Cognitive Radio Networks with Full-duplex Capabilities

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Abstract

Inspired by recent developments in full-duplex (FD) communications, we consider an opportunistic spectrum access (OSA) network in which secondary users (SUs) are capable of partial/complete self-interference suppression (SIS). This enables them to operate in either simultaneous transmit-and-sense (TS) or simultaneous transmit-and-receive (TR) modes, with the goal of achieving improved primary user (PU) detection and/or higher SU throughput. We first consider an overlay OSA setup, and we study the TS and TR modes. We also explore the spectrum awareness/efficiency tradeoff and determine an efficient adaptive strategy for the SU link. We then consider a spectrum underlay model, with the objective of optimizing SUs' transmission powers so as to maximize the sum-throughput of K FD secondary links subject to a PU outage constraint. Operating in an FD fashion is not always efficient for SUs. Hence, we propose an optimal policy for switching between FD and half-duplex. The criteria for this policy depend mainly on the SIS capabilities of SUs. Finally, we propose a mode selection algorithm for the switching process. Numerical results indicate that operating in the TS mode can reduce the PU outage probability by up to 100% compared with the classical listen-before-talk scheme.

Index Terms

Cognitive Radio, full-duplex, self-interference cancellation, spectrum awareness/efficiency tradeoff.

I. Introduction

Until recently, the concept of *simultaneous transmission and reception* over the *same* frequency channel, i.e., operating in full-duplex (FD) mode, was deemed impossible. A traditional radio is half-duplex (HD), i.e., the radio can either transmit or receive over a given channel, but not simultaneously. The problem of achieving FD communications over the same channel is that the transmitted power from a given node is typically much larger than the received power of another signal to be captured by the same node. While the node is receiving, its transmitted signal is considered as self-interference. The infeasibility of FD communications has recently been challenged in several works (see [1] for a survey), which showed that various of self-interference suppression (SIS) techniques (e.g., RF analog cancellation, digital baseband interference cancellation, circulators, phase shifters, etc.) can be combined to enable FD communications. In fact, it has been demonstrated that a node's transmission can be suppressed at its receive chain by up to 110 dB, depending on the underlying SIS schemes [2].

In this paper, we consider an opportunistic spectrum access (OSA) system in which a secondary user (SU) employs SIS techniques to mitigate the undesirable interference of its own transmission. In this setup, SIS can be used to increase the SU's throughput by enabling bidirectional simultaneous *transmission-and-reception (TR)*. It can also be used to increase the SU's awareness of primary user (PU) activity by allowing the SU to sense while transmitting, a capability that we refer to as *transmission-sensing (TS)*. We study two main scenarios. First, we consider a spectrum overlay model (i.e., the SU must first sense the spectrum for any PU activity) and analyze the TS and TR modes at the SU. We investigate the switching policy at the SU link, taking into consideration the tradeoff between spectral efficiency (throughput) and spectrum awareness (PU detection). Our objective here is to determine the optimal action for an SU link that maximizes its throughput subject to a given PU outage probability. We also obtain the optimal sensing and transmission durations that achieve this objective. Second, we consider a spectrum underlay model and determine the optimal SU transmission powers that maximize the throughput of K secondary links, operating in FD fashion (TR mode). In this case, sensing is not used, as SUs transmit concurrently with PUs, but controlling SUs' interference onto the PU is the main challenge. We determine the SUs' optimal transmission powers, taking into account the residual self-interference and the outage constraint for the PU link.

Exploiting FD/SIS in dynamic spectrum access (DSA) systems has been discussed in [3]–[7]. In [5], we studied the overlay model of DSA systems and explored the spectrum awareness/efficiency tradeoff. In this paper, we extend our work in [5] to address the power control problem. In addition, we allow for a more realistic formulation of the SU collision probability, PU outage probability, and SU throughput. Instead of the energy-based technique used in [5], we consider waveform-based

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sensing . The authors in [3] focused on deriving the false-alarm and detection probabilities, and the PU and SU throughput under TS mode assuming energy-based detection and perfect SIS. Our work is different in that we consider waveform-based spectrum sensing and imperfect SIS. Because energy-based detection cannot differentiate between different types of signals, it exhibits poor sensing accuracy under low SIS capabilities. In [8] the authors focused on the cooperation between primary and secondary systems in cellular networks. They proposed allowing the secondary base station to relay the primary signal in an FD/TR fashion to enhance the system throughput. To enable the TS mode, the authors in [4], [9], [10] focused on studying SIS techniques from an antenna perspective. Other spectrum sharing protocols based on relaying systems can be found in [11], [12].

Power control for the spectrum underlay setting was addressed before (e.g., [13]–[16]), but only considering HD transmissions. Centralized and distributed power control algorithms were proposed in [17], where SUs utilize PU feedback to control the interference at the primary receiver. For the sake of comparison with the HD case, in our analysis of the underlay model we consider a similar power-control setup to [17]. In [18], the authors proposed an optimal dynamic power allocation scheme for FD devices that maximizes the sum-rate in a multi-user system. Our power control approach is different from [18] in that we address the problem in an OSA setting subject to a PU outage constraint. Furthermore, switching between FD and HD modes was not considered in [18], which is important for nodes with partial SIS capabilities.

The contributions of this paper are as follows. First, we derive the detection and false-alarm probabilities for the TS mode, assuming waveform-based sensing. We analyze the SU collision probability, the SU throughput, and the PU outage probability for both TS and TR modes. Based on our analysis, we compare the performance of the two modes with the traditional HD Listen-Before-Talk scheme (also refereed to as *transmission-only* (TO) mode). Second, we study the sensing/throughput tradeoff for SUs in both TS and TR modes. For both modes, we determine the "optimal" sensing and transmission durations that maximize the SU throughput subject to a constraint on the PU outage probability. Third, we explore the spectrum awareness/efficiency tradeoff that arises due to the competing goals of minimizing the collision probability with the PU (TS mode) and maximizing the SU throughput (TR mode). Given this tradeoff, we determine an adaptive strategy for the SU link that enhances its throughput subject to a given outage probability. Fourth, considering a spectrum underlay setting, we study the power control problem for SUs that are capable of perfect/imperfect SIS and that operate in FD fashion. Our objective is to find the optimal SU transmission powers that maximize the sum-throughput of K FD secondary links, subject to a PU outage constraint. Fifth, we determine the optimal policy for SUs to switch between TR and TO modes.

The rest of the paper is organized as follows. The system model is described in Section II. In Section III we study waveform-based sensing for the TS mode and formulate the collision/outage probabilities for both TS and TR modes. The sensing/throughput tradeoff and the spectrum awareness/efficiency tradeoff are discussed in Section IV. In Section V, we study the power control problem for the underlay model. Numerical results are given in Section VI, followed by conclusions in Section VII.

II. System Model and Operation Modes

A. System Model

As shown in Figure 1, we consider an OSA network where SUs opportunistically access PU-licensed channels. SUs have partial/complete SIS capability, allowing them to transmit and receive/sense at the same time. Let χ_i be a factor that represents the degree of SIS at an SU node i, $\chi_i \in [0,1]$. Specifically, χ_i is the ratio between the residual self-interference and the original self-interference before suppression. If $\chi_i = 0$, SIS is perfect; otherwise, the SU can only suppress a fraction $1-\chi_i$ of its self-interference (imperfect SIS). For example, if the residual self-interference is 1% of the power of the original self-interference signal, $\chi_i = \sqrt{0.01} = 0.1$. χ_i may differ from one node to another, depending on the employed SIS technique.

We assume that interference between different SU links is resolved by implementing an appropriate multiple access scheme (e.g., [19], [20]). For SU i, let P_i denote its transmission power. We consider a path-loss channel model, where the channel gain between a transmitter i and a receiver j at distance d_{ij} is given by $h_{ij} = A\, d_{ij}^{-\eta}$. Here, A is a frequency-dependent constant and η is the path-loss exponent.

The PU activity on a given channel (hence, channel availability for the SU) is characterized by an alternating busy/idle (ON/OFF) process. Let the ON and OFF durations be denoted by T_{ON} and T_{OFF}, with corresponding probability distributions f_{ON} and f_{OFF}, and means $\bar{T}_{ON}$ and $\bar{T}_{OFF}$, respectively. A PU/SU collision occurs whenever an SU transmission overlaps with a PU transmission. However, the PU/SU may still be able to decode uncorrupted packets in the non-overlapping periods [21]. Hence, in defining the SU collision probability and the PU outage probability, we consider the ratio of the overlapping duration of the SU/PU transmissions to the total transmission duration. We also assume a saturated traffic scenario, i.e., the SU always have data to transmit.

Let p be the SU belief that the PU is idle, $p \in [0,1]$. The SU decides the optimal action according to this belief, which is updated after each SU action. Since the PU ON/OFF periods are typically much longer than an SU transmission period, we ignore the small probability that the PU switches its state multiple times during a single SU transmission. Specifically, we only consider the case where the PU may switch its state at most once during a single SU transmission. In the analysis, we use bold-font letters to denote vectors. The symbols $\mathrm{E}[.]$, $\mathrm{Var}[.]$, and $F(.)$ indicate the expectation, variance, and CDF of random variables, respectively.

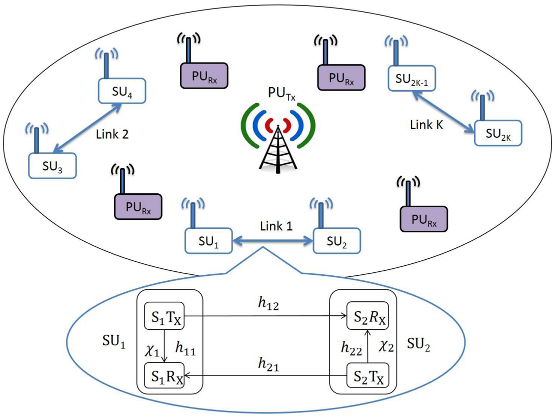
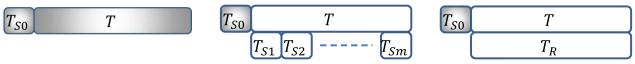


Fig. 1. System model for an OSA network. Each SU i consists of a transceiver with a given SIS factor χ_i $(0 \leq \chi_i \leq 1)$.



(a) TO mode (b) TS mode (c) TR mode

Fig. 2. Modes of operation for the SU.

B. SU Modes of Operation

1) Transmission-Only (TO) Mode: As shown in Figure 2(a), in the TO mode the SU senses the spectrum for a duration T_{S0} (which we refer to as *HD sensing*) and then carries out data transmission. The transmission duration is denoted by T.

2) Transmission-Sensing (TS) Mode: To check channel availability, the SU will initially sense in a HD fashion for a duration T_{S0}, as shown in Figure 2(b). Based on the sensing outcome, the SU will decide whether to transmit for T seconds or not. If it decides to transmit, it will continue to sense for the return of a PU. This sensing process may be split into m (consecutive) short FD sensing periods T_{Si}, $i = 1, 2, \ldots, m$. After each T_{Si}, the SU decides whether the PU is active or not. The motivation behind this approach is to account for the tradeoff between sensing efficiency and timeliness in detecting PU activity. On the one hand, increasing the sensing duration improves the sensing efficiency. However, such an increase implies delaying the time to make a decision regarding the presence/absence of PU activity. Thus, in the TS mode, we have a total of $m + 1$ sensing durations. If at the end of any given sensing period, PU activity is detected, the SU aborts its transmission.

3) Transmission-Reception (TR) Mode: Instead of sensing while transmitting, the SU may receive data from its peer SU while transmitting to that same peer, as shown in Figure 2(c). As before, an initial sensing period of length T_{S0} is needed to determine channel availability. Let T_R be the reception duration. Without loss of generality, we assume that $T_R = T$.

III. Sensing Metrics and Outage/Collision Probabilities

A. Waveform-based Spectrum Sensing in the TS Mode

Due to its simplicity, energy-based spectrum sensing has been studied extensively in literature. However, this technique cannot differentiate between different types of signals. In the TS mode, residual self-interference from the SU transmission can cause energy detection to wrongly indicate PU activity. Waveform-based sensing was studied in [22], [23] for the HD case. To detect the presence of a PU signal, waveform-based sensing correlates a known pattern in the PU signal (e.g., preambles or pilot symbols) with the received signal. In this section, we analyze waveform-based sensing for the TS mode. To simplify the notation, we use χ to denote the SIS factor at an arbitrary SU.

The hypothesis test of whether the channel is free or not can be formulated as follows:

$$r(n) = \begin{cases} \chi\, s(n) + w(n), & H_0 \text{ (if PU is idle)} \quad (1a) \\ l(n) + \chi\, s(n) + w(n), & H_1 \text{ (if PU is busy)} \quad (1b) \end{cases}$$

where $r(n)$ is the discretized nth sample of the received signal at the SU, $w(n)$ is additive white Gaussian noise (with variance σ_w^2), $l(n)$ is the received PU signal, and $s(n)$ is the self-interfering SU signal before carrying out SIS. $s(n)$ is assumed to be a zero-mean complex random signal with variance σ_s^2. We assume that the self-interference channel coefficient is one. Given the proximity of the transmit and receive antennas on the same RF device, this assumption is justified. We also assume that all signal samples are independent, hence $r(n)s$ are independent.

