Non-Gaussian Signal Detection: How much can massive MIMO help?

Ture Peken Ravi Tandon Tamal Bose
Department of Electrical and Computer Engineering
University of Arizona, Tucson, Arizona 85719
Email: {*turepeken*, *tandonr*, *tbose*}@email.arizona.edu

***Abstract*—The radio frequency spectrum is occupied with authorized and unauthorized user activities which might include noise and interference. Detection of signals-of-interest (SOI) and differentiation from non-signals-of-interest (NSOI) are therefore crucial for frequency use management. There is a wide variety of signals in a desired radio spectrum band, which leads to the application of Signal Intelligence (SIGINT) to detect and identify signals in real-time. In this paper, we study the problem of non-Gaussian signal detection when the receivers are configured with a large number of antennas (or the massive antenna regime). First, we investigate the performance of signal detection with massive MIMO when the transmitted signals are generated from a Gaussian distribution. For the detection of Gaussian signals, we consider the Neyman-Pearson (NP) detector. Then, we focus on the performance of non-Gaussian signal detection with massive MIMO, which is one of the main objectives of this paper. We show that the NP detector gives poor performance for non-Gaussian signals in low signal-to-noise-ratio (SNR). Therefore, we propose to use a bispectrum detector, which contains the Gaussian noise and reveals the non-Gaussian information that exists in the signal. We present the theoretical analysis for asymptotic behavior of Probability of False Alarm (P_{FA}) and Probability of Detection (P_D) when the transmitter sends Gaussian and non-Gaussian signals. We show the performance of signal detection (for both Gaussian and non-Gaussian signals) as a function of the number of antennas and sampling rate. We also obtain the scaling behavior of the performance in the massive antenna regime.**

I. Introduction

Detecting an unknown, non-Gaussian signal in Gaussian noise is crucial for both commercial and military usage. The problem of signal detection occurs in various commercial applications including smart city, environmental monitoring, smart homes, intelligent vehicular systems [1]. There is a rapid growth in wireless data traffic due to increase in wireless devices and applications. Therefore, efficient utilization of radio-spectrum is essential from the commercial perspective. Revealing the information about the spectrum usage is very critical for military, since it allows to understand the availability in adversarial environments.

Various studies on spectrum sensing has been done in the case when the transmitted signals are Gaussian [2], [3]. Effect of using multiple antennas on spectrum sensing has been also investigated [4], [5]. The optimal detector in NP sense has been proposed when the statistics of both noise and signal are known [6]. In general, the statistics of noise and transmitted signals are unknown, hence suboptimum detectors such as energy detector, sliding window matched filtering have been studied in the literature [7]. The application of higher-order statistics has been also proposed for suboptimum detection [8], [9]. The detection problem in the case of Gaussian signals when the receiver has the massive MIMO capability has been also considered [10], [11]. To the best of our knowledge, the problem of sensing, when the signal is non-Gaussian and the receiver is equipped with a large number of antennas has not been studied.

Main Contributions of this paper: We focus on the non-Gaussian signal detection in Gaussian noise within the massive MIMO framework. We investigate how much massive MIMO can help improve the performance of signal detection. We assume that the observed spectrum has active and inactive bands in which the user transmits signals actively and remains silent, respectively. In our analysis, we focus on one active band as shown in Fig. 1. The main contributions of this paper can be highlighted as follows.

First, we study the problem of detecting an unknown Gaussian signal in Gaussian noise by using NP detector. We present the asymptotic analysis in the large antenna regime (i.e., massive MIMO), and analyze the asymptotic behavior of P_{FA} and P_D. Then, we show the relationship between P_{FA} and P_D when the number of antennas (M) and observed samples (L) go to infinity under the constraint of $\sqrt{L}ln(M) \geq M$. We show the convergence of P_{FA} and P_D with M and L for different values of SNR by using MATLAB based simulations.

Second, we focus on the main problem of this paper. We investigate the non-Gaussian signal detection in Gaussian noise. In most practical situations, the signal is non-Gaussian or becomes non-Gaussian after going through a nonlinear propagation media. If the signal is non-Gaussian, NP detector does not give promising results. Therefore, we accomplish the non-Gaussian signal detection by using bispectrum. The bispectrum is defined as the Fourier transform of the third-order cumulant function. If a process is non-Gaussian, it has an infinite number of non-vanishing cumulants [12]. On the other hand, the bispectrum of a Gaussian signal is zero in the principal domain [13]. Therefore, a non-Gaussian signal can be detected by using its bispectrum, which will be nonzero. First, we estimate the bispectrum of the received L samples

