new alternative description, based on electromagnetic potentials.

[10] We consider a hierarchy of NR current distributions which encompasses classes of NR sources not considered before, including certain nonlocalized NR current distributions. The latter class of NR sources is of interest in extended particle models (e.g., an extended NR electron cloud [*Bohm and Weinstein*, 1948]). A fundamental physical application that has been suggested from time to time [*Devaney and Wolf*, 1973; *Bohm and Weinstein*, 1948; *Goedecke*, 1964] is to create extended atom models with stable atomic states corresponding to NR source modes.

[11] We consider first longitudinal current distributions. They form the simplest class of NR sources. Later we describe the more general class of NR current distributions with localized curl that encompasses the special cases of transverse NR current distributions with

localized curl and localized NR current distributions. The developed classification leads to interesting new relations for the fields, the potentials and the transverse and longitudinal components of a source that lend themselves to isolating the role of the potentials for the objectives of the present study.

[12] Henceforth we shall employ the usual relations for the electromagnetic fields and potentials in the Coulomb gauge as given, e.g., by *Jackson* [1975]. We summarize these relations for the reader's convenience in Appendix A of this paper. Note that, in the Coulomb gauge, the vector potential is transverse, i.e., $\mathbf{A}(\mathbf{r}) = \mathbf{A}_T(\mathbf{r})$. Hence we shall refer to $\mathbf{A}(\mathbf{r})$ as the transverse vector potential.

3.1. Longitudinal Current Distributions

[13] It follows from Equations (A5), (A7), and (A8) in Appendix A of this paper that, for a longitudinal current distribution $\mathbf{J}_L(\mathbf{r})$, the transverse vector potential $\mathbf{A}(\mathbf{r}) = 0$ so that the magnetic field $\mathbf{H}(\mathbf{r}) = 0$. Since there is no magnetic field, a longitudinal current distribution generates no radiation fields. The quasi-static electric field is related to the scalar potential $\Phi(\mathbf{r})$ and to $\mathbf{J}_L(\mathbf{r})$ via

$$\mathbf{E}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = \frac{4\pi}{i\omega}\mathbf{J}_L(\mathbf{r}). \tag{2}$$

3.2. Nonradiating Current Distributions With Localized Curl

[14] We consider next the class of NR current distributions that have a localized curl. First we show that the magnetic field produced by the most general NR current distribution whose curl is localized within a certain simply or multiply connected region σ (such that $\nabla \times \mathbf{J}(\mathbf{r}) = 0$ for $\mathbf{r} \notin \sigma$) must vanish everywhere outside σ. Second, we consider the electric field and the transverse vector potential of a special type of NR current distributions with localized curl: transverse ones. Finally we complete our description of the fields and potentials associated with the most general NR current distribution with localized curl by combining the results corresponding to the longitudinal and transverse cases.

[15] Consider a NR current distribution $\mathbf{J}(\mathbf{r})$ whose curl is localized within a certain region σ. It follows from Equation (A1) that

$$\left(\nabla^2 + k^2\right)\mathbf{H}(\mathbf{r}) = -\frac{4\pi}{c}\nabla \times \mathbf{J}(\mathbf{r}) \tag{3}$$

so that when $\nabla \times \mathbf{J}(\mathbf{r})$ is localized

$$\left(\nabla^2 + k^2\right)\mathbf{H}(\mathbf{r}) = 0 \quad \text{if } \mathbf{r} \notin \sigma. \tag{4}$$

Now we note that, since we require $\mathbf{J}(\mathbf{r})$ to be NR, its magnetic field $\mathbf{H}(\mathbf{r})$ must decay faster than $1/r$ outside of σ. It follows immediately from Equation (4) and a theorem on solutions of the homogeneous Helmholtz equation that decay sufficiently rapidly at infinity [see *Müller*, 1969, pp. 87–88] that the magnetic field $\mathbf{H}(\mathbf{r})$, produced by a NR current distribution $\mathbf{J}(\mathbf{r})$ whose curl is localized within σ, will vanish everywhere outside σ, i.e.,

$$\mathbf{H}(\mathbf{r}) = 0 \quad \text{if } \mathbf{r} \notin \sigma. \tag{5}$$