The performance of any spectrum sensing technique is quantified by the false-alarm and detection probabilities, P_f and P_d, which are the probabilities that the SU declares the sensed channel to be busy given hypothesis H_0 and H_1, respectively. A good sensing technique exhibits high P_d (to reduce collisions between SUs and PUs) and low P_f to enhance the utilization

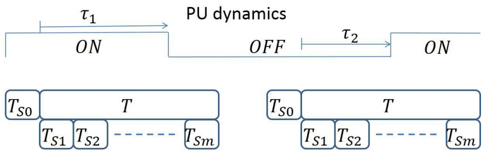


Fig. 3. Two possible scenarios that lead to a PU/SU collision in the TS mode (similar scenarios arise in the TR mode).

of the available spectrum. Let N be the number of samples taken during a given sensing period. Define the decision metric M as follows:

$$M \overset{\text{def}}{=} \text{Re}\left[\sum_{n=1}^{N} r(n)\, l^{*}(n)\right]. \tag{2}$$

In waveform-based sensing, the metric M correlates the received samples with the samples of a static part of the PU signal. The value of M is then compared with some threshold γ to determine the presence/absence of a PU signal. Substituting (1a) and (1b) into (2), we obtain the value of M for H_0 and H_1, respectively. Let M_i be the resulting M under hypothesis $H_i, i = 0, 1$. Then, M_0=Re$\left[\sum_{n=1}^{N} (\chi s(n) l^{*}(n)+w(n) l^{*}(n))\right]$ and M_1=$\sum_{n=1}^{N} |l(n)|^2$+Re $\left[\sum_{n=1}^{N} (\chi s(n) l^{*}(n)+w(n) l^{*}(n))\right]$. Hence, P_f and P_d can be expressed, respectively, as follows: $P_f = \Pr[M_0 > \gamma] = 1 - F_{M_0}(\gamma)$, and $P_d = \Pr[M_1 > \gamma] = 1 - F_{M_1}(\gamma)$, where $F_{M_0}(\gamma)$ and $F_{M_1}(\gamma)$ are the CDFs of M_0 and M_1, respectively.

***Proposition** 1:* Using the Central Limit Theorem (for large N), the pdf of M_0 can be approximated by a Gaussian distribution with mean μ_{M_0}=0 and variance $\sigma^2_{M_0}$=$\frac{N}{2}\left[\chi^2 \text{E}\, |s(n)|^2\, \text{E}\, |l(n)|^2 + \text{E}\, |w(n)|^2\, \text{E}\, |l(n)|^2\right]$ (See the Appendix for the proof).

Accordingly, $P_f = Q\left(\frac{\gamma - \mu_{M_0}}{\sigma_{M_0}}\right)$ for a large N. Substituting with μ_{M_0} and $\sigma^2_{M_0}$, we get P_f for FD sensing:

$$P_f = Q\left(\frac{\gamma}{\chi^2\sigma_s^2 + \sigma_w^2}\sqrt{\frac{2}{N\, \mathit{SNR}^{(FD)}}}\right) \tag{3}$$

where $\mathit{SNR}^{(FD)} \overset{\text{def}}{=} \text{E}\, |l(n)|^2 / \left(\chi^2 \text{E}\, |s(n)|^2 + \text{E}\, |w(n)|^2\right)$ is the SNR at the secondary receiver of the sensing node in the FD case. Note the existence of a self-interference term, along with the noise term. The number of samples, $N = T_S f_S$, is a function of the sensing duration (T_S) and the sampling rate f_S.

***Proposition** 2:* For a large N, the pdf of M_1 can be approximated by a Gaussian distribution with mean $\mu_{M_1} = N\, \text{E}\, |l(n)|^2$ and variance $\sigma^2_{M_1} = N\left[\text{E}\, |l(n)|^4 - \text{E}^2 |l(n)|^2 + \frac{1}{2}\left(\chi^2 \text{E}|s(n)|^2\, \text{E}\, |l(n)|^2 + \text{E}\, |w(n)|^2\, \text{E}\, |l(n)|^2\right)\right]$ (See the Appendix for the proof).

Hence, $P_d = Q\left(\frac{\gamma - \mu_{M_1}}{\sigma_{M_1}}\right)$. Substituting for μ_{M_1} and $\sigma^2_{M_1}$, we obtain P_d under hypothesis H_1:

$$P_d = Q\left(\frac{\gamma / \left(\chi^2\sigma_s^2 + \sigma_w^2\right) - N\, \mathit{SNR}^{(FD)}}{\sqrt{N\left[(\alpha - 1)\left(\mathit{SNR}^{(FD)}\right)^2 + \mathit{SNR}^{(FD)}/2\right]}}\right) \tag{4}$$

where α is a parameter of the PU signal that relates to its randomness [22]. For example, $\alpha = 2$ for complex Gaussian signals. For constant-amplitude signals such as BPSK and QPSK, $\alpha = 1$. Generally, $\alpha \overset{\text{def}}{=} E\, |l(n)|^4 / E^2\, |l(n)|^2$. The expressions for P_f and P_d in (3) and (4) for FD sensing converge to their HD counterparts ($\tilde{P}_f$ and $\tilde{P}_d$) under perfect SIS (i.e., $\chi = 0$):

$$\tilde{P}_f = Q\left(\frac{\gamma}{\sigma_w^2}\sqrt{\frac{2}{N\, \mathit{SNR}^{(HD)}}}\right) \tag{5}$$

$$\tilde{P}_d = Q\left(\frac{\gamma / \left(\sigma_w^2\right) - N\, \mathit{SNR}^{(HD)}}{\sqrt{N\left[(\alpha - 1)\left(\mathit{SNR}^{(HD)}\right)^2 + \mathit{SNR}^{(HD)}/2\right]}}\right) \tag{6}$$

where $\mathit{SNR}^{(HD)} \overset{\text{def}}{=} \text{E}\, |l(n)|^2 / \text{E}\, |w(n)|^2$ is the SNR at the secondary receiver of the sensing node in the HD case. Note that the optimal sensing threshold γ^* can be determined according to the system requirements on P_f and $(1 - P_d)$.

B. SU Collision Probability

In this section, we analyze the SU collision probability for both FD modes. This probability is defined as the ratio of time overlap between PU and SU transmissions to the duration of one SU transmission period. Practically, the SU can still benefit from the uncorrupted received packets that do not fall in the overlapping period [21].

Generally, there are two possible events that could lead to a PU/SU collision, as shown in Figure 3. First, due to its imperfect sensing, the SU may wrongly decide that the PU is idle and proceed to transmit data when the PU is actually ON. Second, the SU may start transmitting while the PU is idle, but later on the PU becomes active during the SU's transmission. Both events are considered in the following analysis. Let τ_1 and τ_2 be the forward recurrence time for the PU ON and OFF periods, respectively, observed at the end of the initial sensing period T_{S0} (see Figure 3). The pdfs of τ_1 and τ_2 are given by $f_{\tau_1}(t) = \int_t^{\infty} f_{ON}(u)\,\mathrm{d}u/\bar{T}_{ON}$, and $f_{\tau_2}(t) = \int_t^{\infty} f_{OFF}(u)\,\mathrm{d}u/\bar{T}_{OFF}$. Define $P_{Si}^{(OFF)}, i = 1, 2, \ldots, m$ as the probability that the PU switches from ON to OFF during T_{Si}. Similarly, $P_{Si}^{(ON)}$ is the probability that the PU switches from OFF to ON during T_{Si}. Formally,

$$P_{Si}^{(OFF)} = F_{\tau_1}\left(\sum_{k=1}^{i} T_{Sk}\right) - F_{\tau_1}\left(\sum_{k=1}^{i-1} T_{Sk}\right) \tag{7}$$

$$P_{Si}^{(ON)} = F_{\tau_2}\left(\sum_{k=1}^{i} T_{Sk}\right) - F_{\tau_2}\left(\sum_{k=1}^{i-1} T_{Sk}\right). \tag{8}$$

1) TS Mode: Without loss of generality, we assume that if the PU is active for any part of a sensing duration $T_{Si}, i = 1, 2, \ldots, m$, then the SU's transmission during the whole T_{Si} period will be corrupted. Note that an SU's sensing duration is typically much smaller than the ON/OFF periods of the PU. Define $\boldsymbol{P_f} = [P_{f,0}\; P_{f,1}\; \ldots\; P_{f,m}]$ and $\boldsymbol{P_d} = [P_{d,0}\; P_{d,1}\; \ldots\; P_{d,m}]$ as $(m+1)$-dimensional vectors that represent the false-alarm and detection probabilities for the $m+1$ sensing periods in the TS mode. By definition, $P_{f,0} = \tilde{P}_f$ and $P_{d,0} = \tilde{P}_d$. As shown in Figure 3, there are two scenarios that lead to a collision. First, if the SU mis-detects the PU activity after T_{S0}. Although the SU has collided with the PU, it still has the opportunity to detect the PU transmission through any of the parallel sensing durations. The second scenario for collision occurs when the SU correctly decides that the PU is OFF after T_{S0}, but the PU later switches from OFF to ON during the period T. This may happen during any of the FD sensing periods T_{Si}, $i = 1, 2, \ldots, m$. The duration of the overlap between the SU and PU transmissions depends on two parameters: the probability that the PU switches its activity during period T and the outcomes of the consecutive sensing periods $T_{Si}, i = 1, 2, \ldots, m$.

***Proposition** 3:* The conditional probability that the SU collides with the PU given that the SU decides to transmit in the TS mode can be expressed as follows (see the Appendix for the proof):

$$P_{coll}^{(TS)} = \frac{(1-p)\,(1-P_{d,0})}{w} P_{C1}^{(TS)} + \frac{p\,(1-P_{f,0})}{w} P_{C2}^{(TS)} \tag{9}$$

where $w = (1-p)\left(1-\tilde{P}_d\right) + p\left(1-\tilde{P}_f\right)$ is the probability that the initial sensing process results in H_0 (i.e., the probability that the SU will attempt a transmission). $P_{C1}^{(TS)}$ is the SU collision probability given that the SU mis-detects the PU transmission after T_{S0}. It accounts for different possibilities about the PU leaving the channel during any sensing period $T_{Si}, i = 1, 2, \ldots, m$ and different corresponding sensing outcomes. $P_{C2}^{(TS)}$ is the SU collision probability under the second scenario, where the PU becomes active during T. It accounts for different cases about the PU return and different corresponding sensing outcomes. The expressions for $P_{C1}^{(TS)}$ and $P_{C2}^{(TS)}$ are shown in (10) and (11), respectively:

$$\begin{aligned} P_{C1}^{(TS)} = & \sum_{i=1}^{m} \Bigg[P_{Si}^{(OFF)} \Bigg\{ \sum_{j=1}^{m} \left(\frac{\sum_{k=1}^{\min(i,j)} T_{Sk}}{\sum_{k=1}^{j} T_{Sk}} P_{d,j\le i} \prod_{k=1}^{\min(i,j-1)} (1-P_{d,k})\, P_{f,j>i} \prod_{k=i+1}^{j-1} (1-P_{f,k}) \right) + \frac{\sum_{k=1}^{i} T_{Sk}}{\sum_{k=1}^{m} T_{Sk}} \prod_{k=1}^{i} (1-P_{d,k}) \\ & \prod_{k=i+1}^{m} (1-P_{f,k}) \Bigg\} \Bigg] + (1-F_{\tau_1}(T)) \left\{ \sum_{j=1}^{m} \left(P_{d,j} \prod_{k=1}^{j-1} (1-P_{d,k}) \right) + \prod_{k=1}^{m} (1-P_{d,k}) \right\} \end{aligned} \tag{10}$$

$$P_{C2}^{(TS)} = \sum_{i=1}^{m} \left[P_{Si}^{(ON)} \prod_{k=1}^{i-1} (1-P_{f,k}) \left\{ \sum_{j=i}^{m} \left(\frac{\sum_{k=i}^{j} T_{Sk}}{\sum_{k=1}^{j} T_{Sk}} P_{d,j\ge i} \prod_{k=i}^{j-1} (1-P_{d,k}) \right) + \frac{\sum_{k=i}^{m} T_{Sk}}{\sum_{k=1}^{m} T_{Sk}} \prod_{k=i}^{m} (1-P_{d,k}) \right\} \right] \tag{11}$$

where $P_{d,j\le i} = P_{d,j}$ if $j \le i$, otherwise $P_{d,j\le i} = 1$. Similarly, $P_{f,j>i}$ and $P_{d,j\ge i}$ are defined. The ith term in the outer-most summation of (10) represents the probability that the PU becomes idle during the sensing period T_{Si} and the corresponding overlapping ratio between the SU and PU transmissions. The last term of (10) represents the case where the PU stays ON throughout the whole SU transmission period. Note that the outcome of T_{Sm} will not affect the SU collision probability for the current SU transmission session. However, it will affect the next SU's action. Since the SU is capable of monitoring the PU activity while transmitting, it can abort its communication once such an activity is detected. As a result, the collision probability in the TS mode is smaller than that of the TR/TO modes.

