ECE (435/535a) Spring 2016 Quiz 4
April 19

Name: Solutions **Signature:** ____________________

Instructions: Answer all questions and show all work. Answers which are not justified with appropriate work will receive 0 points. Students who cheat will receive zero points on the exam and will be subject to the university's disciplinary procedure for academic dishonesty. Cheating includes, but is not limited to, collaborating of conferring in any way with anyone. Use of the internet is strictly forbidden. Your signature above attests that you are in compliance with these rules.

1. M-ary Frequency-Shift Keying (FSK) is used to transmit a block of $K = \log_2 M$ bits per signal waverform of duration T. The signal waveforms are

$$x_m(t) = \sqrt{\frac{2\mathcal{E}_S}{T}} \cos(\omega_c t + m\Delta\omega t),$$

where $\mathcal{E} =$ is energy per symbol, $\omega_c \gg \frac{1}{T}$ and $1 \leq m \leq M$ and $0 \leq t < T$. The frequency separation $\Delta\omega$ determines the degree to which we can discriminate among M possible transmitted symbols. As a measure of similarity between the signals $x_m(t)$ and $x_n(t)$, we define the correlation coefficients $\gamma_{m,n}$ as

$$\gamma_{m,n} = \frac{1}{\mathcal{E}_S} \int_0^T x_m(t) x_n(t) dt.$$

(a) (20 points) Find the minimum frequency separation $\Delta\omega$ so that the signals $x_m(t)$, $1 \leq m \leq M$ are orthogonal.

We need to find minimum $\Delta\omega$ that gives $\gamma_{m,n} = 0$

$$\gamma_{m,n} = \frac{1}{\mathcal{E}_s} \int_0^T \frac{2\mathcal{E}_s}{T} \cos(\omega_c t + m\Delta\omega t)\cos(\omega_c t + n\Delta\omega t)dt =$$

$$= \frac{1}{T}\int_0^T \cos((m-n)\Delta\omega t)\, dt + \frac{1}{T}\int_0^T \cos(2\omega_c t + (m+n)\Delta\omega t)\, dt$$

$$= \frac{\sin((m-n)\Delta\omega T)}{(m-n)\Delta\omega T} = 0 \Rightarrow \Delta\omega_k = \frac{2k\pi}{2T} \rightarrow \boxed{\Delta\omega_{min} = \frac{\pi}{T}}$$

(b) (5 points) For this $\Delta\omega$ determine the dimensionality of the signal space.

M-ary orthogonal FSK waveforms have a geometric representation as M, M-dimensional orthogonal vectors. $\Rightarrow$ M-dimensions

(c) (5 points) Let $\mathbf{x}_1$ be a vector in the signal constellation corresponding to $x_1(t)$. Find its coordinates.