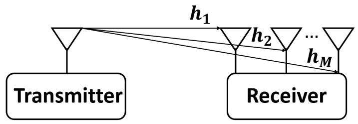

Fig. 1: An active band which has a transmitter with single antenna and a receiver with M antennas.

at the each of the M antennas of the receiver. Then, we get the average of bisectrum estimates over the antennas. After that, a generalized maximum likelihood ratio test (GLRT) is applied to the received signal in the bispectral domain to derive the P_{FA} and the P_D. Finally, we provide an asymptotic analysis of the non-Gaussian signal detection when M and L go to infinity. We show that the P_{FA} and the P_D converge to 0 and 1, respectively if we set $M >> L$. We conduct MATLAB simulations to verify the results of non-Gaussian signal detection with high sample rate in large antenna regime.

II. Gaussian Signal Detection in Gaussian Noise with Massive MIMO

A. System Model

Suppose that the receiver has M antennas and each antenna receives L samples. We assume that transmitted signal samples are independent zero-mean random variables with complex Gaussian distribution. Let $\boldsymbol{x} = [x_1, ..., x_L]$ be the transmitted symbols vector and x_i is the ith sample with distribution $\mathcal{CN}(0, \sigma_s^2)$. The hypothesis of the transmitted signal being active and inactive is shown by H_0 and H_1, respectively. The additive noise samples at different antennas are independent zero-mean Gaussian random variables. Let $\boldsymbol{Y} = [\boldsymbol{y}_1, ..., \boldsymbol{y}_L] \in \mathbb{C}^{M\times L}$ be a complex matrix containing the observed signals at M antennas, $\boldsymbol{N} = [\boldsymbol{n}_1, ..., \boldsymbol{n}_L]$ be a complex matrix and the vector $\boldsymbol{n}_i \in \mathbb{C}^{M\times 1}$ represents additive white Gaussian noise at the receiver in ith sensing time with distribution $\mathcal{CN}(0, \sigma_n^2 I_M)$. The received signal at the M-antenna receiver under the two hypotheses is:

$$H_0 : \boldsymbol{y}_i = \boldsymbol{n}_i, \qquad i = 1, ..., L, \tag{1}$$

$$H_1 : \boldsymbol{y}_i = \boldsymbol{h} x_i + \boldsymbol{n}_i, \qquad i = 1, ..., L, \tag{2}$$

where $\boldsymbol{h} \in \mathbb{C}^{M\times 1}$ denotes the channel gain vector between the transmitter and M antennas of the receiver. The channel gain vector, i.e., $\boldsymbol{h}$, is assumed to be constant at each sensing time. The probability density function (PDF) under the two hypotheses can be written as:

$$p\left(\boldsymbol{Y}|H_0, \sigma_n^2\right) = \prod_{l=1}^{L} \frac{e^{\left(-\frac{1}{2}\boldsymbol{y}_l^H \boldsymbol{C}_w^{-1} \boldsymbol{y}_l\right)}}{(2\pi)^{\frac{M}{2}}\, det^{\frac{1}{2}}\left(\boldsymbol{C}_w\right)}, \tag{3}$$

$$p\left(\boldsymbol{Y}|H_1, \boldsymbol{h}, \sigma_n^2, \sigma_s^2\right) = \prod_{l=1}^{L} \frac{e^{\left(-\frac{1}{2}\boldsymbol{y}_l^H (\boldsymbol{C}_s + \boldsymbol{C}_w)^{-1} \boldsymbol{y}_l\right)}}{(2\pi)^{\frac{M}{2}}\, det^{\frac{1}{2}}\left(\boldsymbol{C}_s + \boldsymbol{C}_w\right)}, \tag{4}$$

where $\boldsymbol{C}_w = \sigma_n^2 \boldsymbol{I}_M$ and $\boldsymbol{C}_s = \sigma_s^2 \boldsymbol{h}\boldsymbol{h}^H$. $\boldsymbol{I}_M$, $det(.)$, and e denote the $M \times M$ identity matrix, the determinant of a matrix and Euler's number, respectively. The NP detector is given as:

$$L\left(\boldsymbol{Y}\right) = \frac{p\left(\boldsymbol{Y}|H_1, \boldsymbol{h}, \sigma_n^2, \sigma_s^2\right)}{p\left(\boldsymbol{Y}|H_0, \sigma_n^2\right)} > \gamma \tag{5}$$

for a given threshold γ.

B. Asymptotic Behavior of Probability of False Alarm and Probability of Detection

In this section, we give the asymptotic analysis of the Gaussian signal detection by using NP detector.