2) TR Mode: Since the SU is carrying out m sensing actions while transmission in the TS mode, we derive the SU collision probability by considering different possibilities about the PU switching process during an SU action (with a precision of T_{Si}

duration). For the sake of comparison, we assume that the period T is divided into m equal durations $T_i, i = 1, 2, \ldots, m$ in the TR mode. This assumption is just to simplify the derivation, and does not have any effect on the physical SU operation. Similar to the TS mode, two scenarios can lead to a collision in the TR mode. Hence, the probability that the SU collides with the PU given that the SU decides to transmit data after T_{S0} is given in (12). Note that, according to our definition, the SU's collision probability in the TO mode is similar to that of the TR mode.

$$P_{coll}^{(TR)} = \frac{(1-p)\left(1-\tilde{P}_d\right)}{w}\left(\sum_{i=1}^{m}\left[P_{Si}^{(OFF)}\frac{\sum_{k=1}^{i}T_k}{T}\right]+(1-F_{\tau_1}(T))\right)+\frac{p\left(1-\tilde{P}_f\right)}{w}\sum_{i=1}^{m}\left[P_{Si}^{(ON)}\frac{T-\sum_{k=1}^{i-1}T_k}{T}\right]. \tag{12}$$

C. PU Outage Probability

Although the overlap duration is the same for the colliding SU and PU transmissions, the two have different collision probabilities, as their transmission durations may be different.

1) TS Mode: To illustrate the difference between the SU collision probability and the PU outage probability in the TS mode, consider the first collision scenario. According to Figure 3, the SU's decision after the instant where the PU becomes idle during T (i.e., $\tau_1 < T$) will not have any impact on the PU outage probability, as the PU ON period is already determined (in contrast to the SU collision probability).

***Proposition** 4:* The conditional PU outage probability in the TS mode given that the SU decides to transmit can be expressed as follows (see the Appendix for the proof):

$$P_{out}^{(TS)} = \frac{(1-p)(1-P_{d,0})}{w}P_{O1}^{(TS)} + \frac{p(1-P_{f,0})}{w}P_{O2}^{(TS)} \tag{13}$$

where $P_{O1}^{(TS)}$ and $P_{O2}^{(TS)}$ represent the PU outage probability under the first and second PU/SU collision scenarios, respectively. The ratio of the overlap duration to the total PU ON period is determined by the instant where the PU switches its activity during period T (if any) and the SU sensing outcomes. All these different possibilities are accounted for in $P_{O1}^{(TS)}$ and $P_{O2}^{(TS)}$, which are given in (14) and (15).

$$\begin{aligned} P_{O1}^{(TS)} = &\sum_{i=1}^{m-1}\left[P_{Si}^{(OFF)}\sum_{j=1}^{i}\left(\frac{\sum_{k=1}^{j}T_{Sk}}{\bar{T}_{ON}}P_{d,j<i}\prod_{k=1}^{j-1}(1-P_{d,k})\right)\right] \\ &+\left(1-F_{\tau_1}\left(\sum_{k=1}^{m-1}T_{Sk}\right)\right)\left\{\sum_{j=1}^{m-1}\left(\frac{\sum_{k=1}^{j}T_{Sk}}{\bar{T}_{ON}}P_{d,j}\prod_{k=1}^{j-1}(1-P_{d,k})\right)+\frac{T}{\bar{T}_{ON}}\prod_{k=1}^{m-1}(1-P_{d,k})\right\}, \end{aligned} \tag{14}$$

$$P_{O2}^{(TS)} = \sum_{i=1}^{m}\left[P_{Si}^{(ON)}\prod_{k=1}^{i-1}(1-P_{f,k})\left\{\sum_{j=i}^{m}\left(\frac{\sum_{k=i}^{j}T_{Sk}}{\bar{T}_{ON}}P_{d,j\geq i}\prod_{k=i}^{j-1}(1-P_{d,k})\right)+\frac{\sum_{k=i}^{m}T_{Sk}}{\bar{T}_{ON}}\prod_{k=i}^{m}(1-P_{d,k})\right\}\right]. \tag{15}$$

2) TR Mode: In this mode, the PU outage probability $P_{out}^{(TR)}$, given in (16), has a very similar structure to $P_{coll}^{(TR)}$ except for the replacement of the SU transmission duration by the average PU ON period. Note that the PU outage probability under the TO mode is the same as that under the TR mode due to the similarity in the sensing-transmission structure.

$$P_{out}^{(TR)} = \frac{(1-p)\left(1-\tilde{P}_d\right)}{w}\left(\sum_{i=1}^{m}\left[P_{Si}^{(OFF)}\frac{\sum_{k=1}^{i}T_k}{\bar{T}_{ON}}\right]+(1-F_{\tau_1}(T))\frac{T}{\bar{T}_{ON}}\right)+\frac{p\left(1-\tilde{P}_f\right)}{w}\sum_{i=1}^{m}\left[P_{Si}^{(ON)}\frac{T-\sum_{k=1}^{i-1}T_k}{\bar{T}_{ON}}\right]. \tag{16}$$

IV. Adaptive SU Communication Strategy

In this section, we study important tradeoffs in the TR and TS modes and introduce an adaptive strategy for the SU link.

A. Sensing/Throughput Tradeoff

First, we analyze the SU's throughput under different modes of operation. Given our definition of a successful SU transmission (portion of the SU transmission duration where no overlap between the SU and PU transmissions take place), we formulate the SU throughput as follows:

1) TS Mode: The SU may gain some throughput during the non-overlapping portions of T. To compute the SU throughput under the TS mode, we need to consider two cases. First, if the SU mis-detects the PU activity, it will not gain any throughput, unless the PU completes its transmission before the end of T. Second, if the SU correctly identifies an idle channel, it will gain $\log(1 + SNR_{TS})$, where $SNR_{TS} = P_i |h_{ij}|^2 / \sigma_j^2$ is the SNR at a receiving SU node j from a transmitting SU node i under the TS mode. However, this throughput gain may be reduced by the PU transmission if the PU decides to access the same channel currently used by the SU. To formulate the SU throughput under the two possible scenarios, we need to address different cases where the PU switches its activity during any of the parallel sensing periods T_{Si} and consider different outcomes from the imperfect sensing durations.

***Proposition** 5:* The total SU throughput under the TS mode is as follows (See the Appendix for the proof):

$$R_{TS}=\left[\frac{(1-p)\left(1-\tilde{P}_d\right)}{w}R_{TS}^{(1)}+\frac{p\left(1-\tilde{P}_f\right)}{w}R_{TS}^{(2)}\right]\log\left(1+SNR_{TS}\right) \quad (17)$$

where $R_{TS}^{(1)}$ and $R_{TS}^{(2)}$ are the ratios of the non-overlapping durations (between the SU and PU transmissions) to the summation of the initial sensing period and the "actual" SU transmission period under the first and second collision scenarios, respectively. Note that the actual transmission period in the TS mode does not have to be exactly T as the SU may abort communication if any PU activity is detected. The expressions for $R_{TS}^{(1)}$ and $R_{TS}^{(2)}$ are given as follows:

$$R_{TS}^{(1)}=\sum_{i=1}^{m-1}\left[P_{Si}^{(OFF)}\prod_{k=1}^{i}\left(1-P_{d,k}\right)\left\{\sum_{j=i+1}^{m}\left(\frac{\sum_{k=i+1}^{j}T_{Sk}}{\sum_{k=0}^{j}T_{Sk}}P_{f,j}\prod_{k=i+1}^{j-1}\left(1-P_{f,k}\right)\right)+\frac{\sum_{k=i+1}^{m}T_{Sk}}{\sum_{k=0}^{m}T_{Sk}}\prod_{k=i+1}^{m}\left(1-P_{f,k}\right)\right\}\right], \quad (18)$$

$$\begin{aligned}R_{TS}^{(2)}=&\sum_{i=2}^{m}\left[P_{Si}^{(ON)}\left\{\sum_{j=1}^{m}\left(\frac{\sum_{k=1}^{\min(i-1,j)}T_{Sk}}{\sum_{k=0}^{j}T_{Sk}}P_{f,j<i}\prod_{k=1}^{\min(i-1,j-1)}\left(1-P_{f,k}\right)P_{d,j\geq i}\prod_{k=i}^{j-1}\left(1-P_{d,k}\right)\right)\right.\right.\\&\left.\left.+\frac{\sum_{k=1}^{i-1}T_{Sk}}{\sum_{k=0}^{m}T_{Sk}}\prod_{k=1}^{i-1}\left(1-P_{f,k}\right)\prod_{k=i}^{m}\left(1-P_{d,k}\right)\right\}\right]+\left(1-F_{\tau_2}(T)\right)\left\{\sum_{j=1}^{m}\left(\frac{\sum_{k=1}^{j}T_{Sk}}{\sum_{k=0}^{j}T_{Sk}}P_{f,j}\prod_{k=1}^{j-1}\left(1-P_{f,k}\right)\right)+\frac{T}{T_{S0}+T}\prod_{k=1}^{m}\left(1-P_{f,k}\right)\right\}.\end{aligned} \quad (19)$$

2) TR Mode: The benefit of using SIS in this mode is to achieve higher SU throughput by enabling bidirectional communications over the same channel. The total SU throughput, shown in (20), is the sum of the throughput of the two directions,

$$R_{TR}=G\left[\frac{(1-p)\left(1-\tilde{P}_d\right)}{w}\sum_{i=1}^{m}\left[P_{Si}^{(OFF)}\frac{T-\sum_{k=1}^{i}T_k}{T+T_{S0}}\right]+\frac{p\left(1-\tilde{P}_f\right)}{w}\left(\sum_{i=2}^{m}\left[P_{Si}^{(ON)}\frac{\sum_{k=1}^{i-1}T_k}{T+T_{S0}}\right]+\left(1-F_{\tau_2}(T)\right)\frac{T}{T+T_{S0}}\right)\right] \quad (20)$$

where $G=\log\left(1+SNR_{TR}^{(j)}\right)+\log\left(1+SNR_{TR}^{(i)}\right)$ is the SU throughput gain, $SNR_{TR}^{(j)}=P_i\left|h_{ij}\right|^2/\left(\sigma_j^2+\chi_j^2P_j\left|h_{jj}\right|^2\right)$ is the SNR in the TR mode at SU node j for a transmission from SU node i, h_{jj} is the channel gain from transmitter j to receiver j at the same node (i.e., the self-interference channel), and σ_j^2 is the noise variance at node j. R_{TR} is basically formulated by multiplying the bidirectional SU throughput by the ratio of the non-overlapping SU transmission duration to the total initial sensing plus transmission durations. Note that, the SU throughput in the HD mode, R_{TO}, can be formulated similar to R_{TR}. However, R_{TO} includes only the throughput of the forward link $\log\left(1+SNR_{TO}\right)$, where $SNR_{TO}=SNR_{TS}$.

Now that the SU throughput is obtained for each mode, we proceed to optimize the SU operation. Two optimization problems ($P1$ and $P2$) are considered, which explore the sensing/throughput tradeoff in the TS and TR modes. Specifically, our objective in $P1$ is to determine the optimal sensing and transmission durations, $\boldsymbol{T_S}$ and T, so as to maximize the SU throughput in the TS mode subject to a constraint on the PU outage probability. Formally,

$$\begin{aligned}P1:\underset{\boldsymbol{T_S},T}{\text{maximize}}\quad & R_{TS}\\ \text{subject to}\quad & P_{out}^{(TS)}\leq P_{out}^{*(TS)},\quad \sum_{i=1}^{m}T_{Si}\leq T\\ & T_{min}\leq T\leq T_{max},\quad T_{Si,min}\leq T_{Si}\leq T_{Si,max}\quad\forall i\end{aligned}$$

where $\boldsymbol{T_S}=[T_{S0}\ T_{S1}\ \ldots\ T_{Sm}]$ is an $(m+1)$-dimensional vector whose elements are the sensing durations in the TS mode and $P_{out}^{*(TS)}$ is a desired bound on the PU outage probability under the TS mode. $T_{Si,min}$, $T_{Si,max}$, T_{min}, and T_{max} represent constraints on the minimum and maximum values of the optimization parameters. $P1$ addresses the sensing/throughput tradeoff from different perspectives. First, with regard to $\boldsymbol{T_S}$, we have $m+1$ optimization parameters. For T_{S0}, there is an optimal solution that maximizes R_{TS} for any given $T_{Si}, i=1,2,\ldots,m$ and a given T. The detection probability increases monotonically with T_{S0}, ultimately satisfying constraint $P_{out}^{*(TS)}$. At the same time, increasing T_{S0} will reduce the transmission duration, hence reducing the throughput (assuming that the SU either senses or transmits over a channel). The confluence of the two factors ensures that there exists one local optimal point. Generally, the optimal values for the m sensing periods $T_{Si}, i=1,2,\ldots,m$ depend on two competing goals. First, increasing these durations will improve the sensing accuracy of the SU, and hence reduce the PU outage probability and enhance the SU throughput. Second, if these parallel sensing durations increase beyond a certain point, this may delay the SU decisions, taken at the end of the sensing durations, which may affect the SU performance negatively. With regard to the SU transmission duration, Increasing T will increase the SU throughput. However, if T is increased beyond a certain limit, it will cause a reduction in the throughput due to the high probability that the PU becomes active in the currently used channel. Note that m can be determined using the second constraint in $P1$, after determining the optimal values for $\boldsymbol{T_S}$ and T.

Next, we consider optimizing the parameters of the TR mode, namely, T_{S0} and T in $P2$, to maximize the SU throughput subject to a given PU outage probability:

$$\begin{aligned} P2: \underset{T_{S0},T}{\text{maximize}} \quad & R_{TR} \\ \text{subject to} \quad & P_{out}^{(TR)} \leq P_{out}^{*(TR)} \\ T_{min} \leq T \leq T_{max}, \quad & T_{S0,min} \leq T_{S0} \leq T_{S0,max}. \end{aligned}$$

Using a similar argument as in $P1$, it is easy to see that the sensing/throughput tradeoff exists in $P2$ w.r.t. both parameters T_{S0} and T. However, in $P2$ we only have T_{S0} instead of $\boldsymbol{T_S}$.

