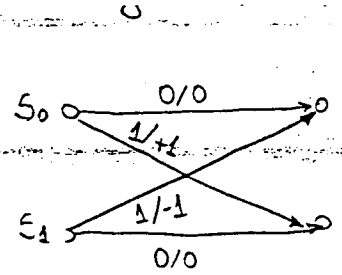
Viterbi Algorithm for the dicode channel $h(D) = (1-D)$



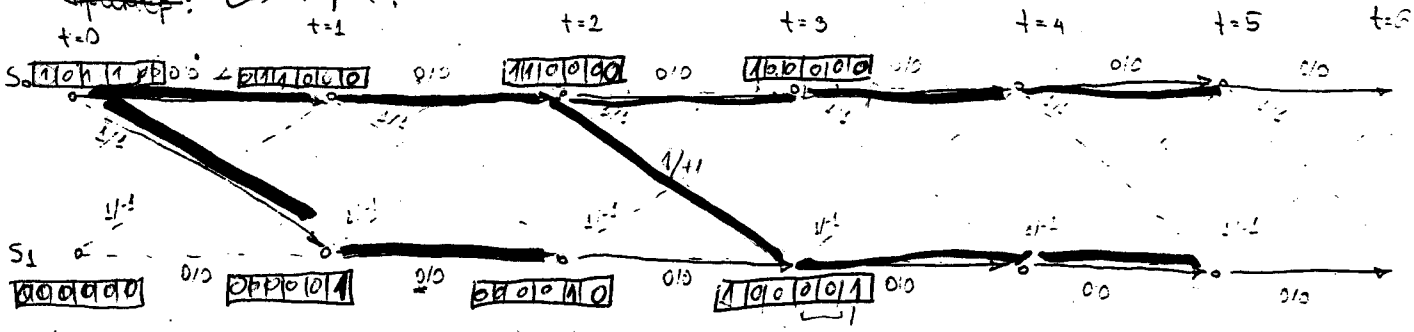
$$D_n(S_0) = \min\{D_{n-1}(S_0) + \ell(S_0,S_0),\ D_{n-1}(S_1) + \ell(S_1,S_0)\}$$

n - time

$\ell(S_0,S_0)$ - branch metric

$$D_n(S_1) = \min\{D_{n-1}(S_0) + \ell(S_0,S_1),\ D_{n-1}(S_1) + \ell(S_1,S_1)\}$$

Example:



0.2, 0.1, 0.1, -0.7, 0.1, -0.2, 0.3, 0.8

$D_0(S_0) = 0$

$t=1$ $D_1(S_0) = D_0(S_0) + (0.2-0)^2 = 0.04$

$D_1(S_1) = D_0(S_0) + (0.2-1)^2 = 0.64$

$t=2$ $D_2(S_0) = \min[D_1(S_0) + (0.1-0)^2,\ D_1(S_1) + (0.1+1)^2] = \min[0.05, 1.85] = 0.05$ (via S_0)

$D_2(S_1) = \min[D_1(S_0) + (0.1-1)^2,\ D_1(S_1) + (0.1-0)^2] = \min[0.85, 0.65] = 0.65$ (via S_1)

$t=3$ $D_3(S_0) = \min[D_2(S_0) + (0.1-0)^2,\ D_2(S_1) + (0.1+1)^2] = \min[0.06, 1.86] = 0.06$ (via S_0)

$D_3(S_1) = \min[D_2(S_0) + (0.1-1)^2,\ D_2(S_1) + (0.1-0)^2] = \min[1.46, 0.66] = 0.66$ (via S_1)

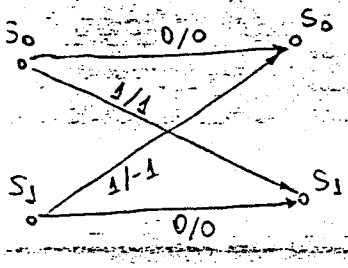
$t=4$ $D_4(S_0) = \min[D_3(S_0) + (-0.7-0)^2,\ D_3(S_1) + (-0.7+1)^2] = \min[0.55, 0.75] = 0.55$ (via S_0)

$D_4(S_1) = \min[D_3(S_0) + (-0.7-1)^2,\ D_3(S_1) + (-0.7-0)^2] = \min[2.95, 1.15] = 1.15$ (via S_1)

$t=5$ $D_5(S_0) = \min[D_4(S_0) + (0.1-0)^2,\ D_4(S_1) + (0.1+1)^2] = \min[0.56, 2.36] = 0.56$ (via S_0)

$D_5(S_1) = \min[D_4(S_0) + (0.1-1)^2,\ D_4(S_1) + (0.1-0)^2] = \min[1.36, 1.16] = 1.16$ (via S_1)

