PERFORMANCE OF LDPC CODED SPREAD OFDM WITH CLIPPING

Sara Sandberg[1], *Cormac de Fréin*[2], *James P. LeBlanc*[1], *Bane Vasic*[3], *and Anthony D. Fagan*[2]

[1]Dept. of Computer Science and Electrical Engineering, Luleå University of Technology, Sweden
[2]Dept. of Electronic and Electrical Engineering, University College Dublin, Ireland
[3]Electrical and Computer Engineering Dept., University of Arizona, USA
sara.sandberg@ltu.se *cdefrein@ee.ucd.ie* *leblanc@ltu.se* *vasic@ece.arizona.edu* *tony.fagan@ucd.ie*

ABSTRACT

We present the performance of OFDM systems with coding, spreading, and clipping. Low-Density Parity-Check (LDPC) codes give coding gains and spreading by the Walsh Hadamard transform gives gains in terms of increased frequency diversity as well as reduced peak-to-average power ratio (PAPR) of the transmitted OFDM signal. By evaluating both the IFFT transform (OFDM) and the Walsh Hadamard transform in a single step, the number of operations needed for the spread OFDM system is actually *less* than for the conventional OFDM system. Reducing the PAPR is important in systems with clipping since it is related to the probability of clips. Each clip introduces clipping noise to the system which reduces the performance. Results of a clipped OFDM system with LDPC coding and spreading for an ETSI indoor wireless channel model are presented and compared with other systems. It is shown that there is a gain by spreading for LDPC coded OFDM systems, and especially for systems with clipping.

1. INTRODUCTION

In wireless communications, the channel is often time-varying due to relative transmitter-receiver motion and reflections. This time-variation, called fading, reduces system performance. With a high data rate compared to the channel bandwidth, multipath propagation becomes frequency-selective and causes intersymbol interference (ISI). A multicarrier OFDM system is known to transform a frequency-selective fading channel into parallel flat-fading subchannels if a cyclic prefix is used for preventing inter-block interference. The receiver complexity is thereby significantly reduced, since the equalizer can be implemented as a number of one-tap filters. In such a system, the data transmitted on some of the carriers might be strongly attenuated and could be unrecoverable at the receiver. Lately, spread spectrum techniques have been combined with the conventional OFDM to better exploit frequency diversity, [1][2]. This combination implies spreading information across all (or some of) the carriers by precoding with a unitary matrix and is in the following referred to as spread OFDM (SOFDM).

Another way to resist data corruption on fading subchannels is to use error-correcting codes. In [3], joint precoding and coding of the OFDM system is suggested with convolutional codes or turbo-codes and it is shown that there is a significant performance gain offered by introducing precoding to the coded transmission. In the last decade low-density parity-check (LDPC) codes, first invented by Gallager in 1962 [4], have attracted attention, see e.g. [5]. Serener *et al.* investigate the performance of SOFDM with LDPC coding in [6][7].

One of the major drawbacks with the OFDM system is its high peak-to-average power ratio (PAPR). A high PAPR corresponds to a high probability of clipping in the power amplifier in the transmitter or, alternatively, a large input power backoff. This implies reduced signal power, degrading bit error rate and for clipping even spectral spreading. There has been much research in the area of reducing the PAPR for OFDM systems, [8][9]. It is shown in [10] that precoding by the Walsh Hadamard (WH) matrix reduces the PAPR of the OFDM signal and the associated reduced probability of clipping distortion will increase the performance of the system. This precoding scheme has also been suggested for spreading, [1]. Surprisingly, the joint WH spreading and OFDM modulation can be performed by one single transformation that requires less operations than the IFFT alone, [11]. In this paper, the total performance gain of the WH spreading is investigated for an OFDM system with LDPC coding and clipping. In particular, the gain in bit-error-rate performance is analyzed for an ETSI channel model and results for clipped OFDM signals are provided.

The conventional and SOFDM system as well as the

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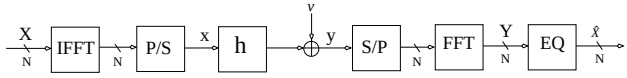

Figure 1: Conventional OFDM.