After some mathematical manipulations, the NP detector in (5) becomes equivalent to deciding H_1 if

$$\frac{\sigma_s^2}{2} \sum_{l=1}^{L} |\boldsymbol{h}^H \boldsymbol{y_l}|^2 > \gamma^{'} \tag{6}$$

where $\gamma^{'} = \frac{2\sigma_n^4}{\sigma_s^2}\left(1 + \sigma_s^2 \boldsymbol{h}^H \boldsymbol{C}_w^{-1} \boldsymbol{h}\right) ln \left[\gamma \left(1 + \sigma_s^2 \boldsymbol{h}^H \boldsymbol{C}_w^{-1} \boldsymbol{h}\right)^{\frac{L}{2}}\right]$.

For the NP detector in (6), P_D can be rewritten as:

$$P_D = Prob\left(\sum_{l=1}^{L} \tilde{\boldsymbol{y}}_l{}^H \boldsymbol{\Lambda} \tilde{\boldsymbol{y}}_l > \gamma^{'} | H_1\right). \tag{7}$$

where $Prob(.|.)$ denotes conditional probability and $\tilde{\boldsymbol{y}}_l = \left(\boldsymbol{C}_s + \boldsymbol{C}_w\right)^{-\frac{1}{2}} \boldsymbol{y}_l \sim \mathcal{CN}\left(0, \boldsymbol{I}_M\right)$. Here, $\boldsymbol{\Lambda} = diag\left\{\boldsymbol{h}^H\left(\boldsymbol{C}_s + \boldsymbol{C}_w\right)\boldsymbol{h}, 0...0\right\}$ and $diag\left\{d_1, ..., d_M\right\}$ denotes a $M \times M$ diagonal matrix with d_i as the ith diagonal element. In (7), $\sum_{l=1}^{L} \tilde{\boldsymbol{y}}_l{}^H \boldsymbol{\Lambda} \tilde{\boldsymbol{y}}_l$ has a scaled Chi-square distribution with $2L$ degrees of freedom. Therefore, the P_D can be simplified as:

$$P_D = \frac{\Gamma\left(L, \frac{\gamma^{'}}{\boldsymbol{h}^H (\boldsymbol{C}_s + \boldsymbol{C}_w) \boldsymbol{h}}\right)}{\Gamma(L)}, \tag{8}$$

where $\Gamma\left(\alpha\right) = \int_0^\infty t^{\alpha-1} e^{-t} dt$ and $\Gamma\left(\alpha, x\right) = \int_x^\infty t^{\alpha-1} e^{-t} dt$ are the complete and upper incomplete gamma functions, respectively. P_{FA} can be also derived similarly as:

$$P_{FA} = \frac{\Gamma\left(L, \frac{\gamma^{'}}{\boldsymbol{h}^H \boldsymbol{C}_w \boldsymbol{h}}\right)}{\Gamma(L)}. \tag{9}$$

From the law of large numbers [14]:

$$\lim_{M\to\infty} \frac{1}{M} \boldsymbol{h}^H \boldsymbol{h} \xrightarrow{a.s.} \mathbb{E}\left\{|h_i|^2\right\}, \tag{10}$$

where $\mathbb{E}\left\{|h_i|^2\right\}$ is the expected value of the i.i.d random variable $|h_i|^2$. $\mathbb{E}\left\{.\right\}$ denotes the expectation of a random variable. By using the central limit theorem, the definition of $\boldsymbol{C}_s$ and $\boldsymbol{C}_w$, and (10), (8) and (9) can be written as in the following when $L, M \to \infty$:

$$P_D = Q\left(\frac{\frac{\gamma^{'}}{M\mathbb{E}\{|h_i|^2\}(M\sigma_s^2 \mathbb{E}\{|h_i|^2\} + \sigma_n^2)} - 2L}{2\sqrt{L}}\right), \tag{11}$$

$$P_{FA} = Q\left(\frac{\frac{\gamma^{'}}{M\sigma_n^2 \mathbb{E}\{|h_i|^2\}} - 2L}{2\sqrt{L}}\right). \tag{12}$$

Let us define a new threshold such that:

$$\gamma^{''} = \frac{\gamma^{'}}{M\sigma_n^2\mathbb{E}\left\{|h_i|^2\right\}}. \quad (13)$$

By using the relationship between inverse Q-function and inverse error function, the threshold required to achieve $P_{FA} = \epsilon$ becomes:

$$\gamma^{''} = 2\sqrt{L}\left(\sqrt{2}erf^{-1}(1-2\epsilon)+\sqrt{L}\right). \quad (14)$$

By using (13) and (14), (11) can be written as:

$$P_D = Q\left(\frac{\sqrt{2}erf^{-1}(1-2\epsilon)+\sqrt{L}}{(M\sigma_s^2\mathbb{E}\left\{|h_i|^2\right\}+\sigma_n^2)} - \sqrt{L}\right). \quad (15)$$

