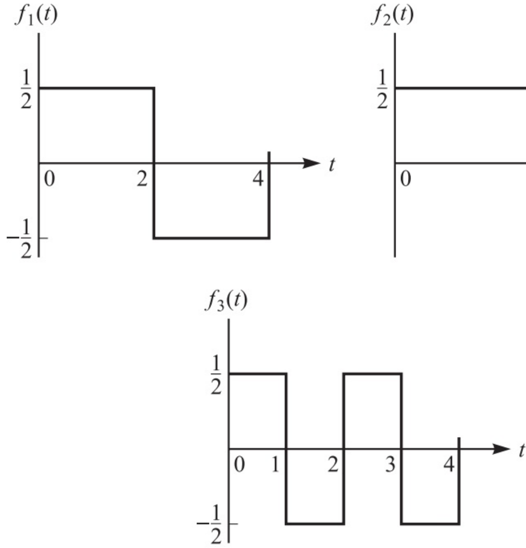
Homework 4 — **ECE 435a/535a (Spring 2016)**

1. Consider the three waveforms $f_n(t)$ shown in the figure below.



   (a) Show that these waveforms are orthonormal.

   (b) Express the waveform $x(t)$ as a linear combination of $f_n(t)$, $n = 1, 2, 3$, if

$$x(t) = \begin{cases} \text{-1}, & 0 \leq \text{t} < 1 \\ 1, & 1 \leq \text{t} < 3 \\ \text{-1}, & 3 \leq \text{t} < 4 \end{cases}$$

   and determine the weighting coefficients.

2. Suppose that s(t) is either a real- or complex-valued signal that is represented as a linear combination of orthonormal functions $\{f_n(t)\}$, i.e.,

$$\hat{s}(t) = \sum_{k=1}^{K} s_k f_k(t),$$

   where

$$\int_{-\infty}^{\infty} f_n(t) f_m^{\star}(t) dt = \begin{cases} 1, & \text{m = n} \\ 0, & \text{otherwise} \end{cases}$$

   Determine the expressions for the coefficients $\{s_k\}$ in the expansion $\hat{s}_i(t)$ that minimize the energy

$$\mathcal{E}_e = \int_{-\infty}^{\infty} \mid s(t) - \hat{s}(t) \mid^2 dt,$$

   and the corresponding residual error $\mathcal{E}_e$.

3. Determine the average energy of a set of M-PAM signals of the form

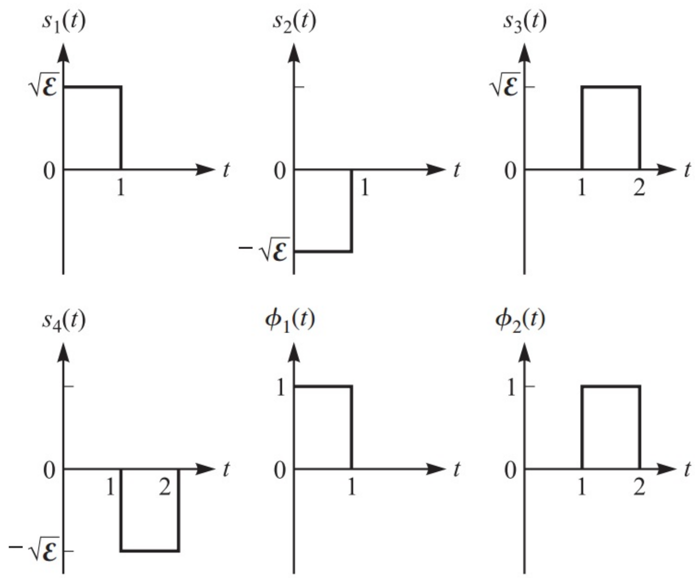
$$s_m(t) = s_m\phi(t), m = 1, 2, \ldots, M \text{ and } 0 \leq t \leq T,$$

where

$$s_m = \sqrt{\mathcal{E}_g}A_m,\ m = 1, 2, \ldots, M$$

The signals are equally probable with amplitudes that are symmetric about zero and are uniformly spaced with distance d between adjacent amplitudes.

4. Determine the signal space representation of the four signals $s_k(t)$, $k = 1, 2, 3, 4$, shown in the figure below, by using as basis functions the orthonormal functions $\phi_1(t)$ and $\phi_2(t)$. Plot the signal space diagram, and show that this signal set is equivalent to that for a four-phase PSK signal.



5. **(Extra-Graduates)** Prove the Cauchy-Schwart inequality

$$\mid \sum_{k=1}^{n} x_k y_k^\star \mid^2 \leq \Big(\sum_{k=1}^{n} \mid x_k \mid^2 \Big)\Big(\sum_{k=1}^{n} \mid y_k \mid^2 \Big).$$