Detection for output-continuous memoryless channels

$$X \longrightarrow \boxed{AWGN} \longrightarrow Y$$

$y = x + n \quad n \sim N(0, \sigma^2)$

and X is a discrete channel, ie, $X \in \{-A, +A\}$

– conection with physics $\quad \sigma^2 = \frac{N_0}{2} \quad A = \sqrt{E} \quad E_b$ - energy per bit

$\frac{N_0}{2}$ - power spectrd density

1) X is a memoryless source

$$p(y|-A) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+A)^2}{2\sigma^2}}$$ corresponding to bit 0 $\quad 0 \to -A$

$$p(y|+A) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-A)^2}{2\sigma^2}}$$ bit 1 $\quad 1 \to +A$

Given the channel output y, the detector wants to find what is more likely: $\{-A$ was transmited$\}$ or $\{+A$ was transmited$\}$

$$\frac{P(-A|y)}{P(+A|y)} \underset{+A}{\overset{-A}{\gtrless}} 1$$

$$\frac{\frac{1}{P(y)} P(-A) \cdot P(y|-A)}{\frac{1}{P(y)} P(+A)\, P(y|+A)} \underset{+A}{\overset{-A}{\gtrless}} 1$$

$$\frac{P(y|-A)}{P(y|+A)} \underset{+A}{\overset{-A}{\gtrless}} \frac{P(+A)}{P(-A)} \quad / \ln \;\to\; \mu = \ln \frac{P(y|-A)}{P(y|+A)}$$ is called log-likelihood ratio (LLR)

$$\ln \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y+A)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-A)^2}{2\sigma^2}}} \underset{+A}{\overset{-A}{\gtrless}} \ln \frac{P(+A)}{P(-A)}$$

$\mu \gtrless \frac{P(+A)}{P(-A)}$ – test

$$-\frac{(y+A)^2}{2\sigma^2} + \frac{(y-A)^2}{2\sigma^2} \underset{+A}{\overset{-A}{\gtrless}} \ln \frac{P(+A)}{P(-A)} \quad / 2\sigma^2$$

$$-(y+A)^2 + (y-A)^2 \underset{+A}{\overset{-A}{\gtrless}} 2\sigma^2 \ln \frac{P(+A)}{P(-A)}$$

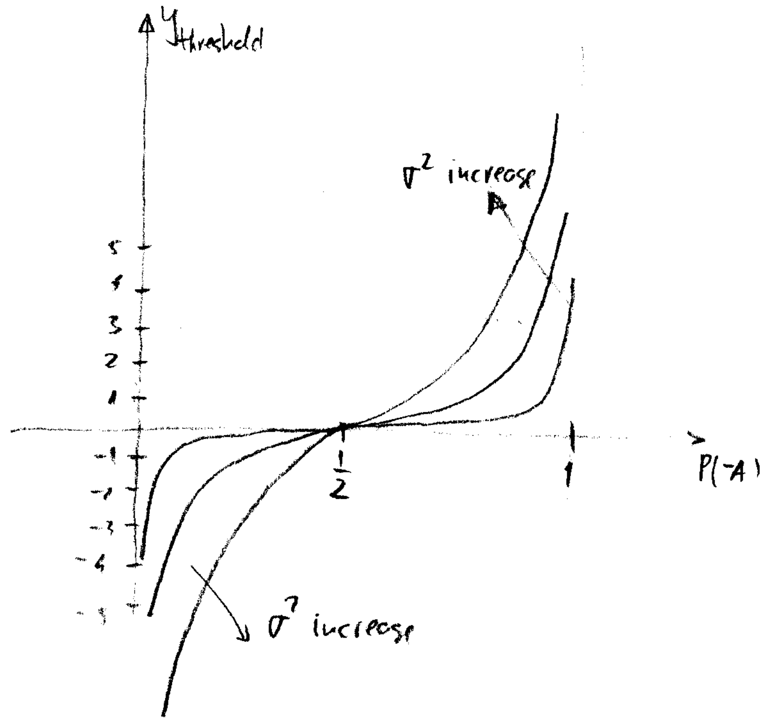
$$-y^2 - 2yA - A^2 + y^2 - 2yA + A^2 \underset{+A}{\overset{-A}{\gtrless}} 2\sigma^2 \ln \frac{P(+A)}{P(-A)}$$

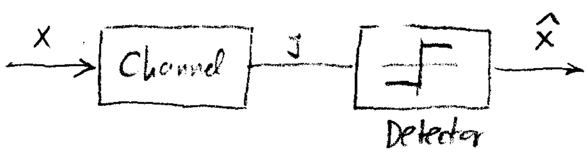
$$-4yA \underset{+A}{\overset{-A}{\gtrless}} 2\sigma^2 \ln \frac{P(+A)}{P(-A)}$$

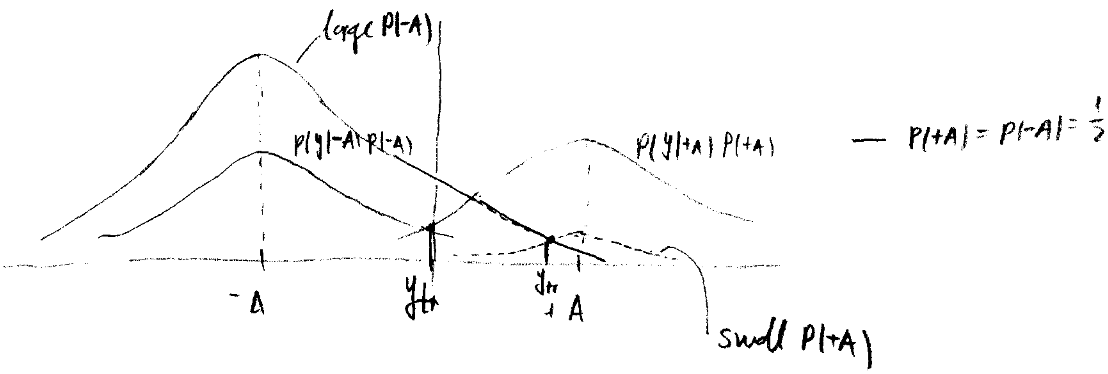
$$y \underset{-A}{\overset{+A}{\gtrless}} -\frac{1}{2}\sigma^2 \ln \frac{P(+A)}{P(-A)}$$

$$y \underset{-A}{\overset{+A}{\gtrless}} \frac{1}{2}\sigma^2 \ln \frac{P(-A)}{P(+A)} = y_{threshold}$$

— log likelihood function







2) Transmission of sequences (vector channel)

X – memoryless

noise – memoryless

$$\bar{x} = (x_1, x_2, \dots x_N) \longrightarrow \boxed{\begin{array}{c}\text{AWGN}\\ \text{channel}\end{array}} \longrightarrow \bar{y} = (y_1, y_2, \dots y_N)$$

$$\bar{y} = \bar{x} + \bar{n}$$

$$E[n_i n_j] = \delta_{i,j} \cdot \frac{N_0}{2}$$

white

$$p(\bar{y} | \bar{x}) = p(y_1, y_2, \dots y_N | x_1, x_2, \dots, x_N) = \prod_{i=1}^{N} p(y_i | x_i)$$

$$p(y_i | x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-x_i)^2}{2\sigma^2}}$$

Independent decision on individual symbols

$$\mu_i = \ln \frac{p(y_i | x_i = -A)}{p(y_i | x_i = +A)}$$ – log likelihood ratio

3) Markov sources – Viterbi algorithm