For the case when $L, M \to \infty$, the term inside Q-function in (12) is dominated by $\sqrt{L}ln(M) - \sqrt{L}$. In this case, (12) can be approximated as:

$$P_{FA} \approx Q\left(\sqrt{L}ln(M) - \sqrt{L}\right). \quad (16)$$

Moreover, the term inside Q-function in (16) goes to ∞. Then, P_{FA} will always converge to 0 when $L, M \to \infty$. Given that $P_{FA} = \epsilon$, (15) can be approximated when $L, M \to \infty$ as:

$$P_D \approx Q\left(\frac{-\sqrt{L}ln(M)}{M} - \sqrt{L}\right). \quad (17)$$

If $\sqrt{L}ln(M) \geq M$ and $L, M \to \infty$, over all term inside (17) goes to $-\infty$. Therefore, P_D converges to 1.

III. Non-Gaussian Signal Detection in Gaussian Noise with Massive MIMO

A. System Model

The receiver has M antennas and each antenna receives L samples. We assume that the transmitted signal samples are independent, identically distributed random variables with non-Gaussian distribution. It is also assumed that the signal has constant variance and bispectrum. Furthermore, $\boldsymbol{x}$, H_0 and H_1 denote the transmitted symbols vector and two hypotheses defined in Section II, respectively. It is assumed that the additive noise samples at different antennas are independent zero-mean Gaussian random variables with variance σ_n^2. The complex matrix $\boldsymbol{Y}$ and the vector $\boldsymbol{n}_i$, which are defined as in Section II, denote the observed signals at the M antennas of the receiver and the additive white Gaussian noise at the M antennas of the receiver in ith sensing time, respectively. The channel gain vector $\boldsymbol{h}$ is assumed to be constant at each sensing time.

B. Bispectrum Estimation for the Receiver with Single Antenna

Signal detection in higher order spectrum domains like the bispectrum and trispectrum gives promising results when the received signal is non-Gaussian due to its ability of suppressing Gaussian noise and retaining the non-Gaussian information simultaneously [15]. Therefore, we use the bispectrum for non-Gaussian signal detection in our analysis.

The bispectrum of a random signal is generated by the Fourier transform of its third order cumulant, which is given as [16]:

$$C_x(\tau_1, \tau_2) = \mathbb{E}\left\{x(t)x(t+\tau_1)x(t+\tau_2)\right\} \quad (18)$$

The bispectrum of a discrete-time random sequence is given as:

$$B(w_1, w_2) = \mathbb{E}\left\{X(w_1)X(w_2)X(w_3)\right\}, \quad (19)$$

where

$$X(w) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} x(n)e^{-jwn} \quad (20)$$

is the Fourier transform of $\{x(n)\}$, and $w_3 = 2\pi k - w_1 - w_2$, k is 0 or 1. Bispectrum must be estimated from the observed signals at the receiver. In this section, we study the case that the receiver has one antenna. Then, the received symbols is denoted by the complex vector $\boldsymbol{y} \in \mathbb{C}^{L\times 1}$. The raw bispectrum estimate can be calculated for L samples as:

$$\hat{B}(j,k) = \frac{1}{L}Y(j)Y(k)Y(-j-k), \quad (21)$$

where $Y(k)$ is the discrete Fourier transform of L samples at the receiver $\{y(n)\}$, $n = 1, ..., L$:

$$Y(k) = \sum_{n=0}^{L-1} y(n)e^{-j\frac{2\pi}{L}kn}. \quad (22)$$

$Y(j+L) = Y(j)$ and $Y(L-j) = Y^*(j)$. It has been shown in [13] that the bispectrum of a Gaussian signal is zero over the all principal domain. Therefore, the bispectrum estimation is implemented in the principal domain in this paper. The principal domain of $\hat{B}(j,k)$ is the triangular grid:

$$R = \begin{Bmatrix} (j,k) : 0 < j \leq \dfrac{L}{2} \\ 0 < k \leq j \\ 2j + k \leq L \end{Bmatrix} \quad (23)$$

where L is even. The variance of raw bispectrum estimate of L samples is given as [12]:

$$var\left(\hat{B}(j,k)\right) = L\left(var\left(Y(j)\right)var\left(Y(k)\right)var\left(Y(-j-k)\right)\right). \quad (24)$$

Since the variance increases with L, we perform a time-domain smoothing by dividing L samples to N_B number of blocks where each block has a length of L_B. First, the discrete Fourier transform of each block in L samples is calculated as:

$$Y_m(k) = \sum_{n=0}^{L_B-1} y(n)e^{-j\frac{2\pi}{L_B}kn}, \qquad m = 1, 2, ..., N_B. \quad (25)$$

