STATE OF THE ART AND FUTURE DIRECTIONS IN CONSTRAINED MULTI TRACK CODE RESEARCH FOR MAGNETIC RECORDING SYSTEMS

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Abstract – *In this paper we present multi-track coding and signal processing algorithms for multi-track recording channels subject to ISI and ITI that enable extremely high data transfer rates and recording densities by the exploiting the two-dimensional nature of recorded data.*

1. INTRODUCTION

In traditional magnetic disk drive systems, the data are recorded in tracks as a sequence of small magnetic domains with two senses of magnetization. Increasing areal density of the data stored on a disk can be achieved by reducing the length of magnetic domains along tracks (increasing linear density) and/or by reducing the track pitch (increasing track, or radial, density). Linear density is limited by the finite sensitivity of the read head, properties of magnetic materials and head designs, and the ability to detect and decode recorded data in the presence of intersymbol interference (ISI) and noise. Most prior research in disk drive systems has focused on increasing linear density. Extremely high densities, viz. 0.5 Tbits/in^2 have been already demonstrated in commercially available systems. However, the rate of linear density increase in the future is not likely to be as high as in the past. This is because, as linear density increases, the magnetic domains on the disk surface become smaller and increasingly thermally unstable. The so-called super-paramagnetic effect fundamentally limits recording density. Given that current linear densities are approaching the super-paramagnetic limit, the obvious alternative to an increase in linear density is an increase in radial density. Such approaches are required to meet the constant demand for increases in data rate and capacity of storage devices.

2. AN OVERVIEW OF MULTI-TRACK DETECTORS AND CODES

A number of multi-track coding and detection schemes have been proposed in the last decade. They can be categorized as follows.

1. The first category is a class of multi-track codes that exploit the idea that the achievable areal density can be increased indirectly. This is done through the relaxation of the per-track maximum runlength constraint (so-called *k*-constraint) in the recording codes by imposing a constraint across multiple tracks instead of on each track. Such codes have been studied by Marcellin et al. [1], [2], Swanson and Wolf [3], and Vasic et al. [4]-[9].
2. The second category is a class of techniques involving multi-track detection combined with partial-response (PR) equalization [10] and maximum likelihood (multiple) sequence detection (MLSD). Such techniques, although maximum likelihood, have computational complexity exponential in *NM*, where *N* is the number of tracks and *M* is the memory of the PR channel. Multi-track codes and reduced complexity detectors for such systems have been studied by Soljanin et al. [11]-[13] and Kurtas et al. [14]-[15]. Recently a combination of equalization and maximum a posteriori (MAP) detection to reduce both ISI and ITI has been considered by Wu et al. [16]. This method combines a multi-track version of the BCJR algorithm [17] with iterative Wiener filtering, and performs very well in conjunction with low-density parity-check (LDPC) codes, but is very complex.
3. The third class of techniques uses the idea of imposing a constraint on a recorded bit sequence in such a way that ITI on each PR channel is either completely eliminated or reduced. Recently, Ahmed et al. [18] constructed a two-track runlength-limited code for a Class 4 partial response (PR4) channel. The code forbids any transitions of opposite polarity on adjacent tracks, and results in up to a 23% gain in areal density over an uncoded system. Similar two track schemes, but for a different multi-track constraint, have been proposed by Davey et al. [19]-[20] and by Lee and Madisetti [21]. Due to their high complexity, these schemes can be used only for a small number of tracks and low order PR polynomials. Recently Zhang et al. [22] have found bounds on information rates of multi-track channels.
4. In [23] we proposed a low-complexity multi-track soft error-event correcting scheme. The idea is an essentially to design a multi-track version of the "post-processor" [23]. The generalization of the post-processor concept to the multiple track case is non-trivial because the detector must be designed to mitigate the effect of errors caused both by ISI and ITI, as explained below.
5. The last set of results is concerned with information-theoretical properties of two-dimensional ISI channels and two-dimensional runlength constrained channels [24], [25]-[26].

In order to achieve high linear densities, higher-order PR polynomials are necessary. However, the complexity of read-channel chips increases exponentially with the PR polynomial order. Further, most of this increase is due to MLSD, which is already the most complicated sub-system in the "read-channel" electronics and is a primary impediment to high data throughput. Thus, increasing the complexity of a detector by another factor (*N*) in the exponent, which is required for MLSD detection over *N* tracks, is not feasible. It is important to note also that the resulting increase in processing delay due to high-complexity MLSD would result in poor quality preliminary decisions for the timing recovery algorithm. The timing recovery problem is exacerbated by the fact that future recording systems will operate in the low-to-medium SNR regime as a consequence of modern (e.g., turbo-like) coding and detection/decoding techniques.

