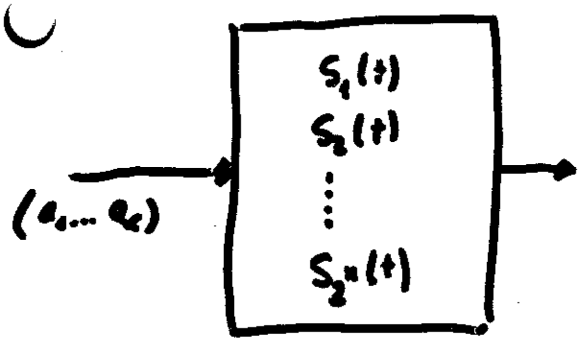
Representation of Digitally Modulated Signals



Modulator
(select one waveform)

- transmitted waveform does not depend on previously transmitted waveforms.
- Linear and nonlinear modulatio
  - superposition principle

↗ Linear memoryless modulation

Pulse-amplitude-modulated signals (PAM), (ASK)

$$s_m(t) = \mathrm{Re}\left\{ A_m \cdot g(t)\, e^{j\omega_c t} \right\}$$

$$= A_m \cdot g(t) \cos \omega_c t \qquad 1 \le m \le M \qquad 0 \le t < T$$

$\{ A_m : 1 \le m \le M \}$ set of possible amplitudes corresponding to $M = 2^k$ possible k-bit input blocks