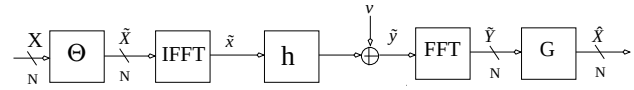

Figure 2: Spread OFDM.

channel model are described in Section 2 and the application of LDPC codes to OFDM in Section 3. Results of the WH spreading applied to clipped and non-clipped OFDM signals follow in Section 4. Finally, Section 5 gives some concluding remarks.

2. THE OFDM SYSTEM

The OFDM modulation is obtained by applying Inverse FFT (IFFT) to N message subsymbols, where N is the number of subchannels in the OFDM system. The baseband signal can be written

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{j2\pi nk/N}, \qquad n = 0, \ldots, N-1 \tag{1}$$

where X_k is symbol number k in a random message stream, also called the frequency domain signal. Thus the modulated OFDM vector can be expressed as

$$\mathbf{x} = IFFT\{\mathbf{X}\} \tag{2}$$

A block diagram describing the conventional OFDM system is shown in Figure 1. h is the channel impulse response and v is a vector of uncorrelated complex Gaussian random variables with variance σ^2. The output from the FFT is

$$Y = CX + W \tag{3}$$

where the diagonal matrix $C = diag(c_1, c_2, \ldots, c_N)$ gives the frequency domain channel attenuations and W is the FFT of the noise v. The elements of W are still uncorrelated complex Gaussian random variables with variance σ^2 due to the unitary property of the FFT. The zero-forcing equalizer is considered for conventional OFDM, but Wiener equalizers are also used. A practical implementation of OFDM usually uses a cyclic prefix in order to avoid intersymbol interference (ISI).

In SOFDM, the frequency domain signal is multiplied by a spreading matrix Θ before it is fed to the IFFT, Figure 2. The spreading considered here is the WH matrix that can be generated recursively for sizes a power of two [12]. The (2×2) WH matrix is given by

$$\mathbf{WH}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{4}$$

The $(2m \times 2m)$ WH matrix is given in terms of the $(m \times m)$ WH matrix,

$$\mathbf{WH}_{2m} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{WH}_m & \mathbf{WH}_m \\ \mathbf{WH}_m & -\mathbf{WH}_m \end{pmatrix}. \tag{5}$$

In the following Θ is assumed to be the $(N \times N)$ WH matrix. The WH transform is an orthogonal linear transform that can be implemented by a butterfly structure as the IFFT and since the WH and IFFT transforms can be combined and calculated with less complexity than the IFFT alone this means that the system complexity is reduced by applying WH spreading. At the receiver side, Wiener filtering is performed and the output of the Wiener filter is the vector $\hat{\mathbf{X}}$ given by $\hat{\mathbf{X}} = G\tilde{\mathbf{Y}}$. G is defined as

$$G = argmin_W \|W^H \hat{\mathbf{X}} - \mathbf{X}\|^2 = \Theta^T F \tag{6}$$

where F is a diagonal matrix,

$$F = diag\left(\frac{c_1^*}{|c_1|^2 + \sigma^2}, \ldots, \frac{c_N^*}{|c_N|^2 + \sigma^2}\right) \tag{7}$$

and $(\cdot)^*$ denotes conjugation. This means that the receiver consists of scalar channel equalization followed by the transpose of the spreading matrix, which in the case of unitary spreading equals the inverse. In the following, the noise power σ^2 and the frequency domain channel attenuations c_i are assumed to be known.

The wireless channel model used in this work is the ETSI indoor wireless channel model for HIPERLAN/2, [13]. Wireless channels can be modeled as Rayleigh fading channels. When a signal is sent from the transmitter across a wireless channel, it travels via many paths (due to reflections, refractions or diffractions) to the receiver. The different path lengths result in time delays or phase differences between the multipath components, which leads to constructive and destructive interference. The received signal y can be written $y = r_y e^{j\phi_y}$ where r_y is the amplitude and ϕ_y is the phase. The received amplitude is Rayleigh distributed with the probability density function

$$p(r_y) = \frac{r_y}{\delta} exp(-\frac{r_y}{2\delta}) \qquad 0 \le r_y < \infty \tag{8}$$