[16] If the NR current is longitudinal (so that σ is the empty set), the connection between the fields, potentials, and currents remains the same as in Section 3.1. On the other hand, if the NR current distribution is transverse and has a localized curl, it follows from Equation (5) and the fourth of Equation (A1) that the electric field

$$\mathbf{E}(\mathbf{r}) = \frac{4\pi}{i\omega}\mathbf{J}_T(\mathbf{r}) \quad \text{if } \mathbf{r} \notin \sigma. \tag{6}$$

By substituting from Equation (6) into Equation (A7), one then obtains the desired connection between the transverse vector potential and the NR transverse current:

$$\mathbf{A}(\mathbf{r}) = -\frac{4\pi c}{\omega^2}\mathbf{J}_T(\mathbf{r}) \quad \text{if } \mathbf{r} \notin \sigma. \tag{7}$$

[17] We can now combine the longitudinal and transverse current results to describe the most general NR current distribution with localized curl. In particular, we note by superposition that the longitudinal and transverse parts can be treated separately, and their fields can be superposed to evaluate the total fields. By means of this procedure, the total electric and magnetic fields and transverse vector potential generated by a NR current distribution $\mathbf{J}(\mathbf{r})$ whose curl is localized within σ are found from Equations (2, 5, 6, 7) to be

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= -\nabla\Phi(\mathbf{r}) + \frac{4\pi}{i\omega}\mathbf{J}_T(\mathbf{r}) = \frac{4\pi}{i\omega}[\mathbf{J}_L(\mathbf{r}) + \mathbf{J}_T(\mathbf{r})] \\ &= \frac{4\pi}{i\omega}\mathbf{J}(\mathbf{r}) \qquad \text{if } \mathbf{r} \notin \sigma,\end{aligned} \tag{8}$$

$$\mathbf{H}(\mathbf{r}) = 0 \quad \text{if } \mathbf{r} \notin \sigma, \tag{9}$$

and

$$\mathbf{A}(\mathbf{r}) = -\frac{4\pi c}{\omega^2}\mathbf{J}_T(\mathbf{r}) \quad \text{if } \mathbf{r} \notin \sigma \tag{10}$$

outside of the curl's region of localization σ. Next we apply these relations to the important special case of a localized NR current distribution.

3.3. Localized Nonradiating Current Distributions

[18] Although it is actually a special case of the NR current distributions considered in Section 3.2, the local-

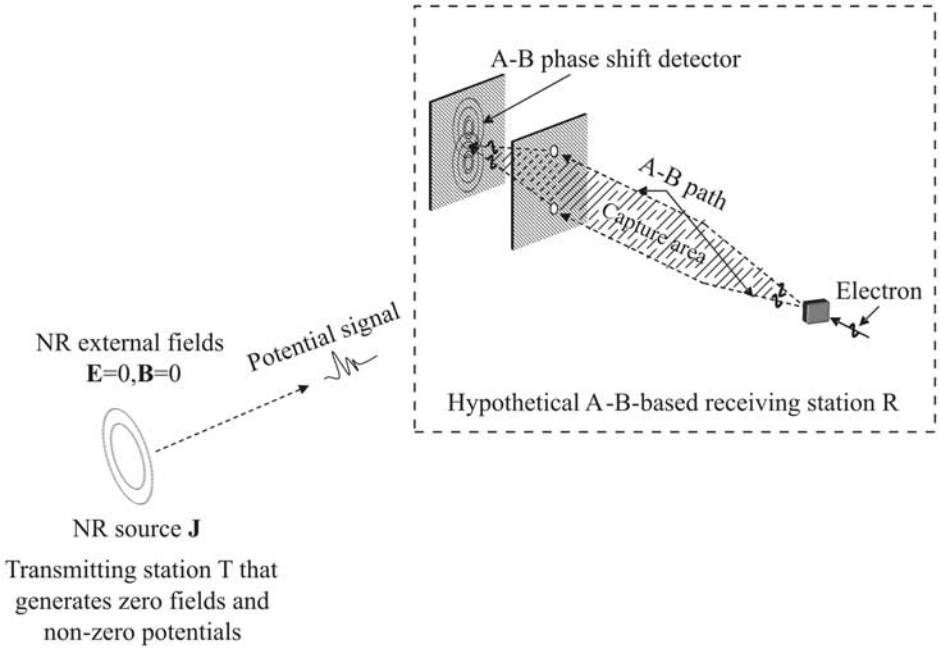

Figure 2. Hypothetical secure communications by NR potentials. The transmitting station T consists of a NR source that generates vanishing external fields but nontrivial potential signals in the direction of a remote receiving station R. The receiving station uses an A-B effect apparatus to detect the transmitted potential signals.