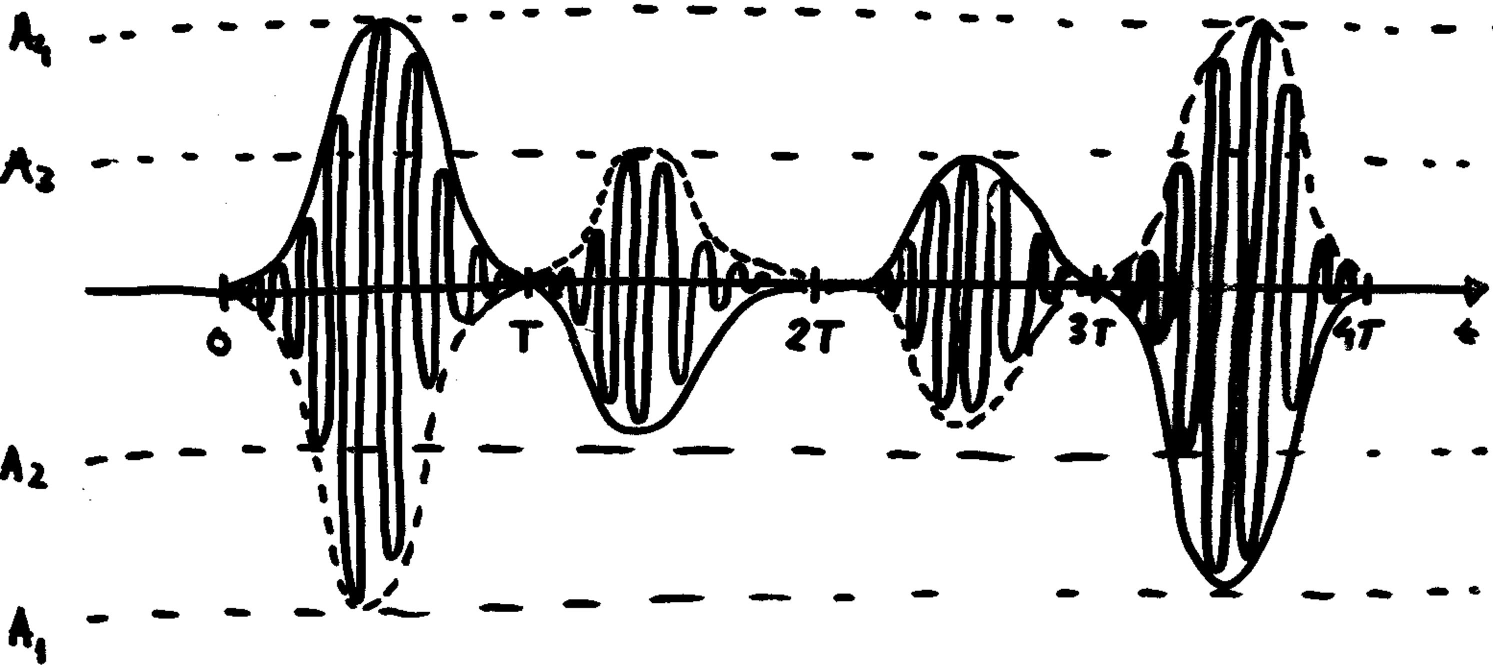
Notice: ↗ is a DSB

- PAM can also be used in the SSB modulation

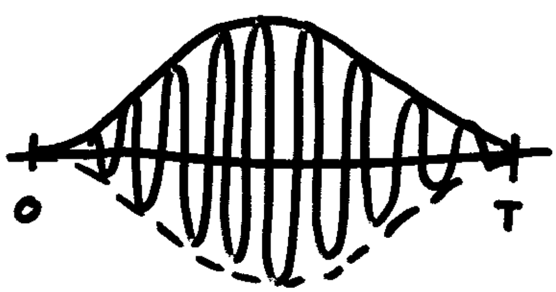
$$s_m(t) = \mathrm{Re}\{ A_m (g(t) \pm j\hat{g}(t)) e^{j\omega_c t} \}$$

- Baseband PAM $\quad s_m = A_m g(t)$

PAM



$M = 2^2 \quad K = 2$



symbol interval

$R = \frac{1}{T_B}$ - bit rate

$\frac{R}{K}$ - symbol rate

rate of changes in modulated signal

$T_B = \frac{T}{K}$

bit interval

PAM

$$\mathcal{E}_m = \int_0^T s_m^2(t)\,dt = \frac{1}{2} A_m^2 \int_0^T g^2(t)\,dt = \frac{1}{2} A_m^2 \cdot \mathcal{E}_g$$

energy of the pulse $g(t)$

Find the orthonormal basis:

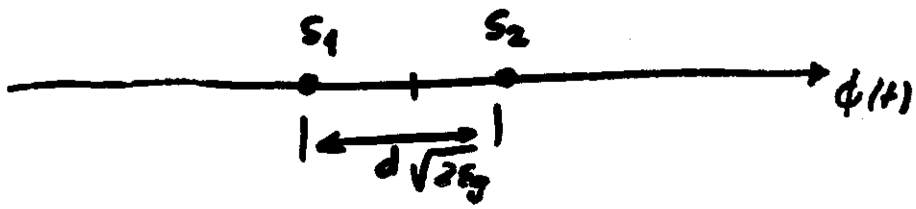
just one dimension:

$$\phi_1(t) = \phi(t) = \frac{g(t)\cos\omega_0 t}{\| g(t)\cos\omega_c t \|} = \sqrt{\frac{2}{\mathcal{E}_g}}\, g(t)\cos\omega_c t$$

$$s_m(t) = s_m \cdot \phi(t)$$

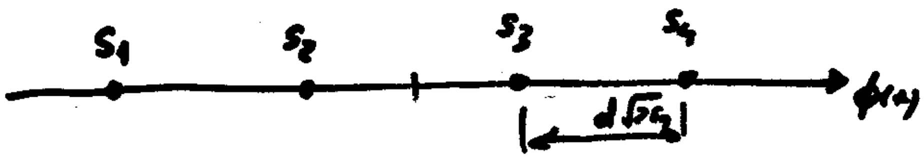
$$s_m = (A_m g(t)\cos\omega_0 t, \phi(t)) = A_m \cdot \sqrt{\frac{\mathcal{E}_g}{2}} \quad 1 \le m \le M$$