$Y_m(j+L) = Y_m(j)$ and $Y_m(L-j) = Y_m^*(j)$. From this transform of the signal, the bispectrum estimate of the signal is evaluated as in the following:

$$\hat{B}_m(j,k) = \frac{1}{L_B}Y_m(j)Y_m(k)Y_m(-j-k). \quad (26)$$

Then, the individual bispectrum estimates are averaged for all the L samples which yields the smoothed estimate:

$$\hat{B}_{avg}(j,k) = \frac{1}{L}\sum_{m=1}^{N_B} Y_m(j)Y_m(k)Y_m(-j-k). \quad (27)$$

C. Bispectrum Estimation for the Receiver with Multiple Antennas

In this section, we study bispectrum estimation when the receiver has multiple antennas. In this case, bispectrum is estimated from the L observed samples at the M antennas of the receiver. The received symbols stored in the complex matrix $\boldsymbol{Y}$ are divided into M blocks, each of length L. Raw bispectrum estimate can be calculated for L samples in mth antenna as follows:

$$\hat{B}_m(j,k) = \frac{1}{L}Y_m(j)Y_m(k)Y_m(-j-k), \quad (28)$$

where $Y_m(k)$ is the discrete Fourier transform of L samples in mth antenna $y_m(n)$ for $m = 1, ..., M$:

$$Y_m(k) = \sum_{n=0}^{L-1} y_m(n)e^{-j\frac{2\pi}{L}kn}. \quad (29)$$

$Y_m(j+L) = Y_m(j)$ and $Y_m(L-j) = Y_m^*(j)$. The principal domain of $\hat{B}_m(j,k)$ is given as in (23) and the variance of $\hat{B}_m(j,k)$ can be calculated similar to (24). The smoothed bispectrum estimate is evaluated by averaging the raw estimates for all M antennas as:

$$\begin{aligned}\hat{B}_{avg}(j,k) &= \frac{1}{M}\sum_{m=1}^{M}\hat{B}_m(j,k) \\ &= \frac{1}{ML}\sum_{m=1}^{M} Y_m(j)Y_m(k)Y_m(-j-k).\end{aligned} \quad (30)$$

The variance of the smoothed bispectrum estimate becomes:

$$\begin{aligned}var(\hat{B}_{avg}(j,k)) &= var(\frac{1}{M}\sum_{m=1}^{M}\hat{B}_m(j,k)) \\ &= \frac{1}{M}var(\hat{B}_m(j,k)).\end{aligned} \quad (31)$$

$\hat{B}_{avg}(j,k)$ has independent real and imaginary parts with the following variance:

$$\begin{aligned}var(Re(\hat{B}_{avg}(j,k))) &\approx var(Im(\hat{B}_{avg}(j,k))) \\ &\approx \frac{var(\hat{B}_{avg}(j,k))}{2}.\end{aligned} \quad (32)$$

Let us define independent and normally distributed random variables $X_1, X_2, ..., X_n$ with mean μ_i and unit variances. Then, the random variable $\sum_{i=1}^{k} X_i^2$ is distributed according to the non-central chi-squared distribution with k degrees of freedom and a non-centrality parameter λ. Here, λ is related to the mean of the random variables X_i as $\lambda = \sum_{i=1}^{k}\mu_i^2$. Moreover, the asymptotic distribution of $\hat{B}_{avg}(j,k)$ is complex normal and independent for each frequency pair (j,k) [12]. By using these facts, the distribution of the statistic t:

$$\begin{aligned}t &= \frac{Re(\hat{B}_{avg}(j,k))^2 + Im(\hat{B}_{avg}(j,k))^2}{\frac{var(\hat{B}_{avg}(j,k))}{2}} \\ &= \frac{2\left|\hat{B}_{avg}(j,k)\right|^2}{var(\hat{B}_{avg}(j,k))},\end{aligned} \quad (33)$$

is non-central chi-square with two degrees of freedom and non-centrality parameter:

$$\hat{\beta}(j,k) = \frac{2\left|\hat{B}_{avg}(j,k)\right|^2}{\frac{L}{M}(var(Y_m(j))var(Y_m(k))var(Y_m(-j-k)))} \quad (34)$$

where $\hat{\beta}(j,k)$ is defined as skewness function.

D. Asymptotic Behavior of Probability of False Alarm and Probability of Detection

In this section, we give the asymptotic analysis of the non-Gaussian signal detection by using bispectrum based detector.