Since $P1$ and $P2$ are non-convex in $\boldsymbol{T_S}$ and T, an exact optimal solution cannot be obtained in polynomial time. Instead, we rely on a discretization approach to obtain a near-optimal solution using a brute-force search method. To analyze the computational complexity, we start with the simpler problem $P2$, where the decision variables are scalars. The decision region consists of the combination of the two vectors $\boldsymbol{D_1} = [T_{S0,min}, T_{S0,min}+\Delta_1, T_{S0,min}+2\Delta_1, \ldots, T_{S0,max}]$ and $\boldsymbol{D_2} = [T_{min}, T_{min}+\Delta_2, T_{min}+2\Delta_2, \ldots, T_{max}]$, where $T_{S0,min}, T_{S0,max}, T_{min}$ and T_{max} are the minimum and maximum possible values for T_{S0} and T, respectively. Δ_1 and Δ_2 are the step values for $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$, respectively. Hence, the computational complexity of $P1$ is $\mathcal{O}\left(\left(\frac{T_{S0,max}-T_{S0,min}}{\Delta_1}+1\right)\left(\frac{T_{max}-T_{min}}{\Delta_2}+1\right)\right)$ with a maximum error of Δ_1 and Δ_2 in detecting T_{S0} and T, respectively. Note that T_{S0} is in the order of hundreds of msecs, and T is also in the order of few seconds, hence the error in computing the optimal values is vanishing, assuming $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ are long enough. The decision variables for $P1$ are $\boldsymbol{T_S}$ and T. Following a similar argument as in $P2$, and assuming the lengths of the decision vectors for $T_{Si}, i = 0, 1, 2, \ldots, m$ are equal, the solving complexity of $P1$ is $\mathcal{O}\left((m+1)\left(\frac{T_{S0,max}-T_{S0,min}}{\Delta_1}+1\right)\left(\frac{T_{max}-T_{min}}{\Delta_2}+1\right)\right)$.

In problems $P1$ and $P2$, we impose a limit on the PU outage probability. By examining this outage probability, we found that it is a monotonically increasing function of the miss-detection and false-alarm probabilities. Hence, constraints on the false-alarm and detection probabilities are already taken into account in the PU outage constraint. One can simply adjust the outage probability constraint threshold in $P1$ and $P2$ to achieve a certain limit on the false-alarm (or miss-detection) probability. In fact, imposing constraints on the false-alarm and detection probabilities in $P1$ and $P2$ (without constraining the outage probability) makes the optimization problems much easier, but less informative. The reason for our choice is that the PU outage probability is much important from a PU perspective than the false-alarm and detection probabilities. In the TS mode, for example, this outage probability takes into account different possibilities for the PU to switch activity during each and every parallel sensing period.

B. Spectrum Awareness/Efficiency Tradeoff

The TS and TR modes give rise to a spectrum awareness/efficiency tradeoff. Specifically, the SU may select the TS mode to continuously sense the channel while transmitting. This way, it decreases the probability of colliding with the PU. On the other hand, the SU may decide to utilize the spectrum efficiently by transmitting and receiving data over the same channel (TR mode). Our objective is to determine the optimal strategy π^* for the SU. To do that, we consider a combined $P1/P2$ formulation as follows:

$$P3: \underset{\pi}{\text{maximize}} \quad R = \max\left(R_{SO}^*, R_{TS}^*, R_{TR}^*\right)$$

where $R_{SO}^* \overset{\text{def}}{=} 0, R_{TS}^*$, and R_{TR}^* are the optimal SU's throughput in the sensing-only (SO), TS, and TR modes, respectively. In the SO mode, the SU carries out in-band sensing-only or out-of-band sensing-only process. In some cases, when the probability that the PU becomes active is too large[1], or the SU is sure that the PU is active (e.g., multiple consecutive busy sensing outcomes), it is better for the SU to operate in the SO mode as the TR/TS modes will not satisfy the PU outage constraint (in this case, TS/TR modes will not be available in $P3$ due to the violation of the PU outage constraint). In $P1$ and $P2$, the SU calculates the maximum achievable throughput in the TS and TR modes under the specified constraints, then in $P3$ it selects the action that provides the higher throughput as long as it satisfies the outage constraint. Denote the action space of the SU by $A = \{2(\text{TR}), 1(\text{TS}), 0(\text{SO})\}$.

***Theorem** 1:* The optimal SU strategy π^* is given by (See the Appendix for the proof):

$$\pi^* = \begin{cases} 2 \quad (\text{TR}), & \text{if } p \geq p_2^* \\ 1 \quad (\text{TS}), & \text{if } p_1^* \leq p < p_2^* \\ 0 \quad (\text{SO}), & \text{if } p < p_1^* \end{cases} \tag{21}$$

[1]For some probability distributions (e.g., Gaussian, Uniform, etc), the probability that the PU becomes active increases with time

where p_1^* and p_2^* are two threshold values:

$$p_1^* = \min\left\{p : P_{out}^{(TS)} \leq P_{out}^{*(TS)}\right\} \tag{22}$$

$$p_2^* = \max\left(\min\left\{p : P_{out}^{(TR)} \leq P_{out}^{*(TR)}\right\}, \min\left\{p : R_{TR}^* > R_{TS}^*\right\}\right).$$

The scheme has a threshold-based structure that depends on the SU belief p. The SU selects the TR action if p is larger than p_2^*, as there is a high probability that the PU is idle and hence, it is better for the SU to utilize this opportunity to increases its throughput. On the other hand, if $p_1^* < p < p_2^*$, the SU will not be able to satisfy the PU outage probability constraint under the TR mode. Hence, the SU selects the TS mode to monitor the PU activity while transmitting. However, in some cases the SU has to stop transmitting over the current channel (i.e., operate in the SO mode), though it gets zero throughput. This may happen if the probability that the PU returns to the currently used channel is very high (i.e., $p < p_1^*$), in which case the SU cannot satisfy the PU outage probability constraint even if it operates in the TS mode. Note that this switching policy also accounts implicitly for the SIS capabilities of both communicating SUs. For instance, if χ_1 and χ_2 of both SUs are very low, then $R_{TR}^* > R_{TS}^*$, and the only factor that causes switching from TR to TS will be the violation of the PU outage probability constraint. On the other hand, if SUs have low SIS capabilities, then $R_{TR}^* < R_{TS}^*$ due to the high self-interference power which will dramatically decrease the node's SNR.

V. Power Optimization for an SIS-capable Underlay DSA System

A. Motivation and System Model

In this section, we consider the power optimization problem in an SIS-capable DSA system, operating according to the underlay model. In this model, SUs transmit concurrently with the PU while controlling their interference onto the PU receiver. TR$_u$ (subscript 'u' stands for underlay) is the only FD mode that SUs can use. To control their interference, SUs can adapt their transmission parameters based on feedback information they overhear from the PU receiver (e.g., ACK/NACK).

The secondary network consists of K transmitter-receiver pairs. For notation purposes, the ith SU link will be denoted by $l_i = (2i-1, 2i), i = 1, 2, \ldots, K$. For the primary link, let P_{PU} denotes the transmission power of the primary transmitter and h_p be the channel gain of the primary link. Let h_{ip} be the channel gain from SU i to the primary receiver, $i = 1, 2, \ldots, 2K$. Channels h_p and h_{ip} are modeled as Rayleigh fading channels. Hence, $|h_p|^2$ and $|h_{ip}|^2$ are exponentially distributed random variables with unit mean. In a typical DSA network, the transmission powers of PUs are much higher than those of SUs. Hence, we focus on cases where SUs do not interfere with each others. Existing literature can be used to tackle the issue of secondary-secondary interference.

For the TR$_u$ mode, the SINR δ at the primary receiver can be expressed as $\delta = \frac{P_{PU}|h_p|^2}{\sum_{i=1}^{2K} P_i |h_{ip}|^2 + \sigma_p^2}$, where σ_p^2 is the noise variance at the primary receiver. Note that in the HD case, the summation in the denominator contains K terms only. An outage to the primary link occurs when δ falls below a certain threshold δ_{th}. Even in the absence of SUs, an outage may still occur due to random channel fading. In this case, the outage probability for the primary link can be expressed as $\zeta_0 = \Pr[\delta \leq \delta_{th}] = 1 - \exp\left(-\sigma_p^2 \delta_{th}/P_{PU}\right)$. Hence, the following constraint on the PU outage probability can be imposed to maintain a certain QoS for the primary link in the presence of SUs: $\Pr[\delta \leq \delta_{th}] \leq \zeta$, where ζ is a given parameter. Although we have not considered any constraint on the maximum SU transmission power (to reduce complexity), this can be easily incorporated in the optimization problem.

There exists a tradeoff between limiting SU's interference so as to reduce the PU outage probability (i.e., operating in the TO$_u$, where only one node is active per link) and efficiently utilizing the spectrum (i.e., operating in the TR$_u$ mode, while inducing more interference). The objective of our optimization problem is to determine the optimal SUs' transmission powers that maximize the sum throughput of the bidirectional K SU links while maintaining that the PU outage probability kept below a certain threshold. Formally, the objective function is:

$$f(\boldsymbol{P}) \stackrel{\text{def}}{=} \sum_{i=1}^{2K} \log\left(1 + \frac{P_i \left|h_{i\hat{i}}\right|^2}{\sigma_{\hat{i}}^2 + \chi_{\hat{i}}^2 P_{\hat{i}} \left|h_{\hat{i}\hat{i}}\right|^2 + I_{\hat{i}}}\right) \tag{23}$$

where $\boldsymbol{P} \stackrel{\text{def}}{=} [P_1, P_2, \ldots, P_{2K-1}, P_{2K}]$ is the SU transmission power vector, $\hat{i}$ denotes the peer node of SU node i (i.e., for link $(1,2)$, if $i = 1$, then $\hat{i} = 2$ and vice versa), $I_{\hat{i}}$ and $\sigma_{\hat{i}}$ are the PU interference and the noise power at node $\hat{i}$. Although, we have only K secondary links, the summation in (23) has $2K$ terms because of the bidirectionality of each link.

B. Optimization Problem

In this section, we first convert the underlying non-convex optimization problem to a convex problem using geometric programming techniques [24]. Then, we solve it using a classical Lagrangian approach. The solution of this problem in the

FD case converges to the HD case at perfect SIS. The power control problem for SUs in the TR$_u$ mode is as follows:

$$\begin{aligned} P4\!: \underset{\boldsymbol{P}}{\text{maximize}} \quad & f(\boldsymbol{P}) \\ \text{subject to} \quad & \Pr[\delta \leq \delta_{th}] \leq \zeta. \end{aligned} \tag{24}$$

It was shown in [25] that the outage probability in the assumed Rayleigh fading environment can be expressed analytically using the following well-known result. Let $z_1, z_2, \ldots, z_n$ be independent and exponentially distributed random variables with means $1/\mu_i, \forall i$. Therefore, $\Pr\left[z_1 > \sum_{i=2}^{n} z_i + c\right] = e^{-\mu_1 c} \prod_{i=2}^{n} \left(1 + \frac{\mu_1}{\mu_i}\right)^{-1}$. Applying this result to the PU outage constraint, we get the following:

$$\begin{aligned} \Pr[\delta \leq \delta_{th}] &= \Pr\left[|h_p|^2 \leq \frac{\delta_{th}\sigma_p^2}{P_{\scriptscriptstyle PU}} + \frac{\delta_{th}\sum_{i=1}^{2K} P_i \left|h_{ip}\right|^2}{P_{\scriptscriptstyle PU}}\right] \\ &= 1 - \left[\exp\left(-\frac{\delta_{th}\sigma_p^2}{P_{\scriptscriptstyle PU}}\right) \prod_{i=1}^{2K} \left(1 + \frac{\delta_{th} P_i}{P_{\scriptscriptstyle PU}}\right)^{-1}\right]. \end{aligned}$$

To simplify the notation, define $\psi = (1-\zeta_0)/(1-\zeta)$, which is the ratio of the PU successful transmission probability given that SUs are OFF to that when SUs are ON. Then the PU outage constraint can be expressed as follows:

$$\prod_{i=1}^{2K} \left(1 + \frac{\delta_{th} P_i}{P_{\scriptscriptstyle PU}}\right) \leq \psi. \tag{25}$$