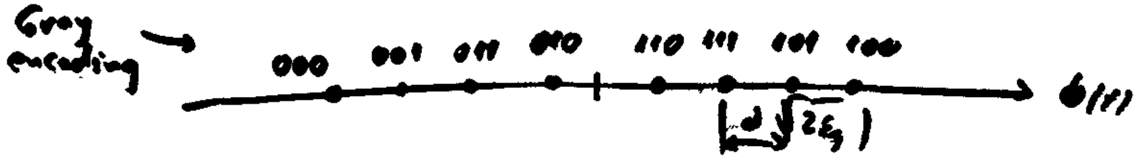
$$A_m = \big(2m - (M+1)\big)\cdot d \quad 1 \le m \le M$$ 2d - the distance



signal space

(signal constellation)





Frequency Shift Keying

$$s_m(t) = \mathrm{Re}\left\{ \tilde{s}_m(t)\, e^{j\omega_c t} \right\} \qquad 1 \le m \le M \qquad 0 \le t < T$$

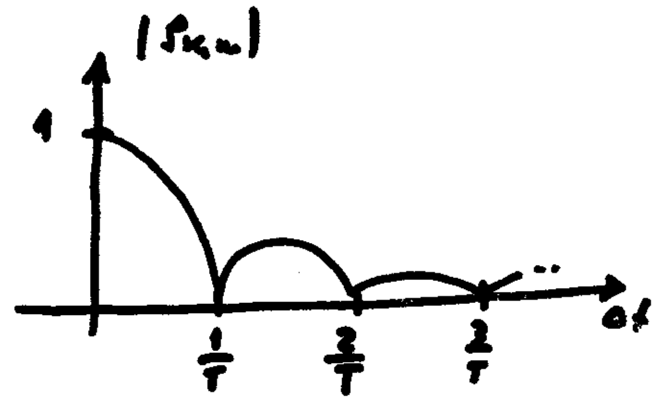
$$= \sqrt{\frac{2\mathcal{E}}{T}} \cos\left(\omega_c t + m\Delta\omega t \right)$$

$$\tilde{s}_m(t) = \sqrt{\frac{2\mathcal{E}}{T}}\; e^{j m \Delta\omega t}$$

$$\rho_{k,m} = \frac{2\mathcal{E}/T}{\mathcal{E}} \int_0^T e^{j\Delta\omega(m-k)t}\, dt = \frac{\sin \frac{(m-k)T\Delta\omega}{2}}{\frac{(m-k)T\Delta\omega}{2}}\, e^{j\frac{(m-k)}{2}T\Delta\omega}$$

$$\mathrm{Re}\{\rho_{k,m}\} = \frac{\sin T(m-k)\Delta\omega}{T(m-k)\Delta\omega}$$

$$\mathrm{Re}\{\rho_{k,m}\} = 0 \quad \text{when } \Delta f = \frac{1}{2T}$$
$$m \neq k$$



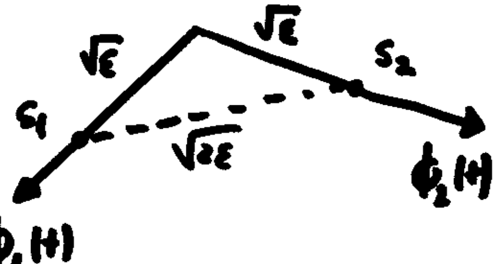
minimal frequency separation between adjacent signals for orthogonality of M-signals.

-FSK- Signal Space Representation

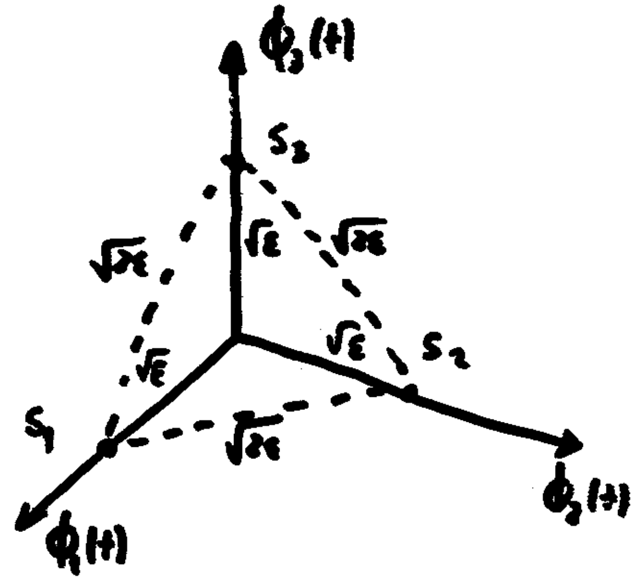
$\Delta f = \frac{1}{2T}$:

$$s_1 = (\sqrt{E}, 0, 0, \dots, 0)$$
$$s_2 = (0, \sqrt{E}, 0, \dots, 0)$$
$$s_3 = (0, 0, \sqrt{E}, \dots, 0)$$
$$\vdots$$
$$s_M = (0, 0, 0, \dots, \sqrt{E})$$

$$d_{km}^{(e)} = \sqrt{2E}$$



$M=2$



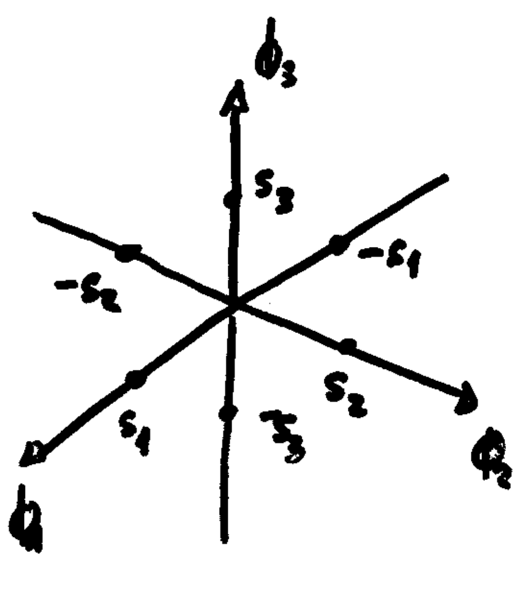
$M=3$

Biorthogonal signals:

- Start from a set of $\frac{M}{2}$ orthogonal signals
- Include negatives

$Re\{s_{m,k}\} = \begin{cases} -1 \\ 0 \end{cases}$

- $d = \begin{cases} 2\sqrt{E} \\ \sqrt{2E} \end{cases}$



Simplex Signals

- Start from a set of M-orthogonal signals
- Subtract the mean

$$s'_m = s_m - \frac{1}{M}\sum_{k=1}^{M} s_k \qquad 1 \le m \le M$$

$$\|s'_m\|^2 = \mathcal{E} - \frac{2}{M}\cdot\mathcal{E} + \frac{1}{M^2}\mathcal{E}M = \mathcal{E}\left(1-\frac{1}{M}\right)$$ - less energy

$$\mathrm{Re}\{\rho_{k,m}\} = \frac{(s'_m, s'_k)}{\|s'_m\|\,\|s'_k\|} = \frac{-\frac{1}{M}}{1-\frac{1}{M}} = -\frac{1}{M-1}$$ - equally correlated

$$(s'_m, s'_n) = \left(s_m - \frac{1}{M}\sum_{k=1}^{M} s_k \,,\, s_n - \frac{1}{M}\sum_{j=1}^{M} s_j\right)$$