and the received phase is uniformly distributed with

$$p(\phi_y) = \frac{1}{2\pi} \qquad 0 \le \phi_y < 2\pi \tag{9}$$

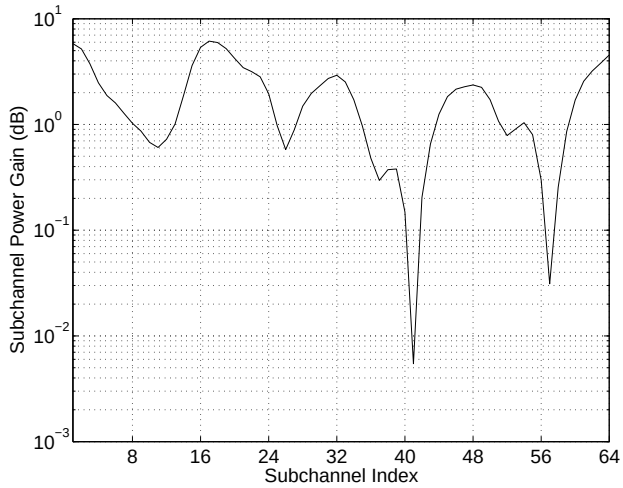

Figure 3: Example of a wireless channel response following the ETSI indoor wireless channel model for HIPERLAN/2.

where δ is the mean power of the waveform. A sample wireless channel response is shown in Figure 3. Two deep spectral nulls are apparent in this example.

In Conventional OFDM, each subchannel is assigned one subsymbol. In the example channel given above, the subsymbols transmitted on the subchannels in the close vicinity of the two deep spectral nulls, will have a high probability of error. However, using the spreading employed in this work, the information transmitted on each subchannel will be a linear combination of the original N subsymbols. This means that instead of a few subsymbols being severely affected by spectral nulls, several subsymbols are lightly affected. This approach leads to improved BER.

3. LDPC CODES FOR OFDM AND SOFDM

Error control codes used in this paper belong to a class known as *low-density parity-check* (LDPC) codes [4]. An LDPC code is a linear block code and it can be conveniently described through a graph commonly referred as a Tanner graph [14]. Such a graphical representation facilitates a decoding algorithm known as the *message-passing* algorithm. A message-passing decoder has been shown to virtually achieve Shannon capacity when long LDPC codes are used. In the next paragraph we will describe a specific class of LDPC codes used here. For more details on message passing decoding the reader is referred to an excellent introduction by Kschischang *et al.* [15].

Consider a linear block code $\mathscr{C}$ of length n defined as the solution-space (in $\mathbb{F}_2$) of the system of linear equations $\mathbf{Hx} = \mathbf{0}$, where $\mathbf{H}$ is an $m \times n$ binary matrix. The bipartite graph representation of $\mathscr{C}$ is denoted by $\mathscr{G}$. $\mathscr{G}$ contains a set of n variable nodes and a set of m check nodes, i.e. nodes corresponding to equations in $\mathbf{Hx} = \mathbf{0}$. A variable node is connected with a check node if it belongs to a corresponding equation. More precisely, The i-th column of $\mathbf{H}$ corresponds to a variable node x_i of the graph $\mathscr{G}$, and the j-th row of the matrix corresponds to a check node S_j of $\mathscr{G}$. The choice of a parity check matrix that supports the message-passing algorithm is a problem that has been extensively studied in recent years, and many random [16] and structured codes have been found [17]. We have chosen codes from a family of rate-compatible array codes, [18][19], because they support a simple encoding algorithm and have low implementation complexity.

The general form of the parity check matrix can be written as

$$H = \begin{pmatrix} I & I & I & \dots & P_{1,m/k} & I & \dots & I \\ 0 & I & P_{2,3} & \dots & P_{2,m/k} & P_{2,(m+1)/k} & \dots & P_{2,n/k} \\ 0 & 0 & I & & P_{3,m/k} & P_{3,(m+1)/k} & \dots & P_{3,n/k} \\ 0 & 0 & 0 & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & I & P_{m/k,(m+1)/k} & \dots & P_{m/k,n/k} \end{pmatrix}$$

where each of submatrices $P_{i,j}$, is a power of a $k \times k$ permutation matrix (see [19]). Notice that the row and column weights of H vary, i.e. the code is irregular. A column j in a set of m/k leftmost columns has the weight j, while the rest of the columns have weight m/k.