This outage constraint can be converted to a convex function using geometric programming techniques [24]. We can apply variable transformation in the log domain by letting $y_i \stackrel{\text{def}}{=} \log(P_i)$, $i = 1, 2, \ldots, 2K$, resulting in $\sum_{i=1}^{2K} \log\left(1 + \frac{\delta_{th} e^{y_i}}{P_{PU}}\right) \leq \log\psi$. Let $\boldsymbol{Y}{=}(y_1, y_2, \ldots, y_{2K})$. At high SINRs, and after applying the transformation of variables, the objective function in (24) can be reformulated as $\sum_{i=1}^{2K} \left(y_i + \log\left|h_{i\hat{i}}\right|^2 - \log\left(\sigma_{\hat{i}}^2 + \chi_{\hat{i}}^2 e^{y_{\hat{i}}} \left|h_{\hat{i}\hat{i}}\right|^2 + I_{\hat{i}}\right)\right)$. By examining this function, which is a summation of the throughputs of K secondary links, we notice that y_1, for example, (which corresponds to power P_1 in (23)) is present as the desired signal in the throughput of the forward link and as a self-interference in the throughput of the backward direction of the same first link. Rearranging the terms to include all the terms with y_i, we get $\sum_{i=1}^{2K} \left(y_i + \log\left|h_{i\hat{i}}\right|^2 - \log\left(\sigma_i^2 + \chi_i^2 e^{y_i} \left|h_{ii}\right|^2 + I_i\right)\right)$. Without loss of optimality, we ignore the constant $\log\left|h_{i\hat{i}}\right|^2$. Also, to simplify the analysis, we define the following terms which are not functions of $\boldsymbol{Y}$. Let $C_i \stackrel{\text{def}}{=} \sigma_i^2 + I_i, \forall i$ and $C \stackrel{\text{def}}{=} \delta_{th}/P_{\scriptscriptstyle PU}$. Hence, our convex optimization problem can be written in the standard form as follows:

$$\begin{aligned} \tilde{P}4\!: \underset{\boldsymbol{Y}}{\text{minimize}} \quad & f_0(\boldsymbol{Y}) = -\sum_{i=1}^{2K} \left[y_i - \log\left(C_i + \chi_i^2 e^{y_i} \left|h_{ii}\right|^2\right)\right] \\ \text{subject to} \quad & \sum_{i=1}^{2K} \log\left(1 + Ce^{y_i}\right) \leq \log\psi. \end{aligned}$$

***Lemma** 1:* Our optimization problem is now a convex problem which can be solved analytically [26].

We formulate the Lagrangian L with a multiplier $\lambda \geq 0$:

$$\begin{aligned} L(\boldsymbol{Y}, \lambda) = &-\sum_{i=1}^{2K} \left[y_i - \log\left(C_i + \chi_i^2 e^{y_i} \left|h_{ii}\right|^2\right)\right] \\ &+ \lambda \left(\sum_{i=1}^{2K} \log\left(1 + Ce^{y_i}\right) - \log\psi\right). \end{aligned} \tag{26}$$

We define the Lagrange dual function g, which yields a lower bound on the optimal value of the original problem (i.e., $g(\lambda) \leq f_0(\boldsymbol{Y}^*)$).

$$\begin{aligned} g(\lambda) = \inf_{\boldsymbol{Y}} L(\boldsymbol{Y}, \lambda) = \inf_{y_i} &-\sum_{i=1}^{2K} \left[y_i - \log\left(C_i + \chi_i^2 e^{y_i} \left|h_{ii}\right|^2\right)\right] \\ &+ \lambda \left(\sum_{i=1}^{2K} \log\left(1 + Ce^{y_i}\right) - \log\psi\right). \end{aligned}$$

***Lemma** 2:* The optimal value of y_i as a function of λ can be expressed as follows (see the Appendix for the proof):

$$y_i^*(\lambda) = \log\left(\frac{-C_iC(\lambda-1) + \sqrt{C_i^2C^2(\lambda-1)^2 + 4C_iC\lambda\chi_i^2\left|h_{ii}\right|^2}}{2C\lambda\chi_i^2\left|h_{ii}\right|^2}\right)$$
$$\forall\lambda \geq 0, i = 1, 2, \ldots, 2K.$$

The Lagrange dual problem can be formulated as follows:

$$P5: \underset{\lambda}{\text{maximize}} \quad g(\lambda)$$
$$\text{subject to} \quad \lambda \geq 0.$$

Since ψ has to be greater than 1 to give room for secondary access, the constraint of the primal problem can be satisfied with strict inequality by setting $P_i = 0, i = 1, 2, \ldots, 2K$, in (25) (i.e., slater's condition is satisfied). Hence, the duality gap between the primal and dual problems is zero (i.e., strong duality holds), and the solution of the dual problem will be the same as the primal problem.

***Theorem** 2:* The optimal power $P_i^*, i = 1, 2, \ldots, 2K$ for the ith SU operating in a FD fashion can be expressed as follows:

$$P_i^* = \frac{-C_iC(\lambda^*-1) + \sqrt{C_i^2C^2(\lambda^*-1)^2 + 4C_iC\lambda^*\chi_i^2\left|h_{ii}\right|^2}}{2C\lambda^*\chi_i^2\left|h_{ii}\right|^2} \tag{27}$$

where λ^* is the optimal solution to the Lagrange dual problem. Differentiating $g(\lambda)$ with respect to λ and equating the result to zero, we get λ^* by solving $\prod_{i=1}^{2K}(1 + CP_i^*) = \psi$ numerically.

***Corollary** 1:* The optimal transmission power for an SU operating in a FD fashion converges to the HD case at perfect SIS (See the Appendix for the proof). That is, at $\chi = 0$, $P_i^* = \frac{\psi^{\frac{1}{2K}} - 1}{C} \forall i$, which is the same as the optimal solution obtained for the HD case [17], but for $2K$ links (since we have 2 active nodes/link). On the other hand, if SUs operate in HD fashion, their optimal powers are given by $P_i^{*(HD)} = \frac{\psi^{\frac{1}{K}} - 1}{C}$, for $i = 1, 2, \ldots, K$.

C. Communication Mode Selection Algorithm (CMSA)

In the previous section, we derived the optimal transmission powers for SUs communicating in the FD TR$_u$ mode. However, operating in TR$_u$ mode is not always the best option, especially at high values of χ due to the residual self-interference. We would like to determine the threshold values for χ, which determines the optimal communication mode (TR$_u$ or TO$_u$). Let $\chi_{th}^{(i)}$ be this threshold for the ith SU, $i = 1, 2, \ldots, 2K$. Note that these thresholds depends on the estimated channels gain and noise variances. Because χ may differ from one node to another and since these thresholds are time varying, both communicating nodes should negotiate to determine the optimal operation mode and the corresponding transmission powers. This process should be repeated to update the threshold values, and the optimal mode. Hence, we introduce the following mode selection algorithm.

Consider the first SU link $l_1 = (1, 2)$, which consists of two SU nodes 1 and 2. The throughput of l_1 in the TO$_u$ and TR$_u$ modes can be expressed as follows:

$$R_{TO_u} = \log(1 + \frac{P_1^{*(HD)}\left|h_{12}\right|^2}{\sigma_2^2}) \tag{28}$$

$$R_{TR_u} = \log\left(1 + \frac{P_1^*\left|h_{12}\right|^2}{\sigma_2^2 + \chi_2^2P_2^*\left|h_{22}\right|^2}\right) + \log\left(1 + \frac{P_2^*\left|h_{21}\right|^2}{\sigma_1^2 + \chi_1^2P_1^*\left|h_{11}\right|^2}\right) \tag{29}$$

Since the SU has two operation modes, TR$_u$ and TO$_u$, the maximum secondary throughput R_u can be expressed as: $R_u = \max(R_{TR_u}, R_{TO_u})$.

***Theorem** 3:* The optimal mode selection policy is given by (see the Appendix for the proof):

$$a^* = \begin{cases} 1 \quad (\text{TR}_u), & \text{if } (\chi_1, \chi_2) < \left(\chi_{th}^{(1)}, \chi_{th}^{(2)}\right) \\ 0 \quad (\text{TO}_u), & \text{otherwise} \end{cases} \tag{30}$$

where $\left(\chi_{th}^{(1)}, \chi_{th}^{(2)}\right)$ is any point that satisfies equation (31).

This threshold curve described by (31) is obtained by equating R_{TR_u} and R_{TO_u} and finding the optimal regions for both modes. At low values of χ_1 and χ_2, it's better for the SU to operate in the TR$_u$ mode to increase its throughput. However, R_{TR_u} decreases with χ_1 and χ_2 until reaching the threshold curve, where any further increment in the values of χ_1 and χ_2 will force the SU to operate in the TO$_u$ mode.

$$q\left(\chi_{th}^{(1)},\chi_{th}^{(2)}\right)=P_2^*\left|h_{21}\right|^2\left(\sigma_2^2+\left(\chi_{th}^{(2)}\right)^2P_2^*\left|h_{22}\right|^2\right)+P_1^*\left|h_{12}\right|^2\left(\sigma_1^2+\left(\chi_{th}^{(1)}\right)^2P_1^*\left|h_{11}\right|^2\right)+P_1^*P_2^*\left|h_{12}\right|^2\left|h_{21}\right|^2$$
$$-\left(P_1^{*(HD)}\left|h_{12}\right|^2/\sigma_2^2\right)\left(\sigma_2^2+\left(\chi_{th}^{(2)}\right)^2P_2^*\left|h_{22}\right|^2\right)\left(\sigma_1^2+\left(\chi_{th}^{(1)}\right)^2P_1^*\left|h_{11}\right|^2\right)=0. \quad (31)$$

Algorithm 1 CMSA

1: Initialize: $K_{FD}=K$, $K_{HD}=0$
2: Master and slave SUs report system parameters to NC
3: NC broadcasts K_{FD} and K_{HD}
4: SU_i Calculates P_i^* in Theorem (2)
5: Master SU calculates $q\left(\chi_1,\chi_2\right)$, using (31)
if $q\left(\chi_1,\chi_2\right)>q\left(\chi_{th}^{(1)},\chi_{th}^{(2)}\right)$ **then**
  Optimal action: a^* =TR$_u$
  Master and slave SUs: Optimal power is P_i^*
else Optimal action: a^* =TO$_u$
  Master SU: Optimal power is $P_i^{*(HD)}$
  Slave SU: Optimal power is zero
end if
6: Master SU reports a^* and the optimal powers.
7: NC updates K_{FD} and K_{HD} as follows:
if a^* =TO$_u$ **then** Decrement K_{FD}, Increment K_{HD}
end if
8: Go to step 2.

***Corollary** 2:* For two communicating SUs with equal SIS capability factors (i.e., $\chi_1=\chi_2=\chi$), the following policy is optimal:

$$a^*=\begin{cases}1 \quad (\mathrm{TR}_u), & \text{if } \chi<\chi_{th}\\ 0 \quad (\mathrm{TO}_u), & \text{otherwise}\end{cases} \quad (32)$$

where χ_{th} is the point where R_{TR_u}=R_{TO_u}, which can be derived using a similar approach to that used in deriving (31).

Using theorems 2 and 3, secondary nodes can execute Algorithm 1 with the help of a network coordinator (NC) to maximize the sum-throughput. We assume that C, ψ and K are known a priori to all users. For a given secondary link, a master SU is the node that applies CMSA and negotiate with the slave node to determine the optimal communication mode. Note that in theorem 3, we only consider one way traffic from node 1 to 2 in the HD mode assuming that SU 1 is the master node at this time instant. The role of the master/slave can be exchanged between both nodes according to the traffic flow. Define K_{FD} and K_{HD} as the number of active FD and HD links, respectively. Note that $K_{FD}+K_{HD}=K$.

In Algorithm 1, a joint determination of the SUs' transmission powers and operation modes is provided. With the help of a NC (a practical example of this setup is the ECMA 392 standard, where a NC is used to organize the operation of multiple secondary links), each SU calculates its optimal transmission power (using Theorem 2), assuming that the whole secondary network will operate in an FD fashion, while maintaining a probabilistic outage constraint on the PU outage probability. Given these transmission powers and depending on the SIS capabilities of the two communicating SUs of a secondary link, the master SU will check whether operating in an FD or HD fashion will return higher link throughput (using Theorem 3). The master SU will then report the optimal operation mode and the corresponding transmission power to the slave SU. Note that the communication between the NC, master SUs, and slave SUs in this algorithm can be executed simultaneously over different sub-carriers of the control channel or in a time-slotted fashion, depending on the available resources and the used multiple access technique.

VI. Numerical Results

A. Overlay Model

Unless stated otherwise, we use the following parameters. $f_S=6$MHz, $\sigma_s^2=5$, $m=500$, $SNR^{(HD)}=-20$ dB, $\alpha=1$, $p=0.5$, T_{ON} and T_{OFF} are exponentially distributed random variables with means $\bar{T}_{ON}=\bar{T}_{OFF}=5$, and $SNR_{TO}=20$ dB.

1) Performance Metrics: We first evaluate the performance of waveform-based spectrum sensing for the FD TS mode and compare it with the energy-based sensing. Figures 4 and 5 depict P_f and P_d versus the sensing duration for different values of χ. Generally, the performance of any spectrum sensing technique expectedly improves (i.e., P_f decreases and P_d increases) with the sensing duration, as more samples are being used for PU detection. Also, as χ increases the performance

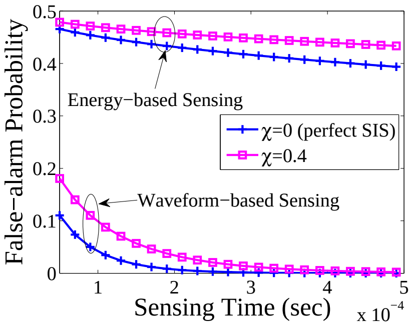


Fig. 4. False-alarm probability vs. sensing time in the FD TS mode.

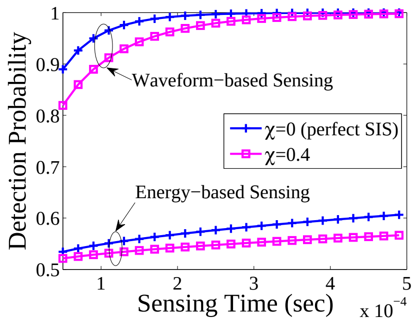


Fig. 5. Detection probability vs. sensing time in the FD TS mode.