ized NR current distribution is considered separately because of its importance in practical applications. In this case, $\mathbf{J}(\mathbf{r}) = 0$ if $\mathbf{r} \notin \sigma$ (where, again, σ can be simply or multiply connected) so that from Equations (8) and (9) the electric and magnetic fields vanish outside of the support of the current, i.e.,

$$\mathbf{E}(\mathbf{r}) = 0, \quad \mathbf{H}(\mathbf{r}) = 0, \quad \text{if } \mathbf{r} \notin \sigma. \tag{11}$$

This result was derived first by *Devaney and Wolf* [1973]. It establishes the vanishing of the electromagnetic field produced by a localized NR current distribution outside its region of localization.

4. Nonradiating Gauge and the Aharonov-Bohm Effect

[19] The question of interest now arises: "Is there a gauge choice for which the external scalar and vector potentials produced by a localized NR source vanish?", or, in contrast, "Can the potentials influence the physics outside of a NR source region?". Aharonov and Bohm taught us to be especially careful when addressing these questions. For this purpose, we consider next two distinct possibilities.

4.1. A Communication Scenario

[20] Figure 2 illustrates schematically the first possibility. We picture a transmitting station T consisting of a hypothetical time-varying NR source that generates vanishing external fields and nonvanishing external potentials, along with a receiving station R consisting of a potential-measuring device based on the A-B effect. The question of interest is whether the receiving station R can (or cannot) acquire signals contained in the potentials produced by the NR source at the transmitting station T. It turns out (we shall show this in Section 4.2) that this question can be addressed in the usual way, i.e., by asking whether a gauge transformation of the form (refer to Equations (A1, A5, A6, A7))

$$\begin{aligned}\Phi'(\mathbf{r}) &= \Phi(\mathbf{r}) + i\frac{\omega}{c}\chi(\mathbf{r}) = 0\\ \mathbf{A}'(\mathbf{r}) &= \mathbf{A}(\mathbf{r}) + \nabla\chi(\mathbf{r}) = 0\end{aligned} \tag{12}$$

exists that suppresses the potentials. The answer to the latter question is found to be "Yes", i.e., a NR gauge exists that eliminates the potentials. In particular, for a

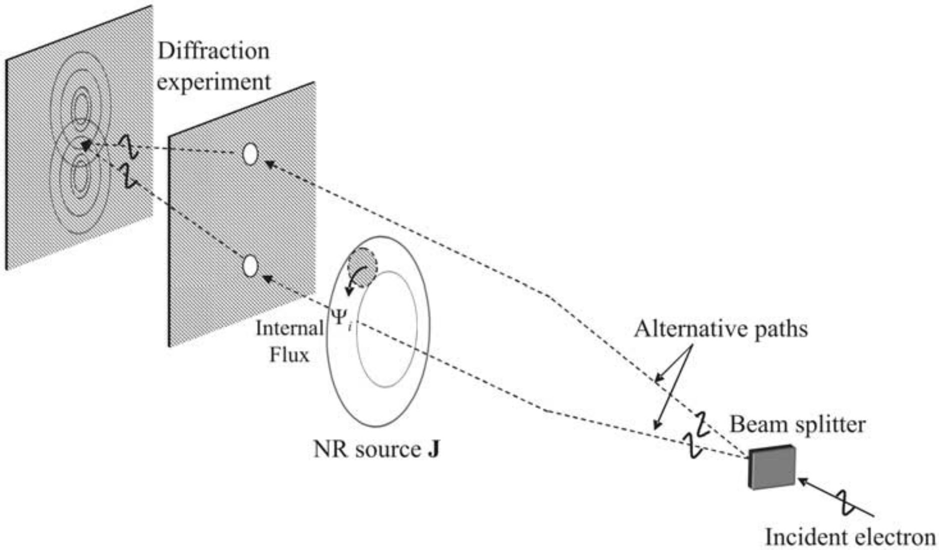

Figure 3. Hypothetical remote sensing by NR potentials. Information about a NR source is extracted by measuring the A-B phase shift associated with split electron paths around the NR source.

localized NR source, the choice $\Phi' = 0$ for the scalar potential in Equation (12) yields from Equations (A3,10)

$$\mathbf{A}'(\mathbf{r}) = \mathbf{A}(\mathbf{r}) + i\frac{c}{\omega}\nabla\Phi(\mathbf{r})$$
$$= -\frac{4\pi c}{\omega^2}[\mathbf{J}_T(\mathbf{r}) + \mathbf{J}_L(\mathbf{r})] = 0 \quad \text{if } \mathbf{r}\notin\sigma. \tag{13}$$

In this NR gauge, both potentials $\mathbf{A}'(\mathbf{r})$ and $\Phi'(\mathbf{r})$ are thus seen to vanish everywhere outside the NR source region.