Figure 4 shows a block diagram describing the SOFDM system with LDPC coding, denoted by LDPC-SOFDM. The message bits are first encoded to codewords of length n and modulated to $n/2$ QPSK symbols. The modulation symbols are $\pm\sqrt{\mathscr{E}_s/2} \pm j\sqrt{\mathscr{E}_s/2}$, where $\mathscr{E}_s$ is the symbol energy. The modulated codeword is partitioned into blocks of N samples, where we assume that $n \geq 2N$, and each block is multiplied by the spreading matrix Θ. The spread signal is sent through the OFDM system and the Wiener filter. Both soft information from the output of the Wiener filter and the SNR for each subchannel are fed to the decoder. Since spreading averages SNR of different subchannels, a theoretical SNR taking the spreading into account is computed and used for the decoding. The theoretical SNR can be calculated from the output of the Wiener filter, which is

$$\hat{X} = G\tilde{Y} = \Theta^T FC\Theta X + \Theta^T FW. \tag{10}$$

Component k of $\hat{X}$ can be written

$$\hat{X}_k = \alpha X_k + \beta_k + \delta_k \tag{11}$$

where

$$\alpha = \frac{1}{N}\sum_{i=1}^{N} \mu_i \tag{12}$$

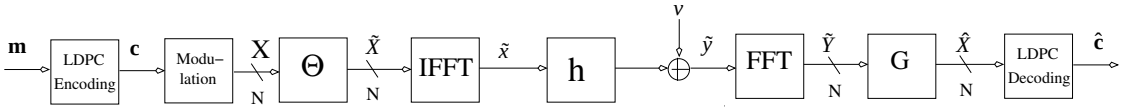

Figure 4: SOFDM with LDPC coding.

$$\beta_k = \sum_{i=1, i\neq k}^{N} \left(\sum_{j=1}^{N} \Theta_{jk}\Theta_{ji}\mu_j \right) X_i \tag{13}$$

and

$$\delta_k = \sum_{i=1}^{N} \Theta_{ik} \frac{c_i^*}{|c_i|^2 + \sigma^2} W_i. \tag{14}$$

Θ_{ij} is element (i, j) in the WH matrix, $\Theta_{ij} = \pm 1/\sqrt{N}$, and $\mu_i = |c_i|^2/(|c_i|^2 + \sigma^2)$. For large N the interference-plus-noise at the output of the Wiener filter $(\beta_k + \delta_k)$ can be considered to be Gaussian, [2], and approximated as noise. The theoretical SNR of subchannel k with spreading is

$$SNR_k = \frac{\mathrm{Var}\{\alpha X_k\}}{\mathrm{Var}\{\beta_k\} + \mathrm{Var}\{\delta_k\}} = \tag{15}$$

$$= \frac{\alpha^2 \mathscr{E}_s}{\sum_{i=1, i\neq k}^{N} \left(\sum_{j=1}^{N} \Theta_{jk}\Theta_{ji}\mu_j \right)^2 \mathscr{E}_s + \frac{\sigma^2}{N} \sum_{i=1}^{N} \frac{|c_i|^2}{(|c_i|^2+\sigma^2)^2}} \tag{16}$$

For conventional OFDM ($\Theta = I$), the SNR is simply

$$SNR_k = \frac{\mathrm{Var}\{c_i X_k\}}{\sigma^2} = \frac{|c_i|^2 \mathscr{E}_s}{\sigma^2} \tag{17}$$

4. RESULTS

To show the performance of LDPC-SOFDM, simulations are performed with rate 0.8 lattice codes and a codeword length of 1024. The maximum column weight of the parity-check matrix is 3. The performance is an average over different channel realizations of the ETSI indoor wireless channel model and the channel realizations are normalized to have energy N, that is,

$$\sum_{i=1}^{N} |c_i|^2 = N \tag{18}$$

Figure 5 shows the BER performance of different OFDM systems without clipping. The OFDM system has 64 subchannels and the channel is assumed to be constant during the transmission of one codeword. Both SOFDM and LDPC coding give a large gain compared with conventional OFDM, but there is also a gain by spreading of about 2.7 dB of the LDPC coded system at 10^{-5} bit-error-rate. Figure 6 shows that the performance does not change much with the number of subcarriers, which is also the size of the precoder. However, with a large number of subchannels, like 512, interleaving is necessary for LDPC-OFDM to get this performance.