These observations motivate research in low-complexity, sub-optimal, multi-track coding and detection schemes as described in this proposal and outlined in the next subsection. We remark that the proposed research focuses on a special case of 2D recording, but many of the results will be applicable also to general 2D recording, including probe and patterned-media recording.

3. CHANNEL MODEL

The magnetic recording channel is modeled by a discrete-time linear filter with a partial response polynomial typically of the form $h(D) = (1-D)(1+D)^M$, or $h(D) = (1+D)^M$, $M \geq 1$ depending on whether the longitudinal or perpendicular recording is employed. We adopt a multi-track system model in which user data are organized in two-dimensional blocks of size $N \times n$, and written on N adjacent tracks. The sequence recorded in the kth track is denoted by

$$\{a_m^{(k)}\}_{m \in Z} . \tag{1}$$

The adjacent tracks are read by an array of N heads. The signal read by the kth head is equalized to some partial response target and is given by

$$r^{(k)}(t) = \sum_{1 \leq l \leq N} \alpha_{|l-k|} \Big(\sum_{m \in Z} a_m^{(l)} h(t - mT) \Big) + n(t) , \tag{2}$$

where $1 \leq k \leq N$ and $h(t)$ is the impulse response of the PR target, $n(t)$ is the colored noise process obtained by filtering additive white Gaussian noise (AWGN) by the equalizer, and the real coefficients α_d, $0 < d \leq N-1$ quantify the cross-talk between two tracks separated by $d-1$ tracks ($\alpha_0 = 0$). For longitudinal recording systems, we assume that the read head response to an isolated (positive-going) transition written on a disk is the Lorentzian pulse, $l(t) = (1 + (2t / PW_{50})^2)^{-1}$. For perpendicular recording, we assume the transition response is the error function,

$$g(t) = (2/\sqrt{\pi}) \int_0^{St} \exp(-x^2) dx = erf(St) \tag{3}$$

where $S = 2\sqrt{\ln 2} \cdot T / PW_{50}$. The dibit response is defined as $(g(t + T/2) - g(t - T/2))$. The channel density is defined as $S_c = PW_{50}/T$, where PW_{50} represents the width of the channel impulse response at a half of its peak value, and T is the channel bit interval.

For the sake of simplicity, we ignore the effects of timing jitter, i.e., we assume that the samples are taken at time instants $t = mT$, where the channel bit interval T is known and fixed. We also ignore the effects of track-following error, i.e., we assume that k^{th} head is perfectly aligned with the kth track. The discrete-time model of the longitudinal recording multi-track system is shown in Figure 1 (an analogous model is valid for perpendicular recording).

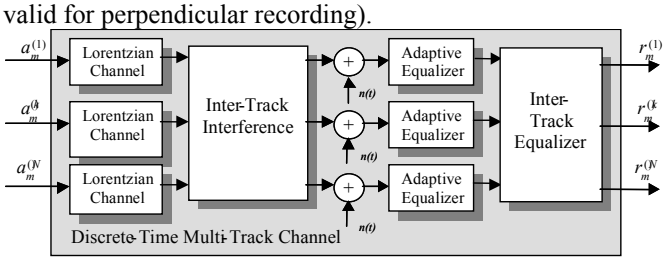

Figure 1. Discrete-Time Multi-track Channel Model

The inter-track interference block is described by matrix $A = [\alpha_{|i-j|}]_{1 \leq i, j \leq N}$.

4. CONSTRAINED CODES FOR ITI REDUCTION

Consider an N-track system in which each track is equalized to a PR channel with polynomial

$$p(D) = h_0 + h_1 D + \ldots h_L D^L . \tag{4}$$

The N-track constrained system is defined as an oriented, strongly connected graph $G = (V, E)$ with vertex set V and edge set E. The vertices (states) are labeled by binary arrays Ψ of dimension $N \times L$, $\Psi = (\Psi_j)_{1 \leq j \leq L}$, while the edges are labeled by binary column vectors $\boldsymbol{x}$ of length N (more details on vector constrained systems can be found [4] and [9]). The response of the multi-track channel to an array of input symbols (Ψ, x) is the vector.

$$y = h_0 x + \sum_{1 \leq j \leq L} h_j \Psi_{L+1-j} \tag{5}$$

where the value y_i, $1 \leq i \leq N$ corresponds to the i-th track, and Ψ_j denotes the jth column of Ψ. In this research, we will be interested in the following types of N-track ITI-reducing constraints imposed on elements of the vector $\boldsymbol{y}$.

1. The number of zeros between two nonzero elements in $\boldsymbol{y}$ is at least d, $d > 0$ ("zero" constraint),
2. Neighboring elements are either of the same sign or at least one of them is zero ("sign" constraint).