[21] Note that in this context the localized NR source is very unique because of the particular form of the associated transverse vector potential $\mathbf{A}(\mathbf{r})$ in Equation (10). This feature of the NR source played a key role in the above cancellation of the associated external potentials. In contrast, for a localized radiating source, $\mathbf{H}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \neq 0$ outside the source's support. Consequently, $\mathbf{A}(\mathbf{r})$ cannot be of the form Equation (10) for a radiating source (recall that $\nabla \times \mathbf{J}_T(\mathbf{r}) = 0$ if $\mathbf{r} \notin \sigma$).

[22] Summarizing, the above result, Equations (12,13), rules out any possibility of using the strategy illustrated in Figure 2 for new secure communications by NR potentials. This conclusion will become more evident after investigating next the more general remote sensing scenario depicted in Figure 3.

4.2. A Remote Sensing Scenario

[23] Figure 3 depicts a yet more tricky scenario. Here one considers the same NR source as shown in Figure 2. However, unlike the situation in Figure 2, the electron path integrals are now allowed to "chain" the NR source. Thus they can cross potentially nonzero magnetic fluxes created in the interior of the NR source. The question of practical interest is whether one can extract NR source information contained in the source's internal magnetic fluxes by measuring the perhaps nontrivial, external NR potentials.

[24] To address this problem, one is forced to consider the path integrals (co-chains) that determine the electron phase shift associated with the A-B effect [*Lee et al.*, 1992; *Peshkin and Tonomura*, 1989; *Carron*, 1995;

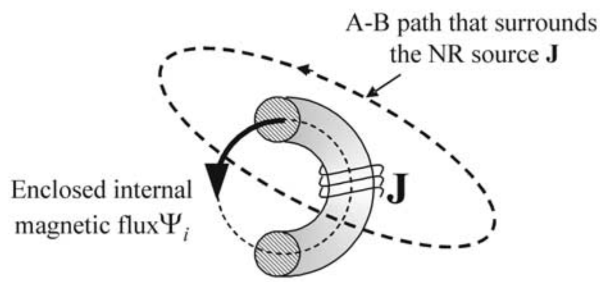

Figure 4. Schematization of the relevant A-B path integral and magnetic internal flux for the NR source in Figure 3. The figure suggests a toroidal-like NR source to ease comprehension, but the general considerations apply to the most general localized NR source.