$\hat{B}_{avg}(j,k)$ is complex Gaussian with zero mean under H_0, so the distribution of the statistic t, which is defined in (33) is a central chi-square with two degrees of freedom:

$$t = \frac{2M}{L\sigma_n^6}\left|\hat{B}_{avg}(j,k)\right|^2. \quad (35)$$

The statistic T_0, which is obtained by summing the individual bin statistics, is an approximation of the central chi-squared with $2P$ degrees of freedom under H_0. P denotes the number of bins in the principal domain [17]. Then, T_0 and P_{FA} are given as:

$$\begin{aligned}T_0 &= \sum t \\ &= \frac{2M}{L\sigma_n^6}\sum_{region}\left|\hat{B}_{avg}(j,k)\right|^2.\end{aligned} \quad (36)$$

$$P_{FA} = Prob(T_0 > \gamma | H_0), \quad (37)$$

When P is large, T_0 can be approximated by the Fisher's approximation [18] as:

$$\sqrt{2T_0} \sim \mathcal{N}(\sqrt{4P-1}, 1). \quad (38)$$

Applying (38) in (37), we obtain the following expression for P_{FA}:

$$P_{FA} = \frac{1}{2} - \frac{1}{2}erf\left(\sqrt{2\gamma} - \sqrt{4P-1}\right). \quad (39)$$

Then, the threshold to achieve $P_{FA} = \epsilon$ becomes:

$$\gamma = \frac{1}{2}\left(erf^{-1}(1-2\epsilon) + \sqrt{2P - \frac{1}{2}}\right)^2. \quad (40)$$

In the principal domain, which is the triangular grid given in (23), there are $\frac{L^2}{12}$ bins. Therefore, (40) can be written as:

$$\gamma = \frac{1}{2}\left(erf^{-1}(1-2\epsilon) + \sqrt{\frac{L^2}{6} - \frac{1}{2}}\right)^2. \quad (41)$$

The statistic T_1, which is obtained by summing the individual bin statistics, is an approximation of the non-central chi-squared with $2P$ degrees of freedom under H_1. By using (33) and (34), the non-centrality parameter is given as:

$$\beta = \sum_{region} \hat{\beta}(j,k). \tag{42}$$

Then, the P_D can be defined as:

$$P_D = Prob(T_1 > \gamma | H_1). \tag{43}$$

If P is large, (43) can be approximated as:

$$\frac{1}{2P}T_1 \sim \mathcal{N}\left(1 + \frac{\beta}{2P}, \frac{P+\beta}{P^2}\right). \tag{44}$$

Therefore,

$$P_D = \frac{1}{2} - \frac{1}{2}erf\left(\frac{\gamma - 2P - \beta}{2\sqrt{P+\beta}}\right), \tag{45}$$

where

$$\beta = \sum_{region} \frac{2\left|\hat{B}_{avg}(j,k)\right|^2}{\frac{L}{M}(var(Y_m(j))var(Y_m(k))var(Y_m(-j-k)))}.$$

If the threshold γ is held as constant, P_D is a monotonically increasing function of the expression:

$$\lambda = \frac{2P + \beta}{2\sqrt{P+\beta}}. \tag{46}$$

If $\beta >> 2P$, $\lambda \approx \frac{\sqrt{\beta}}{2}$. Since we assume that the signal has constant variance and bispectrum, β is linearly proportional with $P\frac{M}{L}$. Let us assume that $\beta = cP\frac{M}{L}$, where c is a constant. Then, $\lambda \approx \frac{\sqrt{\beta}}{2}$ if $\frac{M}{L} >> 1$. Then, from (46), for $\beta >> 2P$:

$$\lambda \approx \frac{1}{2}\sqrt{\frac{(c)PM}{L}}. \tag{47}$$

By using $P = \frac{L^2}{12}$, (45) can be written as:

$$P_D = \frac{1}{2} - \frac{1}{2}erf\left(-\frac{1}{4}\sqrt{\frac{(c)ML}{3}}\right), \tag{48}$$

for $\gamma << 2P + \beta$. Therefore, P_D converges to 1, while M, L go to ∞ and $M >> L$.

