

ECE (435/535a) Spring 2016 Quiz #2
February 18

Name: _____ (Solution Quiz 2) _____ Student Number: _____

Signature: _____

Instructions: Answer all questions and show all work. Answers which are not justified with appropriate work will receive 0 points. Students who cheat will receive zero points on the exam and will be subject to the university's disciplinary procedure for academic dishonesty. Cheating includes, but is not limited to, collaborating or conferring in any way with anyone. Use of the internet is strictly forbidden. Your signature above attests that you are in compliance with these rules.

Problem	Points	Student's Score
1	40	
2	60	
Total:	100	

1. Which of the following codes are
- (20 points) Uniquely decodable?
 - (20 points) Instantaneous?

$$C_1 = \{0, 01\}$$

$$C_2 = \{00, 01, 100, 101, 11\}$$

Def: A code is instantaneous if and only if no codeword is a prefix of some other codeword.

Def: C is uniquely decodable if $Cx_1 = Cx_2$ then $x_1 = x_2$.

(a) $\rightarrow C_1$ is uniquely decodable
 $\rightarrow C_2$ is uniquely decodable

(b) \rightarrow As "0" is the prefix of "01", using the definition, C_1 is not instantaneous.
 $\rightarrow C_2$ is instantaneous.

2. (a) (40 points) Find the binary Huffman code for the random variable X with probabilities

$$P = \left(\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21} \right).$$

(b) (10 points) Find the entropy of X . (Hint: $H(X) = -\sum p_i \log_2 p_i$)

(c) (10 points) Calculate the average length of the Huffman code. (Hint: $\bar{L} = \sum p_i l_i$)

Source	x_1	x_2	x_3	x_4	x_5	x_6
Huffman code (word)	0111	0110	010	11	10	00
length	4	4	3	2	2	2

$$(b) H(X) = -\sum_i p_i \log_2 p_i = 2.398 \text{ bits}$$

$$(c) \bar{L} = \sum_i p_i l_i = 2.429 \text{ bits}$$

$$(\bar{L} > H(X))$$