Problem	Points	Student's Score
1	35	
2	35	
3	30	
4	10	
Total:	110	

$\underline{x}_1 = (\sqrt{E_s}, 0, \cdots, 0)$

$\underline{x}_2 = (0, \sqrt{E_s}, \cdots, 0)$

$\vdots$

$\underline{x}_M = (0, 0, \cdots, \sqrt{E_s})$

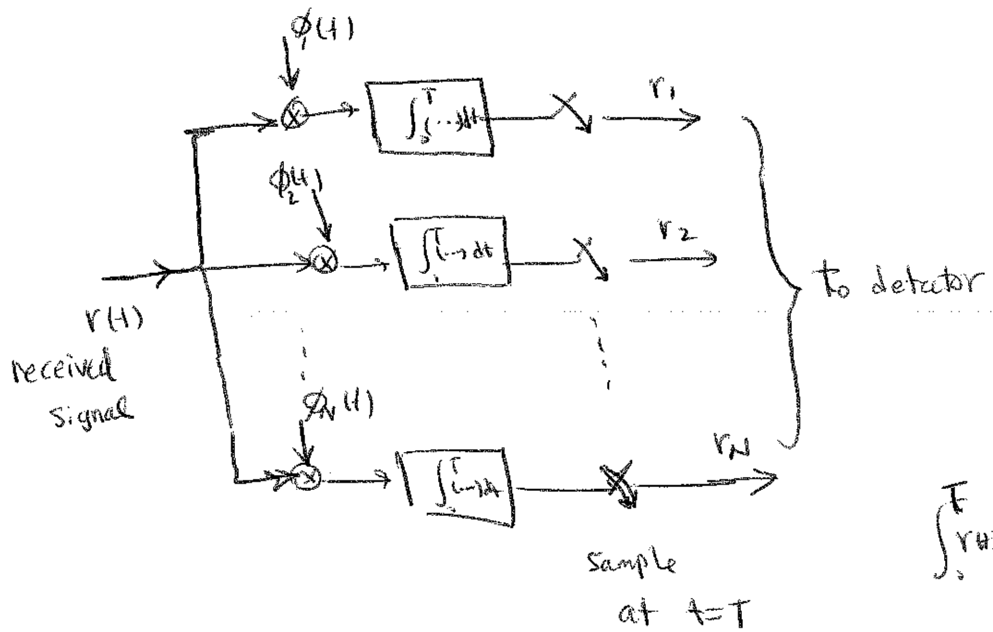
where the basis functions are $\varphi_m(t) = \sqrt{\frac{2}{T}} \cos(w_c + m\Delta\omega_{min})t$

(d) (5 points) What is the minimum distance between constellation points?

For all m, n; we have

$d_{min} = \min(d_{m,n}) = \sqrt{2E_s}$.

2. (35 points) **(Projection Demodulator)** Draw the block diagram of a projection demodulator and explain the function of basic blocks.



It is assumed that the N basis functions $\{\psi_n(t)\}_{n=1}^N$ span the signal space, so that every every $\{s_m(t), 1 \le m \le M\}$ can be represented as a weighted linear combination of $\{\psi_n(t)\}_{n=1}^N$.

$$\int_0^T r(t)\psi_k(t)dt = \int_0^T (s_m(t)+n(t))\psi_k(t)\,dt$$

$$r_k = s_{mk} + n_k, \quad k = 1, 2, \cdots, N.$$

where $s_{mk} = \int_0^T s_m(t)\psi_k(t)\,dt$ $\quad k = 1, \cdots, N$

$n_k = \int_0^T n(t)\psi_k(t)\,dt$

3. (30 points) The N-dimensional signal $x_m(t)$ is transmitted over the Additive White Gaussian Channel whose output $y(t) = x_m(t) + n(t)$ then is projected to the set of N orthonormal functions $\Phi_k(t)$, $1 \le k \le N$, $0 \le t < T$. Show that n_i and n_j are independent for any $i \ne j$, where $n_i = \int_0^T n(t)\Phi_i(t)dt$.

$$E[n_i n_j] = E\left[\left(\int_0^T n(t)\phi_i(t)\,dt\right)\left(\int_0^T n(\tau)\phi_j(\tau)\,d\tau\right)\right]$$

$$= E\left[\int_0^T\int_0^T n(t)\,n(\tau)\,\phi_i(t)\,\phi_j(\tau)\,dt\,d\tau\right]$$

$$= \int_0^T\int_0^T E[n(t)n(\tau)]\,\phi_i(t)\,\phi_j(\tau)\,dt\,d\tau$$

$$= \int_0^T\int_0^T \frac{N_0}{2}\,\delta(t-\tau)\,\phi_i(t)\,\phi_j(\tau)\,dt\,d\tau$$

$$= \frac{N_0}{2}\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \frac{N_0}{2}\,\delta[i-j]$$

where $\delta[i-j] = \begin{cases} 1, & i=j \\ 0, & i \ne j \end{cases}$

So, $E[n_i n_j] = 0$ for $i \ne j$.

$\Rightarrow$ n_i and n_j are independent for $i \ne j$.

4. Extra Credit

(a) (5 points) A prime real estate spot overlooking Monte Carlo is occupied by who?

(b) (5 points) One of the following mathematicians is not German: Kronecker, Schwartz, Hadamard, and Hilbert. Which one?

"Hadamard"