1. Two binary random variables X and Y are distributed according to the joint distribution $p(X = Y = 0) = p(X = 0, Y = 1) = p(X = Y = 1) = \frac{1}{3}$. Compute $H(X), H(Y), H(X|Y), H(Y|X)$, and $H(X, Y)$.

2. Let X and Y denote two jointly distributed discrete valued random variables. Show that

$$H(X) = -\sum_{x,y} p(x, y) \log p(x)$$

, and

$$H(Y) = -\sum_{x,y} p(x, y) \log p(y).$$

3. Let X and Y denote two jointly distributed discrete valued random variables. Show that

$$H(X, Y) \leq H(X) + H(Y).$$

When does the equality hold?

4. Let X denote a binary random variable with $p(X = 0) = 1 - p(X = 1) = p$ and let Y be a binary random variable that depends on X through $p(Y = 1|X = 0) = p(Y = 0|X = 1) = \epsilon$.
   (a) Find $H(X), H(Y), H(X|Y), H(Y|X)$, $H(X, Y)$, and $I(X; Y)$.
   (b) For a fixed p, which ϵ minimizes $I(X; Y)$?

5. **(Extra-Graduates)** Let $X_1, X_2, \ldots, X_n$ be drawn according to $p(x_1, x_2, \ldots, x_n)$. Show that

$$H(X_1, X_2, \ldots, X_n) \leq H(X_1) + H(X_2) + \ldots + H(X_n).$$

When does the equality hold?