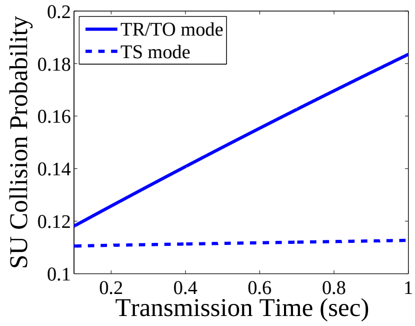


Fig. 6. SU collision probability vs. T at perfect SIS ($T_{S0} = 0.05$ ms).

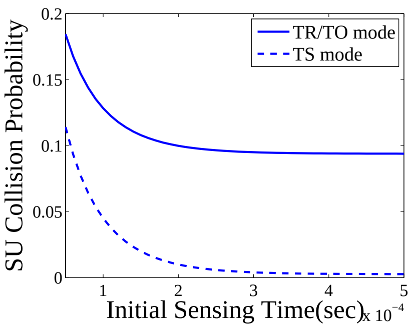


Fig. 7. SU Collision probability vs. T_{S0} at perfect SIS ($T = 1$ s).

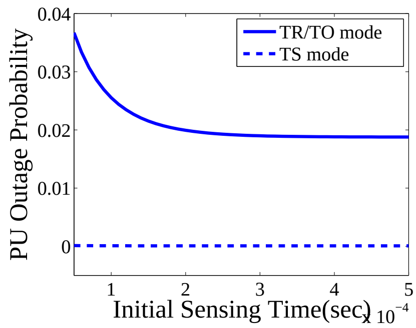


Fig. 8. PU outage probability vs. T_{S0} at perfect SIS ($T = 1$ s).

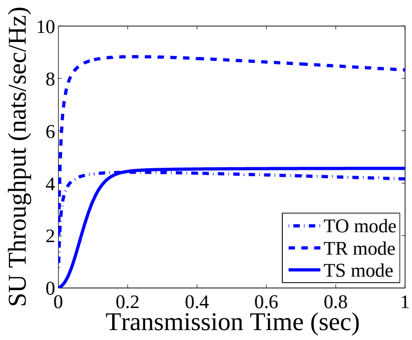


Fig. 9. SU throughput vs. T under perfect SIS ($T_{S0} = 5$ ms).

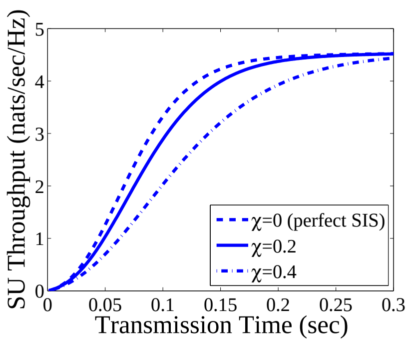


Fig. 10. SU throughput vs. T for the TS mode ($T_{S0} = 5$ ms).

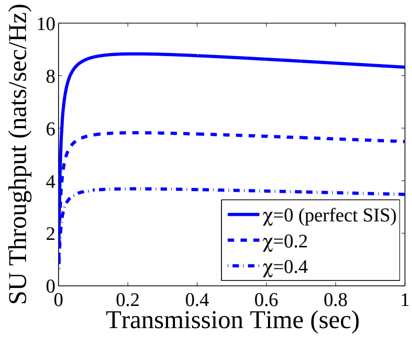


Fig. 11. SU throughput vs. T for the TR mode ($T_{S0} = 5$ ms).

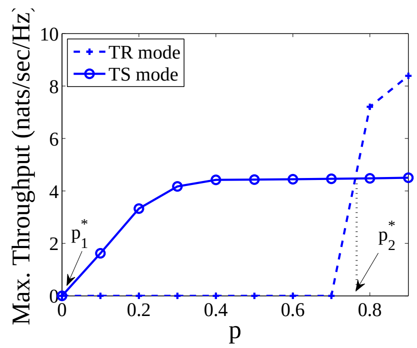


Fig. 12. SU throughput vs. p. ($\bar{T}_{ON} = 500$, $\bar{T}_{OFF} = 100$, $\chi = 0$).

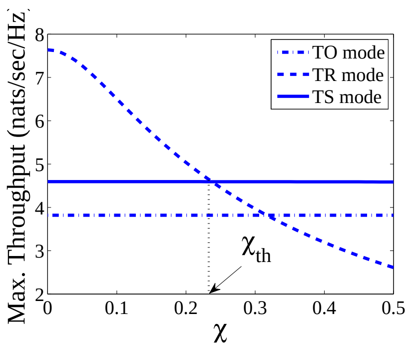


Fig. 13. Maximum throughput vs. χ (PU Outage constraint $= 4 * 10^{-6}$).

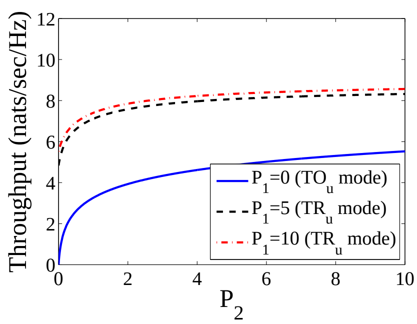


Fig. 14. SU throughput vs. P_2 at different values of P_1 for $\chi = 0.1$.

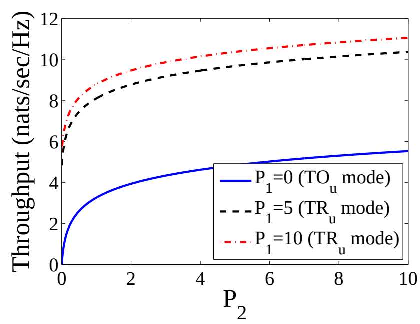


Fig. 15. SU throughput vs. P_2 at different values of P_1 for $\chi = 0$.

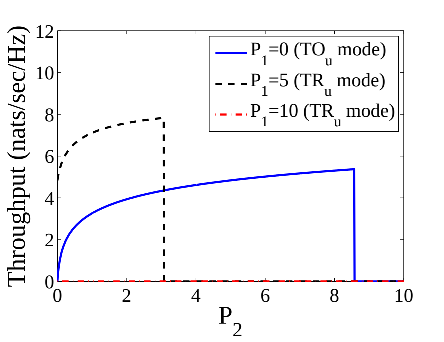


Fig. 16. SU throughput vs. P_2 at different values of P_1 for $\chi = 0.1$ (with PU outage constraint).

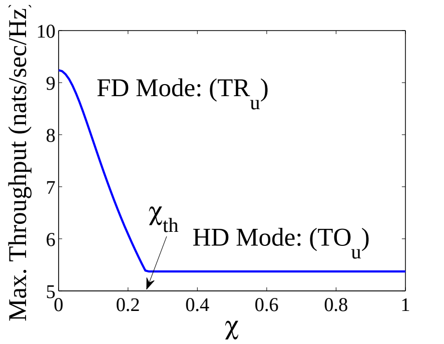


Fig. 17. Maximum throughput vs. χ for a single SU link, where both nodes have the same χ.

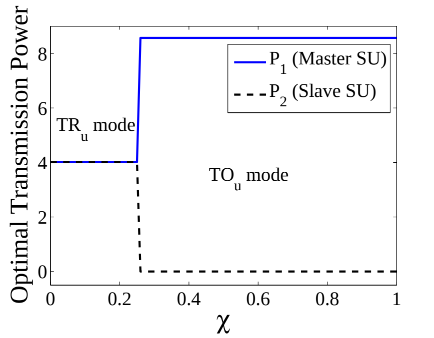


Fig. 18. Optimal SUs transmission powers vs. χ.

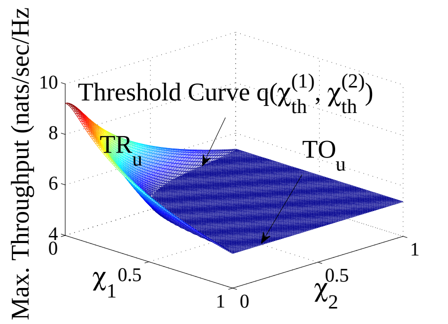


Fig. 19. Maximum SU throughput vs. χ_1 and χ_2.

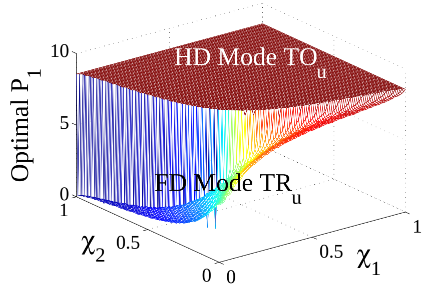


Fig. 20. Optimal transmission power for the master SU vs. χ_1, and χ_2.

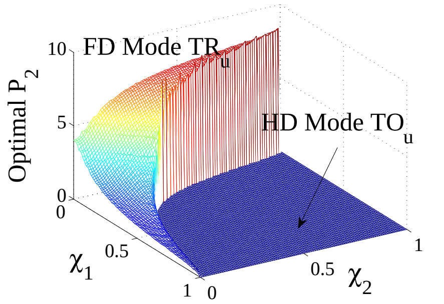


Fig. 21. Optimal transmission power for the slave SU vs. χ_1, and χ_2.

of waveform-based sensing (and similarly for energy-based sensing) degrades due the increase in the residual self-interference. At perfect SIS, P_f and P_d converge to the HD case. As shown in the Figures, SUs need about 20% longer sensing durations to achieve the same sensing accuracy of the HD mode with 20% residual self-interference.

Next, we evaluate the SU collision probability and the PU outage probability for the two FD modes (TS and TR) as well as the TO mode. As shown in Figure 6, with perfect SIS the SU can achieve a lower collision probability in the TS mode than in the TO/TR modes. $P_{coll}^{(TR)}$ increases with T due to the higher probability that the PU will become active again. This effect is negligible in the TS mode, as the SU continuously monitors PU activity while transmitting. As shown in Figure 7, the SU collision probability decreases with T_{S0} because of the increase in the number of samples taken during sensing. Figure 8 demonstrates the benefit of operating in the TS mode, where a reduction of almost 100% in the PU outage probability is possible relative to the TR/TO modes, even in the case of fast varying PU activity. Note that the PU outage probability in the TS mode is in the order of 10^{-5} (not shown in Figure 8 due to the significant difference between $P_{out}^{(TR)}$ and $P_{out}^{(TS)}$).

2) Sensing/Throughput Tradeoff: Figure 9 shows the advantage of the TR mode over other modes. For a given m, increasing T corresponds to longer $T_{Si}, i = 1, 2, \ldots, m$. At very small values of T and with perfect SIS, we notice that R_{TO} is greater than R_{TS}, which happens due to high values of P_f and $(1 - P_d)$ (which cause wrong decisions for the SU). As T increases, the SU throughput in the TS mode becomes higher than that of the TO mode, as $P_{coll}^{(TS)}$ becomes smaller. Note that increasing T initially increases the SU throughput, up to a certain point, where any further increment causes increase in the collision probability, which has a dominant (negative) effect on throughput. Figures 10 and 11 show the effect of imperfect SIS on the SU throughput. As χ increases, R_{TR} decreases due to the additional self-interference. Also, R_{TS} decreases with χ due to the poor sensing performance that occur because of the self-interference.

3) Spectrum Awareness/Efficiency Tradeoff: Next, we consider the optimization problems $P1 - P3$ with a PU outage probability constraint 10^{-9}. Figure 12 shows how the SU can adaptively switch between the TR and TS modes according to p to maximize the throughput. To show the relation between the maximum SU throughput and χ, we solve our optimization problems at different values of χ and for a PU outage probability constraint $= 4 * 10^{-6}$. As shown in Figure 13, at low χ, the best action for the SU is the TR mode. However as χ increases, the throughput achieved at the TR mode decreases due to the increase in the self-interference. In this case, the best action for the SU is the TS mode.

B. Underlay Model

We set a constraint on the maximum SU transmission power. Let $P_i \in [0, P_{max}]$, where P_{max} is the maximum transmission power for an SU. We set $\psi = 1.28$, $C_i = 0.04$, $C = 1/30$, $P_{max} = 10$ and unity channel gains.

1) Secondary throughput for FD/HD modes: We start by the case where the PU outage constraint is loose. Figure 14 shows the variation of the SU throughput with P_2 at different values of P_1 for $\chi = 0.1$. Although χ is very low, a reduction in the SU throughput occurs due to the residual self-interference than the case for perfect SIS shown in Figure 15. Figure 16 shows the variation of the SU throughput versus the SU transmission power at $\chi = 0.1$ after incorporating the PU outage constraint. Adding this constraint causes a truncation in the throughput curve because as the transmission powers of both nodes increase, the interference on the primary receiver increases. At the HD case ($P_1 = 0$ in Figure 16), the SU throughput increases with P_2 until the point that the transmission power value violates the PU outage constraint. The value of this changing point decreases as P_1 increases until it reaches zero for $P_1 = 10$.

2) CMSA for SUs with Same SIS Factor: Figure 17 shows the maximum SU throughput for an SU link at different values of χ. At low χ values, the optimal mode is the TR$_u$ mode. However, if χ exceeds a certain threshold χ_{th}, one of the SUs should keep silent (i.e., optimal mode is TO$_u$). Note also that the throughput at the TR$_u$ mode decreases with χ. Figure 18 shows the optimal transmission powers at different operation modes. At the FD mode, both nodes will have the same optimal power. If $\chi > \chi_{th}$, SUs will operate in a HD fashion, where the slave node will keep silent.