IV. Results

We observe the asymptotic results of P_D and P_{FA} by using MATLAB simulations. First, we obtain the asymptotic results when the transmitted signal is Gaussian. Convergence regions of P_D and P_{FA} for different values of SNR is shown in Fig. 2. When $M > 202$ and $L > \left(\frac{M}{log(M)}\right)^2$, P_D converges to 1 and P_{FA} converges to 0 for -20 dB SNR. For -10 dB SNR, P_D and P_{FA} converges to 1 and 0 respectively when $M > 40$ and $L > \left(\frac{M}{log(M)}\right)^2$. When SNR equals to 0 dB, P_D converges to 1 and P_{FA} converges to 0 for $M > 17$ and $L > \left(\frac{M}{log(M)}\right)^2$. Finally, P_D converges to 1 and P_{FA} converges to 0 for $M > 15$ and $L > \left(\frac{M}{log(M)}\right)^2$ when SNR is 10 dB. According to

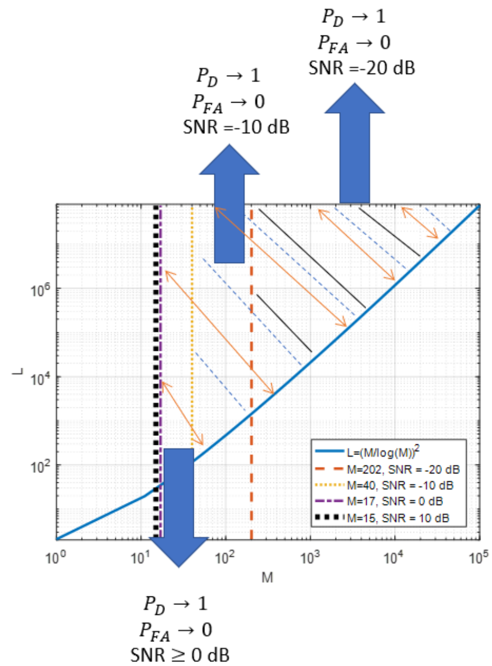

Fig. 2: The convergence of P_D and P_{FA} while L, M go to infinity when the distribution of the transmitted signal is Gaussian.

these results, P_D and P_{FA} converge to 1 and 0 respectively with smaller number of antennas while SNR increases.

Fig. 3 shows P_{FA} versus P_D obtained by derived approximations and simulation when $L = \left(\frac{M}{log(M)}\right)^2$, $L = 2\left(\frac{M}{log(M)}\right)^2$, and $L = 3\left(\frac{M}{log(M)}\right)^2$. $M = 10$ and SNR is -10 dB in all cases. It can be seen that the relationship between P_D and P_{FA} approaches to ideal case while $L > \left(\frac{M}{log(M)}\right)^2$ for both approximations and simulation. It can be also observed that simulation results are upper bounded by derived approximations.

For the non-Gaussian signal case, we compare P_{FA} versus P_D obtained by simulation, when our proposed and NP detectors are used. We also show P_{FA} versus P_D based on the derived approximations of our proposed detector. These results, which can be seen in Fig. 4, are obtained for $M = L = 10$, $M = 2L = 20$, and $M = 3L = 30$. Transmitted signal has a Bernoulli distribution (with parameter 0.5) and the SNR is 0 dB in all cases. It can be seen that P_D converges to 1 while $M >> L$. Simulation results are upper bounded by derived approximations as in the Gaussian signal case. Moreover, it can be observed that our proposed bispectrum based detector outperforms NP detector if the signal is non-Gaussian.

V. Conclusion

We investigated the effect of large number of antennas and sample rate on the quality of non-Gaussian signal detection. First, we studied detection of an unknown Gaussian signal in

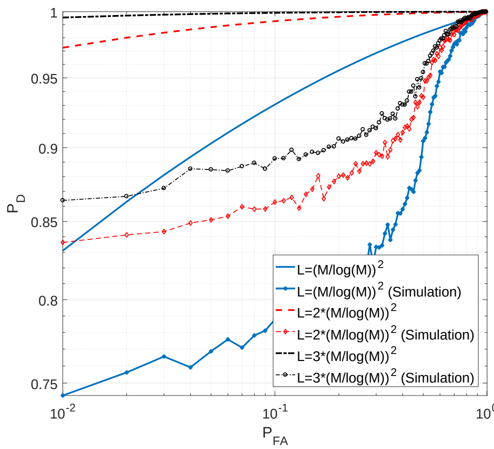

Fig. 3: Approximations and simulation results of P_{FA} versus P_D when the transmitted signal is Gaussian.

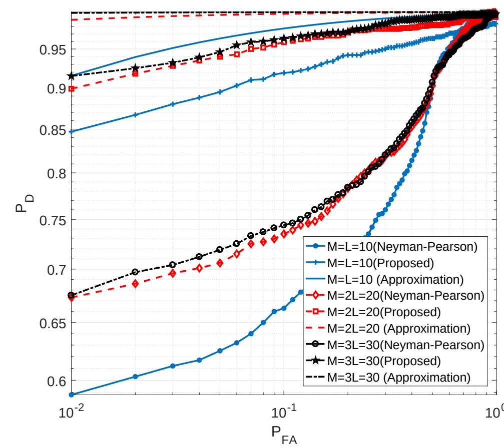

Fig. 4: Approximations and simulation results of the proposed detector and comparison of the results with NP detector when the transmitted signal is non-Gaussian.