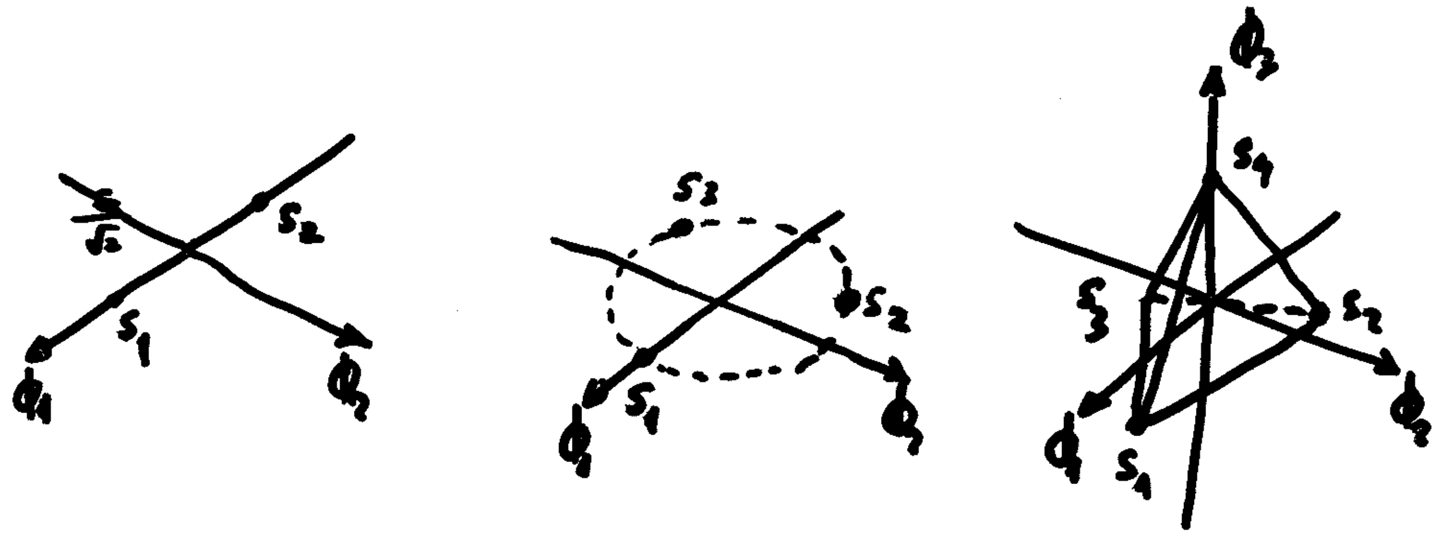
$$= (s_m, s_n) - \frac{2}{M}\sum_{k=1}^{M}(s_k, s_n) + \frac{1}{M^2}\left(\sum_{k=1}^{M} s_k, \sum_{j=1}^{M} s_j\right)$$

$$= (s_m, s_n) - \frac{2}{M}\mathcal{E} + \frac{1}{M^2}\cdot M\cdot\mathcal{E}$$

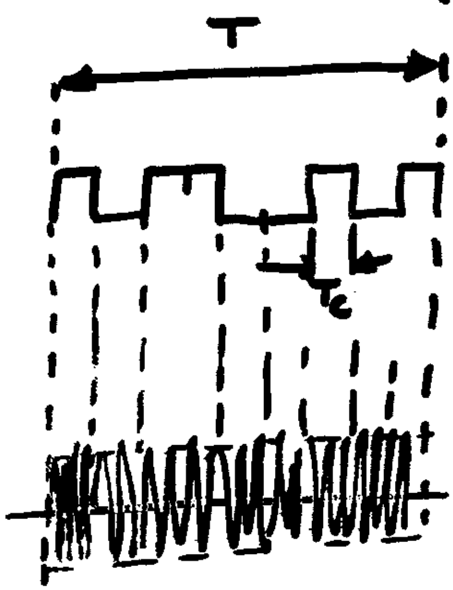
$$= (s_m, s_n) - \frac{1}{M}\mathcal{E}$$

$$(s'_m, s'_n) = \begin{cases} \mathcal{E}\left(1-\frac{1}{M}\right) & m = n \\ -\frac{1}{M}\mathcal{E} & m \neq n \end{cases}$$

Simplex Signals



Signal Waveforms from binary codes



— $c_m = (c_{m,1}, c_{m,2}, \dots, c_{m,N})$

$T_c = \frac{T}{N}$ — block length

$$c_m = 1 \Rightarrow s_{m,j}(t) = \sqrt{\frac{2E_c}{T_c}} \cos w_c t \qquad 0 < t \leq T_c \qquad E_c = \frac{E}{N}$$

$$c_m = 0 \Rightarrow s_{m,j}(t) = -\sqrt{\frac{2E_c}{T_c}} \cos w_c t \qquad 0 \leq t < T_c$$