3) CMSA for SUs with Different SIS Factor: Figure 19 shows the variation of the maximum SU throughput with χ_1, and χ_2. The threshold values, $\chi_{th}^{(1)}$ and $\chi_{th}^{(2)}$, separate between the FD and HD regions, which can be represented by the threshold curve $q(\chi_{th}^{(1)}, \chi_{th}^{(2)})$ shown also in (31). For nodes with χ_1, and χ_2 that are less than the threshold values (or equivalently achieves positive q), the optimal action is the TR$_u$ mode, where the throughput in this case is a decreasing function of χ_1, χ_2. On the other hand, if χ_1, and χ_2 returns negative q, the optimal mode is the TO$_u$. Figures 20 and 21 show the optimal transmission powers for the master and slave SUs, respectively as a function of χ_1, and χ_2. At the region where SUs operate in the FD mode, the optimal transmission power vary according to the SIS capability factors of both nodes. However, when χ_1, and χ_2 go beyond the threshold values, the optimal transmission power will be constant for the master SU and zero for the slave SU.

VII. Conclusions

We proposed and studied a novel application of FD/SIS in the context of DSA systems. Two DSA models were considered: overlay and underlay. For the overlay model, we analyzed two FD modes of operation for an SU device (TS and TR). According to our results, a significant reduction (almost 100% relative to the TO mode) in the PU outage probability can be achieved under the TS mode. On the other hand, the SU throughput can almost be doubled by operating in the TR mode. We studied the effect of imperfect SIS and found that longer sensing durations are needed in the TS mode (under imperfect SIS) than that of the sensing-only phase to achieve the same performance. We studied the sensing/throughput and the spectrum awareness/efficiency tradeoffs of the new FD modes, and proposed an optimal adaptive strategy for the SU link. For the spectrum underlay model, we studied the power control problem for K FD-capable secondary links, derived their optimal transmission powers, and proposed a mode selection algorithm.

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APPENDIX

A. Proofs of Propositions 1 and 2

Proof of Proposition 1: The mean of M_0 can be expressed as follows:

$$\mu_{M_0} = \mathrm{Re}\left[\sum_{n=1}^{N} \mathrm{E}\,\left(\chi\, s(n)\, l^*(n) + w(n)\, l^*(n)\right)\right] = 0. \tag{33}$$

Since $s(n), l(n)$, and $w(n)$ are independent, the above result holds. The SU signal (and similarly for other signals) can be written as a function of the real and imaginary components as follows: $s(n) = s_r(n) + js_i(n)$. Hence, the variance of M_0 is:

$$\begin{aligned}
\sigma^2_{M_0} &= \sum_{n=1}^{N} \mathrm{Var}\,\left(\mathrm{Re}\left[(\chi\, s(n)\, l^*(n) + w(n)\, l^*(n))\right]\right) \\
&= N\left[\chi^2\left\{\mathrm{Var}\,(s_r(n)l_r(n)) + \mathrm{Var}\,(s_i(n)l_i(n))\right\} + \mathrm{Var}\,(w_r(n)l_r(n)) + \mathrm{Var}\,(w_i(n)l_i(n))\right] \\
&= N\left[\chi^2\left\{\mathrm{E}\,\left(s_r^2(n)\right)\mathrm{E}\,\left(l_r^2(n)\right) + \mathrm{E}\,\left(s_i^2(n)\right)\mathrm{E}\,\left(l_i^2(n)\right)\right\} + \mathrm{E}\,\left(w_r^2(n)\right)\mathrm{E}\,\left(l_r^2(n)\right) + \mathrm{E}\,\left(w_i^2(n)\right)\mathrm{E}\,\left(l_i^2(n)\right)\right] \\
&= \frac{N}{2}\left[\chi^2\mathrm{E}\,|s(n)|^2\,\mathrm{E}\,|l(n)|^2 + \mathrm{E}\,|w(n)|^2\,\mathrm{E}\,|l(n)|^2\right].
\end{aligned}$$

■

Proof of Proposition 2: Due to independence, the mean of M_1 is expressed as follows:

$$\mu_{M_1} = \sum_{n=1}^{N} \mathrm{E}\,|l(n)|^2 + \mathrm{Re}\left[\sum_{n=1}^{N} \mathrm{E}\,\left(\chi\, s(n)\, l^*(n) + w(n)\, l^*(n)\right)\right] = N\,\mathrm{E}\,|l(n)|^2 \tag{34}$$

The variance of M_1 can be shown to be:

$$\begin{aligned}
\sigma^2_{M_1} &= \sum_{n=1}^{N}\left[\mathrm{Var}\,\left(|l(n)|^2\right) + \mathrm{Var}\,\left(\mathrm{Re}\left[(\chi\, s(n)\, l^*(n) + w(n)\, l^*(n))\right]\right)\right] \\
&= N\left[\mathrm{Var}\,\left(|l(n)|^2\right) + \chi^2\left\{\mathrm{Var}\,(s_r(n)l_r(n)) + \mathrm{Var}\,(s_i(n)l_i(n))\right\} + \mathrm{Var}\,(w_r(n)l_r(n)) + \mathrm{Var}\,(w_i(n)l_i(n))\right] \\
&= N\Big[\mathrm{E}\,|l(n)|^4 - \mathrm{E}^2\,|l(n)|^2 \\
&\quad + \chi^2\left\{\mathrm{E}\,\left(s_r^2(n)\right)\mathrm{E}\,\left(l_r^2(n)\right) + \mathrm{E}\,\left(s_i^2(n)\right)\mathrm{E}\,\left(l_i^2(n)\right)\right\} + \mathrm{E}\,\left(w_r^2(n)\right)\mathrm{E}\,\left(l_r^2(n)\right) + \mathrm{E}\,\left(w_i^2(n)\right)\mathrm{E}\,\left(l_i^2(n)\right)\Big] \\
&= N\left[\mathrm{E}\,|l(n)|^4 - \mathrm{E}^2\,|l(n)|^2 + \frac{1}{2}\left(\chi^2\mathrm{E}\,|s(n)|^2\,\mathrm{E}\,|l(n)|^2 + \mathrm{E}\,|w(n)|^2\,\mathrm{E}\,|l(n)|^2\right)\right].
\end{aligned}$$

■

B. Proofs of Propositions 3, 4, and 5

Proof of Proposition 3: Consider the first collision scenario (mis-detecting the PU activity). The PU may switch from ON to OFF during T_{S1} which happens with probability $F_{\tau_1}(T_{S1})$. In that case, the SU may detect the PU activity right away at the end of T_{S1} and quit transmission with an overlapping ratio of one. The SU may also quit transmission at the end of T_{S2}

with probability $(1-P_{d,1})\,P_{f,2}$ and an overlapping ratio of $T_{S1}/\left(T_{S1}+T_{S2}\right)$ and so on. This happens with the following probability:

$$P_{S1}^{\text{(OFF)}}\left[P_{d,1}+(1-P_{d,1})\,P_{f,2}\frac{T_{S1}}{T_{S1}+T_{S2}}+(1-P_{d,1})\,(1-P_{f,2})\,P_{f,3}\frac{T_{S1}}{T_{S1}+T_{S2}+T_{S3}}+\ldots\right]. \tag{35}$$

The PU may also switch from ON to OFF during T_{S2}, which has the following corresponding collision probability:

$$P_{S2}^{\text{(OFF)}}\left[P_{d,1}+(1-P_{d,1})\,P_{d,2}+(1-P_{d,1})\,(1-P_{d,2})\,P_{f,3}\frac{T_{S1}+T_{S2}}{T_{S1}+T_{S2}+T_{S3}}+\ldots\right]. \tag{36}$$

Adding different possibilities of collision in the m sensing durations, we get the SU collision probability under the first scenario as follows:

$$P_{C1}^{\text{(TS)}}=\sum_{i=1}^{m}\left[P_{Si}^{\text{(OFF)}}\left\{\sum_{j=1}^{m}\left(\frac{\sum_{k=1}^{\min(i,j)}T_{Sk}}{\sum_{k=1}^{j}T_{Sk}}P_{d,j\le i}\prod_{k=1}^{\min(i,j-1)}(1-P_{d,k})\,P_{f,j>i}\prod_{k=i+1}^{j-1}(1-P_{f,k})\right)+\frac{\sum_{k=1}^{i}T_{Sk}}{\sum_{k=1}^{m}T_{Sk}}\prod_{k=1}^{i}(1-P_{d,k})\right.\right.$$
$$\left.\left.\prod_{k=i+1}^{m}(1-P_{f,k})\right\}\right]+(1-F_{\tau_1}(T))\left\{\sum_{j=1}^{m}\left(P_{d,j}\prod_{k=1}^{j-1}(1-P_{d,k})\right)+\prod_{k=1}^{m}(1-P_{d,k})\right\}.$$

Consider now the second scenario for collision, which occurs when the PU becomes active during period T. Similar to the first case, the time when the PU switches from OFF to ON and the outcomes of the sensing periods determine the overlapping ratio between the SU and PU transmissions. The PU may switch from OFF to ON during T_{S1} which happens with probability $F_{\tau_2}(T_{S1})$. In that case, the SU may detect the PU activity right away at the end of T_{S1} and quit transmission with an overlapping ratio of one. The SU may also quit transmission at the end of T_{S2} with probability $(1-P_{d,1})\,P_{d,2}$ and so on. This happens with the following probability:

$$P_{S1}^{\text{(ON)}}\left[P_{d,1}+(1-P_{d,1})\,P_{d,2}+(1-P_{d,1})\,(1-P_{d,2})\,P_{d,3}+\ldots\right]. \tag{37}$$

The PU may also switch from OFF to ON during T_{S2}, which has the following corresponding collision probability:

$$P_{S2}^{\text{(ON)}}\,(1-P_{f,1})\left[P_{d,2}\frac{T_{S2}}{T_{S1}+T_{S2}}+(1-P_{d,2})\,P_{d,3}\frac{\sum_{i=2}^{3}T_{Si}}{\sum_{i=1}^{3}T_{Si}}+(1-P_{d,2})\,(1-P_{d,3})\,P_{d,4}\frac{\sum_{i=2}^{4}T_{Si}}{\sum_{i=1}^{4}T_{Si}}+\ldots\right]. \tag{38}$$

Adding different possibilities of collision, we get the SU collision probability under the second scenario as follows:

$$P_{C2}^{\text{(TS)}}=\sum_{i=1}^{m}\left[P_{Si}^{\text{(ON)}}\prod_{k=1}^{i-1}(1-P_{f,k})\left\{\sum_{j=i}^{m}\left(\frac{\sum_{k=i}^{j}T_{Sk}}{\sum_{k=1}^{j}T_{Sk}}P_{d,j\ge i}\prod_{k=i}^{j-1}(1-P_{d,k})\right)+\frac{\sum_{k=i}^{m}T_{Sk}}{\sum_{k=1}^{m}T_{Sk}}\prod_{k=i}^{m}(1-P_{d,k})\right\}\right].$$

Putting all together, we get the conditional probability that the SU collide with the PU given that it decides to transmit under the TS mode as shown in (9).

■

Proof of Proposition 4: For the first collision scenario, the PU may switch from ON to OFF during T_{S1} which happens with probability $P_{S1}^{\text{(OFF)}}$. Regardless of the outcomes of the m sensing actions, the corresponding PU outage probability is as follows $P_{S1}^{\text{(OFF)}}\left(T_{S1}/\bar{X}\right)$. The PU may also switch from ON to OFF during T_{S2}, which has the following corresponding collision probability:

$$P_{S2}^{\text{(OFF)}}\left[P_{d,1}\frac{T_{S1}}{\bar{X}}+(1-P_{d,1})\,\frac{T_{S1}+T_{S2}}{\bar{X}}\right]. \tag{39}$$

Adding different possibilities of collision, we get the PU outage probability under the first scenario as follows:

$$P_{O1}^{\text{(TS)}}=\sum_{i=1}^{m-1}\left[P_{Si}^{\text{(OFF)}}\sum_{j=1}^{i}\left(\frac{\sum_{k=1}^{j}T_{Sk}}{\bar{T}_{ON}}P_{d,j<i}\prod_{k=1}^{j-1}(1-P_{d,k})\right)\right]$$
$$+\left(1-F_{\tau_1}\left(\sum_{k=1}^{m-1}T_{Sk}\right)\right)\left\{\sum_{j=1}^{m-1}\left(\frac{\sum_{k=1}^{j}T_{Sk}}{\bar{T}_{ON}}P_{d,j}\prod_{k=1}^{j-1}(1-P_{d,k})\right)+\frac{T}{\bar{T}_{ON}}\prod_{k=1}^{m-1}(1-P_{d,k})\right\}.$$

For the second collision scenario, the PU may switch from OFF to ON during T_{S1} which happens with probability $P_{S1}^{\text{(ON)}}$. In that case, the SU may detect the PU activity right away at the end of T_{S1} and quit transmission. The SU may also quit transmission at the end of T_{S2} with probability $(1-P_{d,1})\,P_{d,2}$ and so on. This happens with the following probability:

$$P_{S1}^{\textit{(ON)}}\left[P_{d,1}\frac{T_{S1}}{\bar{X}}+(1-P_{d,1})\,P_{d,2}\frac{T_{S1}+T_{S2}}{\bar{X}}+(1-P_{d,1})\,(1-P_{d,2})\,P_{d,3}\frac{T_{S1}+T_{S2}+T_{S3}}{\bar{X}}+\ldots\right]. \tag{40}$$

The PU may also switch from OFF to ON during T_{S2}, which has the following corresponding collision probability:

$$P_{S2}^{\textit{(ON)}}\,(1-P_{f,1})\left[P_{d,2}\frac{T_{S2}}{\bar{X}}+(1-P_{d,2})\,P_{d,3}\frac{\sum_{i=2}^{3}T_{Si}}{\bar{X}}+(1-P_{d,2})\,(1-P_{d,3})\,P_{d,4}\frac{\sum_{i=2}^{4}T_{Si}}{\bar{X}}+\ldots\right].$$

Adding different possibilities of collision, we get the PU outage probability under the second scenario as follows:

$$P_{O2}^{\textit{(TS)}}=\sum_{i=1}^{m}\left[P_{Si}^{\textit{(ON)}}\prod_{k=1}^{i-1}(1-P_{f,k})\left\{\sum_{j=i}^{m}\left(\frac{\sum_{k=i}^{j}T_{Sk}}{\bar{T}_{\textit{ON}}}P_{d,j\geq i}\prod_{k=i}^{j-1}(1-P_{d,k})\right)+\frac{\sum_{k=i}^{m}T_{Sk}}{\bar{T}_{\textit{ON}}}\prod_{k=i}^{m}(1-P_{d,k})\right\}\right].$$

Adding $P_{O1}^{\textit{(TS)}}$ and $P_{O2}^{\textit{(TS)}}$ together with the appropriate weights, we get (13), which completes the proof.