Gaussian noise. We analyzed the performance of the Gaussian signal detection by using a NP detector when the number of antennas (M) and sample size (L) are large. We showed the required condition for the relationship between L and M while both of them go to infinity to obtain P_{FA} and P_D close to 0 and 1, respectively. We then studied the performance of the non-Gaussian signal detection in the large antenna regime with a large number of samples. We used a GLRT in bispectral domain to obtain the expressions for P_{FA} and P_D as functions of L and M. We presented an asymptotic analysis for the P_{FA} and the P_D when the observed signal is non-Gaussian. We showed that the P_D goes to 1 while M, L approach to infinity and $M >> L$. Then, we observed the asymptotic behavior of P_{FA} and P_D when the transmitted signal is Gaussian or non-Gaussian by using MATLAB simulations.

In the future, we aim to study the robust signal detection with massive MIMO. In this case, we will assume that all the statistical information related to the transmitted signal is unknown. We aim to investigate how much the large number of antennas at the receiver can improve the quality of signal detection.

VI. Acknowledgements

This work was partially supported by US NSF through grants CCF-1559758 and CNS-1715947. It was also partly supported by the Broadband Wireless Access and Applications Center (BWAC); NSF Award No. 1265960.

References

[1] J. A. Stankovic, "Research directions for the internet of things," *IEEE Internet of Things Journal*, vol. 1, no. 1, pp. 3–9, Feb 2014.

[2] S. Atapattu, C. Tellambura, and H. Jiang, *Energy Detection for Spectrum Sensing in Cognitive Radio*. Springer Publishing Company, Incorporated, 2014.

[3] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum sensing for cognitive radio," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 849–877, May 2009.

[4] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE Transactions on Wireless Communications*, vol. 9, no. 2, pp. 814–823, February 2010.

[5] M. Goldenbaum and S. Staczak, "On multiantenna sensor networks with interference: Energy consumption vs. robustness," in *2012 International ITG Workshop on Smart Antennas (WSA)*, March 2012, pp. 125–132.

[6] J. Neyman and E. S. Pearson, "On the Problem of the Most Efficient Tests of Statistical Hypotheses," *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 231, no. 694-706, pp. 289–337, 1933.

[7] B. Porat and B. Friedlander, "Adaptive detection of signals," in *ICASSP '85. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 10, Apr 1985, pp. 1266–1269.

[8] D. Kletter and H. Messer, "Suboptimal detection of non-Gaussian signals by third-order spectral analysis," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 6, pp. 901–909, Jun 1990.

[9] M. J. Hinich, "Detecting a transient signal by bispectral analysis," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 7, pp. 1277–1283, Jul 1990.

[10] A. Shirazinia, S. Dey, D. Ciuonzo, and P. S. Rossi, "Massive mimo for decentralized estimation of a correlated source," *IEEE Transactions on Signal Processing*, vol. 64, no. 10, pp. 2499–2512, May 2016.

[11] F. Jiang, J. Chen, A. L. Swindlehurst, and J. A. Lpez-Salcedo, "Massive mimo for wireless sensing with a coherent multiple access channel," *IEEE Transactions on Signal Processing*, vol. 63, no. 12, pp. 3005–3017, June 2015.

[12] M. Wolinsky, "Invitation to bispectrum," The University of Texas at Austin, Applied Research Laboratories ARL-TR-88-7, Tech. Rep., Feb 1988.

[13] M. J. Hinich, "Testing for gaussianity and linearity of a stationary time series," *Journal of Time Series Analysis*, vol. 3, no. 3, pp. 169–176, 1982.

[14] H. Cramer, *Random Variables and Probability Distributions*, ser. Cambridge Tracts in Mathematics. Cambridge University Press, 1970.

[15] C. L. Nikias and M. R. Raghuveer, "Bispectrum estimation: A digital signal processing framework," *Proceedings of the IEEE*, vol. 75, no. 7, pp. 869–891, July 1987.

[16] D. R. Brillinger, *Time Series: Data Analysis and Theory*. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics, 2001.

[17] C. M. Pike, J. A. Tague, and E. J. Sullivan, "Transient signal detection in multipath: a bispectral analysis approach," in *Proceedings ICASSP 91: 1991 International Conference on Acoustics, Speech, and Signal Processing*, Apr 1991, pp. 1489–1492 vol.2.

[18] M. G. Kendall and A. Stuart, *The Advanced Theory of Statistics*. New York: Hafner, 1958.