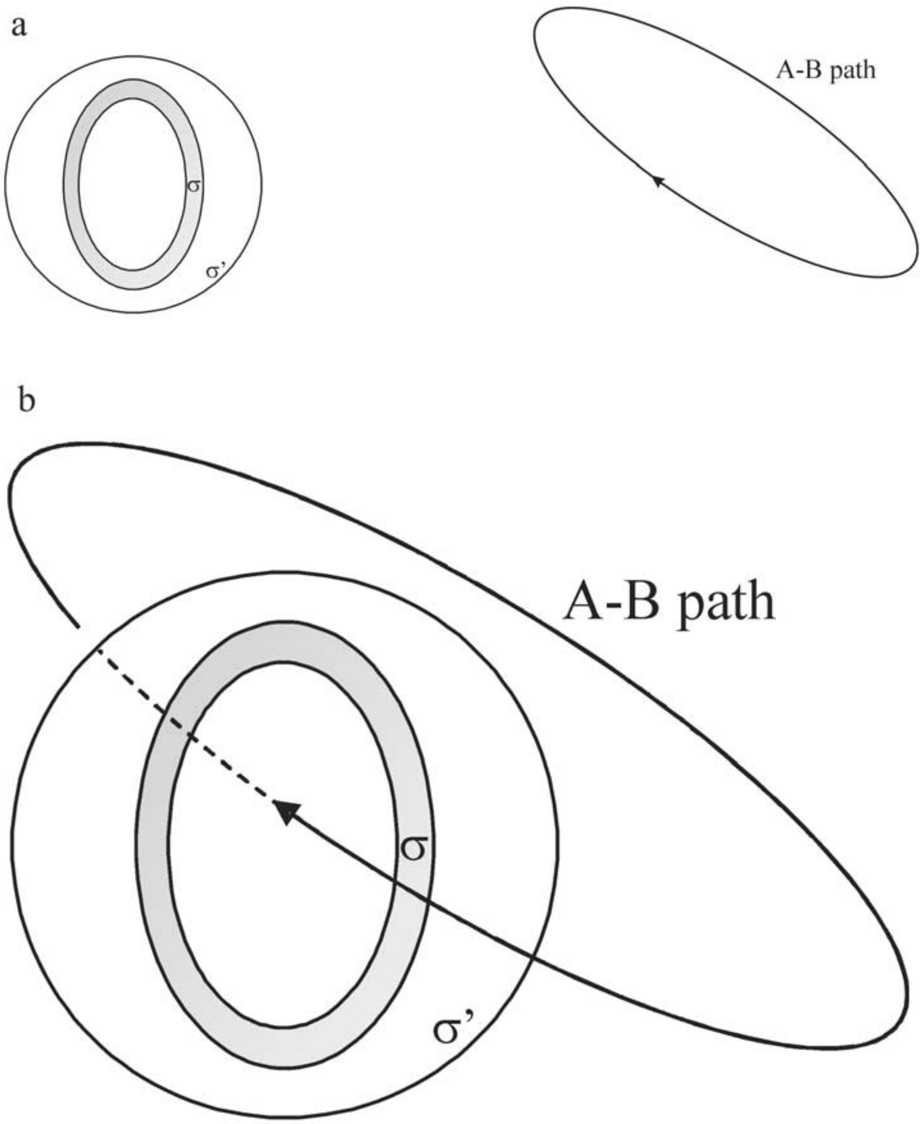

Figure 5. Schematization of the geometrical relations between the NR source support σ, a simply connected region σ′ completely enclosing the NR source support, and the path integrals for the communications and remote sensing examples in Figures 2 and 3, respectively. (a) Communications scenario. The path does not penetrate the simply connected domain σ′ enclosing the NR source support σ. There are no net magnetic fluxes enclosed by the A-B path shown; therefore, no A-B effect is measured. (b) Remote sensing scenario. In this case the path must necessarily penetrate a simply connected domain σ′ enclosing the NR source support σ. Thus the path can enclose internal magnetic fluxes inside the NR source as shown in Figure 4. These fluxes are responsible for the A-B phase shift.

Aharonov and Bohm, 1959]. It is not hard to show from Equation (1) that for A-B experiments in the exterior of a spatially localized NR source, the A-B phase shift is determined by the closed-path integral $\oint_C d\mathbf{x} \cdot \mathbf{A}(\mathbf{x}, t)$ where C is a path that surrounds the NR source. The other, Φ-path integral, vanishes identically for electron paths in the exterior of a NR source. We note from Equations (A1, A5, A7) that, in general, if C is an A-B path that surrounds (with no loss of generality) only one NR source and S_i is an interior surface of that NR source through which one can measure the total magnetic flux $\Psi_i(t)$, where $C_i = \partial S_i$ is the boundary of that interior surface, then the path integral

$$\oint_C d\mathbf{x}\cdot\mathbf{E}(\mathbf{x},t) = \oint_{C_i} d\mathbf{x}\cdot\mathbf{E}(\mathbf{x},t) = -\frac{\partial}{\partial t}\int_{S_i} d\mathbf{S}\cdot\mathbf{B}(\mathbf{x},t) = -\frac{\partial}{\partial t}\Psi_i(t) = -\frac{1}{c}\frac{\partial}{\partial t}\oint_{Ci} d\mathbf{x}\cdot\mathbf{A}(\mathbf{x},t). \quad (14)$$

This connects the electric field co-chain directly with the time derivative of the corresponding NR source vector potential co-chain. Since the fields vanish everywhere outside the NR source, the enclosed magnetic flux is contributed only by the internal fluxes inside the source. Figures 3 and 4 illustrate schematically the relevant A-B path and its associated enclosed magnetic flux Ψ_i.