Trellis Example



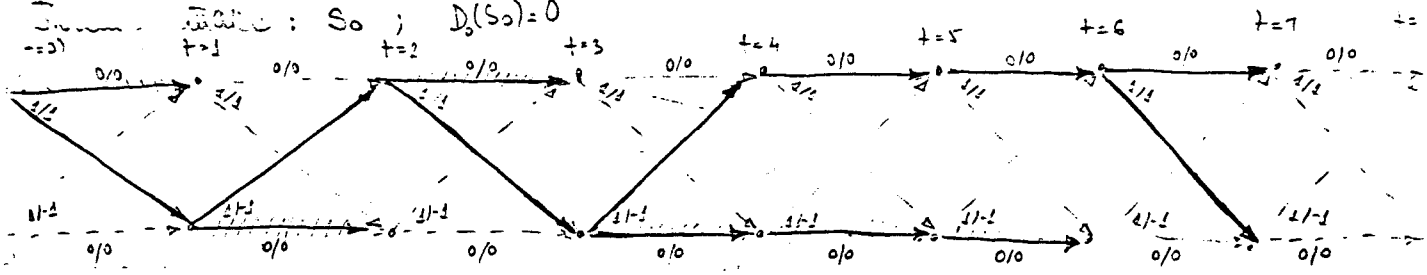
Viterbi Decoder (1-D)

$$D_n(S_i) = \min\{ D_{n-1}(S_j) + \ell(S_i, S_j),\ D_{n-1}(S_u) + \ell(S_i, S_u) \}$$

Input sequence

0.2, -0.9, 0.7, -0.5, 0.2, 0.3, 1.1, ...

Initial state : S_0 ; $D_0(S_0) = 0$



$t=1$

$D_1(S_0) = D_0(S_0) + (0.2-0)^2 = 0.04$

$D_1(S_1) = D_0(S_0) + (0.2-1)^2 = 0.64$

$t=2$

$D_2(S_0) = \min[D_1(S_0) + (-0.9-0)^2,\ D_1(S_1) + (-0.9+1)^2] = \min[0.85, 0.65] = 0.65$ (из S_1)

$D_2(S_1) = \min[D_1(S_0) + (-0.9-1)^2,\ D_1(S_1) + (-0.9-0)^2] = \min[3.65, 1.45] = 1.45$ (из S_1)

Путь $S_0 \to S_0$ ($t=0 \to t=1$) отсекается

$t=3$

$D_3(S_0) = \min[D_2(S_0) + (0.7-0)^2,\ D_2(S_1) + (0.7+1)^2] = \min[1.14, 4.34] = 1.14$ (из S_0)

$D_3(S_1) = \min[D_2(S_0) + (0.7-1)^2,\ D_2(S_1) + (0.7-0)^2] = \min[0.74, 1.94] = 0.74$ (из S_0)

Путь $S_1 \to S_1$ ($t=1 \to t=2$) отсекается

$t=4$

$D_4(S_0) = \min[D_3(S_0) + (-0.5-0)^2,\ D_3(S_1) + (-0.5+1)^2] = \min[1.39, 0.99] = 0.99$ (из S_1)

$D_4(S_1) = \min[D_3(S_0) + (-0.5-1)^2,\ D_3(S_1) + (-0.5-0)^2] = \min[3.39, 0.99] = 0.99$ (из S_1)

Путь $S_0 \to S_0$ ($t=2 \to t=3$) отсекается

$t=5$

$D_5(S_0) = \min[D_4(S_0) + (0.2-0)^2,\ D_4(S_1) + (0.2+1)^2] = \min[1.03, 2.43] = 1.03$ (из S_0)

$D_5(S_1) = \min[D_4(S_0) + (0.2-1)^2,\ D_4(S_1) + (0.2-0)^2] = \min[1.63, 1.03] = 1.03$ (из S_1)

$t=6$

$D_6(S_0) = \min[D_5(S_0) + (0.3-0)^2,\ D_5(S_1) + (0.3+1)^2] = \min[1.12, 2.72] = 1.12$ (из S_0)

$D_6(S_1) = \min[D_5(S_0) + (0.3-1)^2,\ D_5(S_1) + (0.3-0)^2] = \min[1.52, 1.12] = 1.12$ (из S_1)

$t=7$

$D_7(S_0) = \min[D_6(S_0) + (1.1-0)^2,\ D_6(S_1) + (1.1+1)^2] = \min[2.33, 5.53] = 2.33$ (из S_0)

$D_7(S_1) = \min[D_6(S_0) + (1.1-1)^2,\ D_6(S_1) + (1.1-0)^2] = \min[1.13, 2.33] = 1.13$ (из S_0)

Survivor path $S_1 \to S_1 \to S_1 \to S_1$ за $t=3 \to t=4 \to t=5 \to t=6$

Decoded sequence: 1, 1, 1, -1, 0, 0, ...