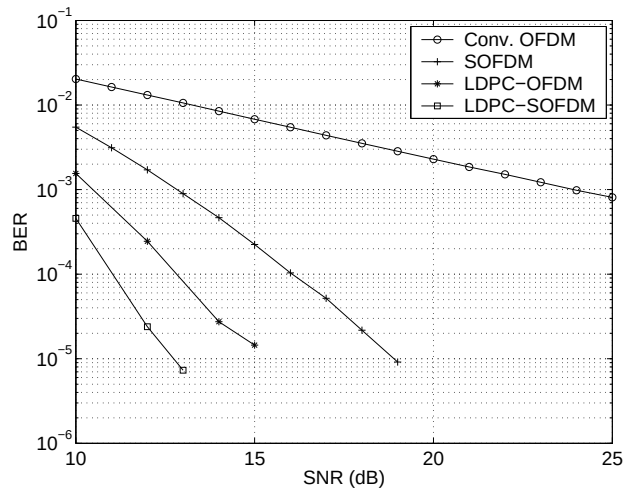

Figure 5: Performance of conventional OFDM, SOFDM, LDPC-OFDM, and LDPC-SOFDM for the case of 64 subchannels.

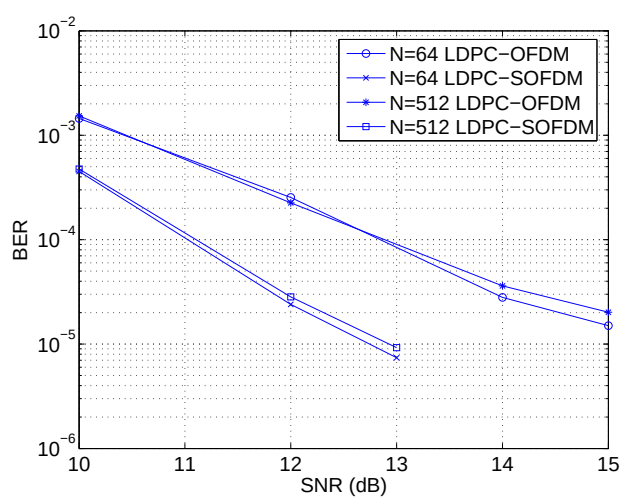

Figure 6: Performance of LDPC-OFDM and LDPC-SOFDM for both 64 and 512 subchannels.

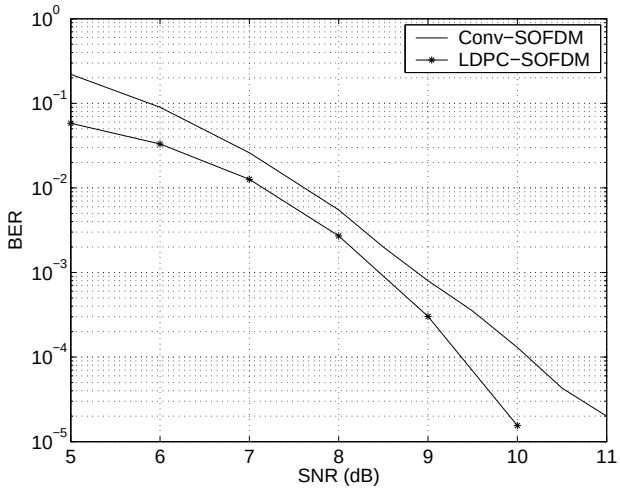

Figure 7: Comparison with a convolutional coded SOFDM system.

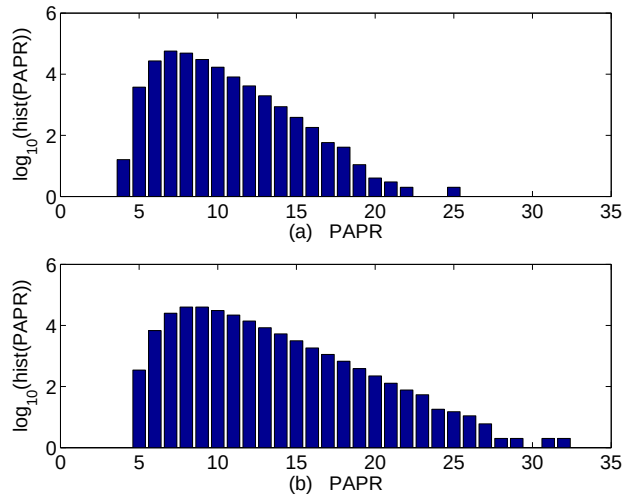

Figure 8: Histograms of the PAPR for an OFDM system with 64 subchannels, with (a) and without (b) spreading.

