ECE 435a/535a (Spring 2016)

1. Find the Huffman D-ary code for $(p_1, p_2, p_3, p_4, p_5, p_6) = (\frac{6}{25}, \frac{6}{25}, \frac{4}{25}, \frac{4}{25}, \frac{3}{25}, \frac{2}{25})$ and the expected word length
   (a) For $D = 2$.
   (b) For $D = 4$.

2. Which of the following codes are
   (a) Uniquely decodable?
   (b) Instantaneous?

$$\mathcal{C}_1 = \{00, 01, 0\}$$
$$\mathcal{C}_2 = \{00, 01, 100, 101, 11\}$$
$$\mathcal{C}_2 = \{0, 00, 000, 0000\}$$

3. A source has an alphabet $\{x_1, x_2, x_3, x_4\}$ with corresponding probabilities $\{0.1, 0.2, 0.3, 0.4\}$.
   (a) Find the entropy of the source.
   (b) Design a Huffman code for the source and compare the average length of the Huffman code with the entropy of the source.
   (c) Design a Huffman code for the second extension of the source (take two letters at a time). What is the average code word length? What is the average required binary letters per each source output letter?
   (d) Which one is a more efficient coding scheme, Huffman coding of the original source or Huffman coding of the second extension of the source?

4. Find the Lampel-Ziv source code for the binary source sequence

   000100100000011000010000000100000010100001000000110100000001100.

5. Design a Huffman code for a source with N symbols whose probabilities are $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^{n-1}}\}$. Show that the average codeword length for such a source is equal to the source entropy.

6. Find the differential entropy of the zero-mean Gaussian memoryless source.

7. The input of the additive white Gaussian noise channel with the noise variance σ_n is the zero-mean Gaussian source X with variance σ_x. Find the mutual information between the channel input X and the channel output Y.

8. **(Extra-Graduates)** For the question 4, recover the original sequence back from the Lampel-Ziv source code. (Hint: You require two passes of the binary sequence to decide on the size of dictionary.)

9. **(Extra-Graduates)** A channel with m input and n output symbols is said to be symmetric if its channel matrix has the property that its each row $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is a permutation of another row, and each column $\mathbf{q} = (q_1, q_2, \ldots, q_m)$ is a permutation of another column. Derive the expression for the channel capacity of such a symmetric channel. (Hint: prove first that conditional entropy $H(Y|X)$ is independent of the input probability distribution).