[25] For a NR source, $\mathbf{E}(\mathbf{r}, t) = 0$ everywhere outside the source support for all times t. Then for any path taken outside a NR source, expression (14) reduces to the fundamental result

$$\oint_{C_i} d\mathbf{x}\cdot\mathbf{A}(\mathbf{x},t) = \Psi_i(t) = \text{constant}, \quad (15)$$

i.e., the co-chain of the NR source vector potential must be a constant for all time. This "static" condition is necessary and applies to the most general, time-varying, localized NR source. In particular, note that if the co-chain value is zero at $t = 0$ as it would be for the general electrodynamic case, i.e., for those cases that can vary with time, then it will be zero for all time. It is only for a static NR source that this co-chain value can be nonzero for $t = 0$, hence, for all time. Thus, only for static NR fields, i.e., those that cannot vary with time, the total internal fluxes available for the relevant noninvasive A-B experiments can be nonzero constants.

[26] The fundamental result Equation (15), therefore, tells us that it is not possible to measure time-dependent, information-carrying aspects of the external NR potentials. Consequently, no electrodynamic information about the source can be detected in the external potentials.

[27] The triviality of the NR potentials described by Equations (12, 13) is now evident in the usual A-B terms. In particular, we note that the formulation leading to Equations (12,13) implicitly assumes that the magnetic flux crossing the A-B electron paths is exactly zero. Such paths do not "invade" the vicinity of the source region (see Figure 5a). Now, since the fields vanish everywhere outside the NR source, it follows at once that the relevant A-B magnetic fluxes are zero for the situation in Figure 5a. This situation is also perfectly addressed locally, i.e., in differential form, by enforcing $\nabla \times \mathbf{A}(\mathbf{r}) = \mathbf{H}(\mathbf{r}) = 0$ for $\mathbf{r} \notin \sigma$, as we required, in fact, in the formulation leading to Equations (12, 13).

[28] On the other hand, the situation depicted in Figures 3, 4 and 5b shows that for the remote sensing application the A-B path essentially "enters" a simply connected region σ' enclosing the NR source support σ. In this case, the A-B measurements can involve internal fluxes of the NR source. Therefore, care must be exercised in evaluating the possible physical significance of the potentials as has been known since the time of *Aharonov and Bohm*'s [1959] original paper. In this case the approach employed in connection with Equations (12,13) is incomplete. Instead, one must investigate the A-B path integrals. By using this general approach, the possibility of observing quantum-mechanical effects of the potentials was found in the present paper (see the discussion in Equation (15)) to be very limited. In particular, only static effects were found to be potentially measurable. One concludes that the A-B effect cannot be used for communications or imaging applications, both of which require dynamic information.

[29] Finally, a connection is worth making to a paper [*Afanasiev and Stepanovsky*, 1995] that presents the opposite view. *Afanasiev and Stepanovsky* [1995] provides a number of examples of time-dependent NR sources with supposedly nonvanishing external potentials. The NR sources in *Afanasiev and Stepanovsky* [1995] are infinitesimally small, and are confined to the origin. They do not involve multiply connected regions and, therefore, cannot induce A-B effects. In other words, the question of measurability of potentials associated with such sources can be addressed directly with the gauge transformation approach presented in Equations (12,13). After some manipulations, one finds that the external potentials of the examples of *Afanasiev and Stepanovsky* [1995] vanish trivially with the NR gauge transformation in Equations (12,13). Finally, the authors of that study argue that perhaps the finite counterparts of their infinitesimal NR sources can exhibit time-dependent A-B effects. However, this contradicts the necessary static condition derived here, Equation (15). This result establishes in the most general case that A-B effects associated with NR potentials are possible only in static situations.

5. Conclusions

[30] In this paper, we presented a new description of NR current distributions from the point of view of electromagnetic potentials. We considered first certain nonlocalized NR current distributions. We then specialized the general results to localized NR current distributions. In the process, we arrived at an interesting hierarchy of NR current distributions.

[31] The general NR source results presented here are relevant to studies and patents addressing the question of (quantum) measurability of electromagnetic potentials

[*Lee et al.*, 1992; *Gelinas*, 1984]. These results conclusively show that electrodynamically not only the fields but also the associated potentials are unobservable everywhere in the exterior of a spatially localized NR source. On the other hand, if the source does radiate, then not only the potentials but also the fields are necessarily nonzero. However, since the electrodynamic fields and potentials can both be expressed in terms of the other, it is obviously questionable whether any device, be it quantum-mechanical or classical, that claims to measure the electromagnetic potentials can actually do so. It follows that in the electrodynamic case encountered, e.g., in communications and imaging systems, measurements of the potentials automatically also measure the fields, and vice versa. We arrive at the fundamental conclusion that only under the static condition derived in this paper, Equation (15), as encountered in the vast majority of A-B experiments [*Peshkin and Tonomura*, 1989], can the potentials possess a measurable physical significance in the exterior of a spatially localized NR source.