■

Proof of Proposition 5: To fully formulate the SU throughput under the first collision scenario, we need to address different cases where the PU switches from ON to OFF during T. If the PU leaves the channel during T_{S1}, the SU will gain a throughput iff it mis-detects the PU activity after T_{S1}. The quantity of throughput gained depends on the false-alarm probabilities of the remaining $m-1$ sensing periods. In that case, the ratio of non-overlapping duration to the actual SU transmission duration (plus initial sensing duration) can be written as follows:

$$P_{S1}^{\textit{(OFF)}}\,(1-P_{d,1})\left[P_{f,2}\frac{T_{S2}}{\sum_{i=0}^{2}T_{Si}}+(1-P_{f,2})\,P_{f,3}\frac{T_{S2}+T_{S3}}{\sum_{i=0}^{3}T_{Si}}+\ldots\right]. \tag{41}$$

The PU may also switch from ON to OFF during T_{S2}, which has the following ratio:

$$P_{S2}^{\textit{(OFF)}}\,(1-P_{d,1})\,(1-P_{d,2})\left[P_{f,3}\frac{T_{S3}}{\sum_{i=0}^{3}T_{Si}}+(1-P_{f,3})\,P_{f,4}\frac{T_{S3}+T_{S4}}{\sum_{i=0}^{4}T_{Si}}+\ldots\right]. \tag{42}$$

Adding different cases for the first scenario, we get the total ratio as follows:

$$R_{\textit{TS}}^{(1)}=\sum_{i=1}^{m-1}\left[P_{Si}^{\textit{(OFF)}}\prod_{k=1}^{i}(1-P_{d,k})\left\{\sum_{j=i+1}^{m}\left(\frac{\sum_{k=i+1}^{j}T_{Sk}}{\sum_{k=0}^{j}T_{Sk}}P_{f,j}\prod_{k=i+1}^{j-1}(1-P_{f,k})\right)+\frac{\sum_{k=i+1}^{m}T_{Sk}}{\sum_{k=0}^{m}T_{Sk}}\prod_{k=i+1}^{m}(1-P_{f,k})\right\}\right].$$

Assuming the SU correctly determines a channel to be idle after T_{S0}, the ratio of non-overlapping duration to the actual SU transmission duration (plus initial sensing duration) under the second collision scenario can be formulated, similarly to the first case, as follows:

$$\begin{aligned}R_{\textit{TS}}^{(2)}=&\sum_{i=2}^{m}\left[P_{Si}^{\textit{(ON)}}\left\{\sum_{j=1}^{m}\left(\frac{\sum_{k=1}^{\min(i-1,j)}T_{Sk}}{\sum_{k=0}^{j}T_{Sk}}P_{f,j<i}\prod_{k=1}^{\min(i-1,j-1)}(1-P_{f,k})\,P_{d,j\geq i}\prod_{k=i}^{j-1}(1-P_{d,k})\right)\right.\right.\\&\left.\left.+\frac{\sum\limits_{k=1}^{i-1}T_{Sk}}{\sum\limits_{k=0}^{m}T_{Sk}}\prod_{k=1}^{i-1}(1-P_{f,k})\prod_{k=i}^{m}(1-P_{d,k})\right\}\right]+(1-F_{\tau_2}(T))\left\{\sum_{j=1}^{m}\left(\frac{\sum\limits_{k=1}^{j}T_{Sk}}{\sum\limits_{k=0}^{j}T_{Sk}}P_{f,j}\prod_{k=1}^{j-1}(1-P_{f,k})\right)+\frac{T}{T_{S0}+T}\prod_{k=1}^{m}(1-P_{f,k})\right\}.\end{aligned}$$

Combining the two cases, we get the total SU throughput under the TS mode as shown in (17).

■

C. Proof of Theorem 1

To proof theorem 1, we will first introduce the following two lemmas:

***Lemma** 3:* $R_{\textit{TS}}$ is a concave and an increasing function of p.

Proof: The first-order derivative of $R_{\textit{TS}}$ can be expressed as follows:

$$R_{\textit{TS}}^{'}=\left(R_{\textit{TS}}^{(2)}-R_{\textit{TS}}^{(1)}\right)\frac{\left(1-\tilde{P}_d\right)\left(1-\tilde{P}_f\right)}{w^2}\log\left(1+\textit{SNR}_{\textit{TS}}\right). \tag{43}$$

Since $T_{\textit{ON}}$ and $T_{\textit{OFF}}$ are much larger than T. Therefore, the probability that the PU switches its state from ON to OFF (or vice versa) during T is relatively small (although we account for it in the analysis). Then, the throughput gain resulting from

$R_{TS}^{(2)}$ is higher than that of $R_{TS}^{(1)}$. Hence, $\grave{R}_{TS}$ is non-negative and therefore R_{TS} is an increasing function of p. Next we find the second-order derivative of R_{TS}.

$$R_{TS}^{''} = -2\left(R_{TS}^{(2)} - R_{TS}^{(1)}\right)\left(\tilde{P}_d - \tilde{P}_f\right)\frac{\left(1-\tilde{P}_d\right)\left(1-\tilde{P}_f\right)}{w^3}\log\left(1+SNR_{TS}\right) < 0. \tag{44}$$

Hence, R_{TS} is a concave function, which completes the proof.

■

***Lemma** 4:* R_{TR} is a concave and an increasing function of p.

Proof: Define $R_{TR}^{(1)}$ and $R_{TR}^{(2)}$ as follows:

$$R_{TR}^{(1)} = \sum_{i=1}^{m}\left[P_{Si}^{(OFF)}\frac{T-\sum_{k=1}^{i}T_k}{T+T_{S0}}\right], \tag{45}$$

$$R_{TR}^{(2)} = \sum_{i=2}^{m}\left[P_{Si}^{(ON)}\frac{\sum_{k=1}^{i-1}T_k}{T+T_{S0}}\right] + \left(1-F_{\tau_2}(T)\right)\frac{T}{T+T_{S0}}. \tag{46}$$

Hence, The SU throughput in the TR mode can be expressed as follows:

$$R_{TR} = G\left[\frac{(1-p)\left(1-\tilde{P}_d\right)}{w}R_{TR}^{(1)} + \frac{p\left(1-\tilde{P}_f\right)}{w}R_{TR}^{(2)}\right].$$

The first-order derivative of R_{TR} is as follows:

$$R_{TR}^{'} = G\left(R_{TR}^{(2)} - R_{TR}^{(1)}\right)\frac{\left(1-\tilde{P}_d\right)\left(1-\tilde{P}_f\right)}{w^2}. \tag{47}$$

Generally, the major throughput the SU gains comes from transmitting data over a free licensed channel due to the low values of the mis-detection/false-alarm probabilities. Hence, the gained throughput (or equivalently the non-overlapping ratio) under $R_{TR}^{(2)}$ is much larger than that of $R_{TR}^{(1)}$. Hence, R_{TR} increases with p. Since, the second-order derivative is negative (as shown next), R_{TR} is a concave function in p.

$$R_{TR}^{''} = -2G\left(R_{TR}^{(2)} - R_{TR}^{(1)}\right)\left(\tilde{P}_d - \tilde{P}_f\right)\frac{\left(1-\tilde{P}_d\right)\left(1-\tilde{P}_f\right)}{w^3} < 0.$$

■

Proof of Theorem 1: Since R_{TR} and R_{TS} are increasing concave functions of the belief p (lemmas 3 and 4). Also, $R_{TR} > R_{TS}$, at $p = 1$ (After all, the values of $R_{TS}^{(2)}$ and $R_{TR}^{(2)}$ are at most 1 since they are the aggregation of disjoint events. Hence, the dominating factors at $p = 1$ are G and $\log\left(1+SNR_{TS}\right)$. Note that $G > \log\left(1+SNR_{TS}\right)$). Since $P_{out}^{(TR)} > P_{out}^{(TS)}$. Therefore, R_{TR} and R_{TS} intersect together at a threshold point p_2^* (where the SU violates the PU outage constraint at the TR mode). Since $R_{TR} \geq R_{TS}$ for $p \geq p_2^*$ and $R_{TR} < R_{TS}$ for $p < p_2^*$, the first two lines of theorem 1 defines the optimal policy. Note that the intersection point at p_2^* is determined by the maximum p that either violates the PU outage probability constraint at the TR mode or the point where R_{TR} goes below R_{TS} (due to low SIS capabilities). Since R_{SO} is constant with p. Therefore, R_{SO} and R_{TS} intersects in at most one point (p_1^*), where the SU violates the PU outage probability constraint at the TS mode. If this point does not exist, then $p_1^* = 0$ and the SO region disappears. Hence, theorem 1 defines the optimal policy.

■

D. Power Optimization Proofs

Proof of lemma 2: Since,

$$g(\lambda) = \inf_{\boldsymbol{Y}} L(\boldsymbol{Y},\lambda) = \sum_{i=1}^{2K}\inf_{y_i}\left[-y_i + \log\left(C_i + \chi_i^2 e^{y_i}\left|h_{ii}\right|^2\right) + \lambda\log\left(1+Ce^{y_i}\right)\right] - \lambda\log\psi.$$

Therefore,

$$-1 + \frac{\chi_i^2 e^{y_i}\left|h_{ii}\right|^2}{C_i + \chi_i^2 e^{y_i}\left|h_{ii}\right|^2} + \frac{\lambda C e^{y_i}}{1+Ce^{y_i}} = 0$$

$$C\chi_i^2\left|h_{ii}\right|^2\lambda e^{2y_i} + C_iC(\lambda-1)e^{y_i} - C_i = 0$$

$$y_i^*(\lambda) = \log\left(\frac{-C_iC(\lambda-1)+\sqrt{C_i^2C^2(\lambda-1)^2+4C_iC\lambda\chi_i^2\left|h_{ii}\right|^2}}{2C\lambda\chi_i^2\left|h_{ii}\right|^2}\right), \qquad \forall\lambda\geq 0, i=1,2,\ldots,2K.$$

■

Proof of corollary 1: At perfect SIS, the SIS capability factor $\chi = 0$. Substituting by $\chi = 0$ in (27), we get $0/0$, which is indeterminate number. Using l'Hopital's rule, we can find the optimal secondary power at perfect SIS as follows:

$$\begin{aligned}
P_i^* &= \lim_{\chi=0}\frac{0.5\left(C_i^2C^2(\lambda^*-1)^2+4C_iC\lambda^*\chi_i^2\left|h_{ii}\right|^2\right)^{-0.5}\left(8C_iC\lambda^*\chi_i\left|h_{ii}\right|^2\right)}{4C\lambda^*\chi_i\left|h_{ii}\right|^2}\\
&= \lim_{\chi=0}\frac{C_i}{\sqrt{C_i^2C^2(\lambda^*-1)^2+4C_iC_\lambda^*\chi_i^2\left|h_{ii}\right|^2}}\\
&= \frac{1}{C(\lambda^*-1)}\\
&= \frac{\psi^{\frac{1}{2K}}-1}{C}
\end{aligned}$$

■

Proof of theorem 3: Since R_{TR_u} is a decreasing function of χ_1 and χ_2, and R_{TO_u} is constant with χ_1 and χ_2. Also, R_{TR_u}, at $\chi_1 = \chi_2 = 0$, is larger than R_{TO_u} (check (29) and (28)). Therefore, R_{TR_u} and R_{TO_u} intersect together at a curve whose equation is described by $q\left(\chi_{th}^{(1)}, \chi_{th}^{(2)}\right)$ in (31). This intersection is called the threshold curve. Since $R_{TR_u} \geq R_{TO_u}$ for $(\chi_1,\chi_2) < \left(\chi_{th}^{(1)}, \chi_{th}^{(2)}\right)$ and $R_{TR_u} \leq R_{TO_u}$ for $(\chi_1,\chi_2) \geq \left(\chi_{th}^{(1)}, \chi_{th}^{(2)}\right)$, equation (30) defines an optimal policy.

■