It is not difficult to see that both of these constraints reduce ITI. The constraint 1) requires that the signals in d tracks neighboring the ith track all have zero crossings when the signal in track i is nonzero, and thus completely cancels the ITI. This constraint is referred also to as a perpendicular minimum runlength constraint or a d-constraint. Constraint 2) does not cancel ITI, but rather allows only "constructive" ITI, which will in fact improve performance. This method is a generalization of approaches of Ahmed [18] and Davey [20].

The Shannon noiseless capacities of such constraints for various PR targets of interest are given in Table 1. It can be seen that the rate penalty for ITI constraints (especially for the "sign" constraint) is not high. We were able to construct a 100% efficient rate 3/4 three-track code with 256 states for the PR2 and PR4 channels for the "zero" constraint.

Table 1. Shannon capacities of ITI-reducing constraints

	PR2	PR2	PR4	PR4	EPR4	EPR4
N	*'sign'*	*'zero'*	*'sign'*	*'zero'*	*'sign'*	*'zero'*
1	0.91625	0.79248	0.91096	0.79248	0.87472	0.66540
2	0.92466	0.77398	0.92190	0.77398	0.87703	0.68491
3	0.92668	0.75000	0.92444	0.75000	0.87680	0.62652
4	0.92910	0.74009	0.92731	0.74009	0.87976	0.62696
5	0.91625	0.79248	0.91096	0.79248	0.87472	0.66540

5. ENUMERATIVE CODING AND LOW-COMPLEXITY ENCODER AND DECODER IMPLEMENTATIONS

The important question in this context is the design of high-rate multi-track codes and low complexity encoders and decoders for such codes. Substantial progress has been made in the theory of constrained codes using symbolic dynamics [27]-[28], as well as in low complexity encoding and decoding algorithms [29]. Despite this progress, the design of constrained codes remains difficult, especially for large constraint graphs such as multi-track constraint graphs. Because of the large graph sizes, novel encoding and decoding algorithms for constrained codes will be sought.

The first class of promising codes is multi-track enumerative encoders and decoders. The idea of enumerative coding

originates in the work of Labin and Aigrain [30], Kautz [31], and has been formulated as a general enumeration scheme for constrained sequences by Tang and Bahl [32] (see also Cover [33], [11]). It has been used by Immink et al. [34] as a practical method for the enumeration of (single-track) (d, k) sequences and by Orcutt and Marcellin [35] for multi-track (d, k) block codes. Our idea is to design such an encoder/decoder that does not require large memory for storing the constraint graph (because it would be prohibitively complex), but rather creates portions of the graph used in different stages of enumerative encoding/decoding on the fly.

The second class of low-complexity codes is based on loose multi-mode codes, as explained by Vasic et al. [36] (in the context of dc-free codes). The idea is to strictly impose the ITI constraint, while the second constraint (k constraint along the tracks) is imposed in a probabilistic fashion. Relaxing the second constraint is beneficial because it enables higher code rates and simplifies the encoder. Another advantage of this approach is it that allows for great flexibility in making tradeoffs between encoder complexity and the strength of the second constraint. To construct multi-mode codes for multi-track constraints, the methodology from [36] will be generalized. As opposed to a mono-mode encoder which is a finite automaton defined by (unique) next-state and output functions, in multi-mode codes there are more than one next-state and more than one output functions. In other words, there are multiple ways of translating an input sequence to an output sequence. The future research will be in improvements and generalizations of the limited tree-search multi-mode encoding algorithm developed in [36] to multi-track codes.

The constrained codes are used in conjunction with iteratively decodable codes such as LDPC codes. Bliss' reverse concatenation topology [37], in which the constrained encoder precedes the error control encoder, CAN be used. This scheme has an advantage over the classical scheme because of the reduced error propagation at the output of the constrained decoder. Good candidates for encoding algorithms are Fair's guided scrambling [38], Vasic's loose sequential constrained encoding algorithm [36], Immink's and van Vingardeen's sequence replacement [39], and loose constraint codes with low complexity encoding and decoding algorithms based on enumeration. A similar enumeration scheme was developed by Milenkovic and Vasic [40] for shaping mark length distribution. Another promissing direction is to combine multi-track constrained codes with codes on graphs. Pearl's probabilistic reasoning theory can be used to develop the belief propagation algorithms for multi-track constrained channels.

6. SUMMARY

In today's systems, the radial density is mostly limited by the mechanical design of the drive and the ability to accurately follow a track. In order to further increase radial density, multiple-head arrays have been developed that enable reading and writing data simultaneously on multiple tracks. Such heads can potentially provide both high density and high speed, but they suffer from inter-track interference (ITI). This ITI is a result of interfering signals induced in the heads from neighboring tracks. Today's recording systems have a large track pitch, and therefore ITI has negligible effect on performance. However, significant advances in coding and signal processing for multi-track recording channels are required before the potential large radial densities together with head arrays may be realized.

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