Appendix A: Review

[32] In Gaussian system of units, the Maxwell equations in free space reduce, under time-harmonic conditions, to [*Jackson*, 1975]

$$\begin{aligned} \nabla \cdot \mathbf{E}(\mathbf{r}) &= 4\pi\rho(\mathbf{r}) \\ \nabla \cdot \mathbf{H}(\mathbf{r}) &= 0 \\ \nabla \times \mathbf{E}(\mathbf{r}) &= i\frac{\omega}{c}\mathbf{H}(\mathbf{r}) \\ \nabla \times \mathbf{H}(\mathbf{r}) &= \frac{4\pi}{c}\mathbf{J}(\mathbf{r}) - i\frac{\omega}{c}\mathbf{E}(\mathbf{r}). \end{aligned} \quad \text{(A1)}$$

In Equation (A1), $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are, respectively, the space-dependent parts of the time-harmonic electric and magnetic fields $\mathbf{E}(\mathbf{r}, t) = \Re\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$ and $\mathbf{H}(\mathbf{r}, t) = \Re\{\mathbf{H}(\mathbf{r})e^{-i\omega t}\}$, where $\Re$ denotes the real part, $\mathbf{r}$ and t denote the position and time, respectively, and ω is the angular frequency of oscillation. In addition, c is the speed of wave propagation. The terms $\rho(\mathbf{r}) = \nabla \cdot \mathbf{J}(\mathbf{r})/(i\omega)$ and $\mathbf{J}(\mathbf{r})$ are, respectively, the space-dependent parts of the time-harmonic charge and current distributions $\rho(\mathbf{r}, t) = \Re\{\rho(\mathbf{r})e^{-i\omega t}\}$ and $\mathbf{J}(\mathbf{r}, t) = \Re\{\mathbf{J}(\mathbf{r})e^{-i\omega t}\}$. The source $\mathbf{J}(\mathbf{r})$ can be written as [*Van Bladel*, 1993]

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_L(\mathbf{r}) + \mathbf{J}_T(\mathbf{r}) \quad \text{(A2)}$$

where $\mathbf{J}_L(\mathbf{r})$ and $\mathbf{J}_T(\mathbf{r})$ are, respectively, the longitudinal and transverse parts of $\mathbf{J}(\mathbf{r})$. They are given, respectively, by the curl-free and divergence-free components

$$\begin{aligned} \mathbf{J}_L(\mathbf{r}) &= -i\frac{\omega}{4\pi}\nabla\Phi(\mathbf{r}) \\ \mathbf{J}_T(\mathbf{r}) &= \nabla \times W(\mathbf{r}) \end{aligned} \quad \text{(A3)}$$

where

$$\begin{aligned} \Phi(\mathbf{r}) &= -\frac{i}{\omega}\int d^3r'\frac{\nabla\cdot\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \\ W(\mathbf{r}) &= \frac{1}{4\pi}\int d^3r'\frac{\nabla\times\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \end{aligned} \quad \text{(A4)}$$

[33] By means of the usual procedure, next we write

$$\mathbf{H}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \quad \text{(A5)}$$

where in the Coulomb gauge

$$\nabla \cdot \mathbf{A}(\mathbf{r}) = 0. \quad \text{(A6)}$$

Then, the Coulomb gauge vector potential is transverse, i.e., $\mathbf{A}(\mathbf{r}) = \mathbf{A}_T(\mathbf{r})$.

[34] It is not hard to show from Equations (A1), (A2), (A3), (A5), and (A6) that the electric field is given by

$$\mathbf{E}(\mathbf{r}) = \frac{4\pi}{i\omega}\mathbf{J}_L(\mathbf{r}) + \frac{i\omega}{c}\mathbf{A}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) + \frac{i\omega}{c}\mathbf{A}(\mathbf{r}) \quad \text{(A7)}$$

where the transverse vector potential $\mathbf{A(r)}$ is related to the transverse part of the current distribution, $\mathbf{J}_T(\mathbf{r})$, by

$$\mathbf{A}(\mathbf{r}) = \frac{1}{c}\int d^3r'\mathbf{J}_T(\mathbf{r}')\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \quad \text{(A8)}$$

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