A comparison to the work in [2] for convolutional codes is shown in Figure 7. The frequency domain channel attenuations c_i in [2] are assumed to be independent identically distributed circular complex Gaussian random variables with variance 1. A convolutional encoder with constraint length 7 is used and the code rate is $3/4$. The spreading and equalizer is the same as in this paper and an OFDM system with 64 subchannels is used. We compare this with our LDPC-SOFDM system with rate 0.8 for the same channel parameters as in [2]. Figure 7 shows that the LDPC-SOFDM system performs better than the system with the convolutional code.

Figure 8 shows the reduction of PAPR that is the result of the WH spreading. The instantaneous PAPR for the kth OFDM block, and the overall PAPR are defined, respectively, by

$$PAPR_k = \frac{\|x(k)\|_\infty^2}{E\{\|x(k)\|_2^2\}/N} \tag{19}$$

$$PAPR = \frac{max_k\|x(k)\|_\infty^2}{E\{\|x(k)\|_2^2\}/N} \tag{20}$$

where $x(k)$ is the kth block of the time domain signal vector. However, in practice it is more useful to know the distribution of the instantaneous PAPR. In Figure 8 the overall PAPR has decreased by 1.1 dB and the mean instantaneous PAPR has decreased by 0.8 dB by spreading.

In many systems clipping occurs in the power amplifier and a reduction of PAPR reduces the number of clips. If $x_i = r_i e^{j\phi_i}$ denotes the input complex signal, the clipping of the baseband signal can be modeled as $\check{s}_i = \check{r}_i e^{j\phi_i}$, with

$$\check{r}_i = \begin{cases} r_i, & \text{for } r_i \le A_{max} \\ A_{max}, & \text{for } r_i > A_{max} \end{cases} \tag{21}$$

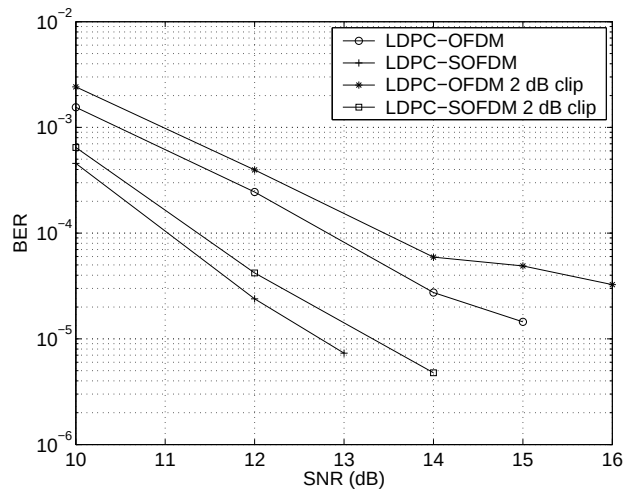

Figure 9: Performance of LDPC-OFDM and LDPC-SOFDM with 64 subchannels. The performance is shown both for no clipping and for a clipping ratio of 2 dB.

where A_{max} is the maximum output amplitude. The clipping ratio γ is defined as

$$\gamma = \frac{A_{max}}{\sqrt{\mathscr{E}_s}}. \tag{22}$$

Figure 9 shows the BER in a case where the signal is clipped with a clipping ratio of 2 dB. The total gain of spreading is around 4 dB at 10^{-5} bit-error-rate, compared to 2.7 dB when there is no clipping, since the PAPR reduction by spreading has reduced the number of clips. The effect of clipping noise to the SNR is not taken into account in the decoder. This result suggests that spreading is of extra value in systems where there is a high probability of clipping.

5. CONCLUSIONS

In this paper the BER performance of LDPC-SOFDM is investigated. The spreading considered is the WH transform which actually can *reduce* the complexity of the system. Results for the ETSI indoor wireless channel model show that using LDPC-SOFDM instead of LDPC-OFDM in a system with clipping gives a gain of 4 dB at a bit-error-rate of 10^{-5}. The gain is due to increased frequency diversity as well as reduced PAPR. The performance is also investigated for different number of subchannels and the results show that systems with different number of subchannels perform almost the same. However, a large number of subchannels increases the PAPR which in turn increases the probability of clips. Our results confirm that spreading enhances the performance of the OFDM system for the ETSI channel model and show that especially the performance in a system with clipping is increased, while the complexity is reduced.

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