Automatic Controls for Transition Maneuver of VTOL MAV

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The MAVs are open-loop unstable and an autopilot’s ability to handle them needs to be established.

Here are two problems:

1. What is the steady state for the MAV under different conditions (attack angle, freestream velocity …)?
2. How to design a stable controller for the horizontal to vertical transition?
### MAV specifications and properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mini-Vertigo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span (cm)</td>
<td>31</td>
</tr>
<tr>
<td>Length (cm)</td>
<td>21</td>
</tr>
<tr>
<td>Height (cm)</td>
<td>16</td>
</tr>
<tr>
<td>Wing area (cm²)</td>
<td>486</td>
</tr>
<tr>
<td>Nacelle area (cm²)</td>
<td>60</td>
</tr>
<tr>
<td>Motor area (cm²)</td>
<td>-</td>
</tr>
<tr>
<td>Rudder area (cm²)</td>
<td>40</td>
</tr>
<tr>
<td>Rudder area (cm²)</td>
<td>35</td>
</tr>
<tr>
<td>Rudder diameter (cm)</td>
<td>14</td>
</tr>
<tr>
<td>Location from apex (cm)</td>
<td>4.16</td>
</tr>
</tbody>
</table>

### Components of MAVs

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airframe</td>
<td>Composite structure</td>
<td>60</td>
</tr>
<tr>
<td>Motor, propeller</td>
<td>MP Jet AC 22/4-60D</td>
<td>50</td>
</tr>
<tr>
<td>Lithium-Polymer Battery</td>
<td>APC 7”-5”</td>
<td></td>
</tr>
<tr>
<td>Battery</td>
<td>3 cell, 11.1V, 910mAh</td>
<td>65</td>
</tr>
<tr>
<td>Autopilot</td>
<td>Paparazzi autopilot</td>
<td>54</td>
</tr>
<tr>
<td>Modem</td>
<td>MaxStream XBee Pro</td>
<td>9</td>
</tr>
<tr>
<td>Servos</td>
<td>Blue Arrow BA-TS-2.5</td>
<td></td>
</tr>
</tbody>
</table>

| Total                   | 249 |
From the appearance, we can see that the MAV is symmetric, so we consider the horizontal to vertical transition process in a x-z.
\[
m \frac{d^2 x}{dt^2} = -W \sin \theta + L \sin \alpha - D \cos \alpha + T
\]

\[
m \frac{d^2 z}{dt^2} = -W \cos \theta - L \cos \alpha - D \sin \alpha
\]

\[
I_{yy} \frac{d^2 \theta}{dt^2} = M_{AW} + z_{AC} \cdot (T + L \sin \alpha - D \cos \alpha) + x_{AC} \cdot (L \cos \alpha + D \sin \alpha)
\]

Where \( M_{AW} \) is the moment generated by the wing around the aerodynamic center, varying by \( L \) and \( \alpha \).
After multiplying the rotation matrix, the state space representation of the system is

\[
\begin{bmatrix}
\dot{x}_E \\
\dot{z}_E
\end{bmatrix} = \frac{\rho VS}{2m} \begin{bmatrix}
-C_D & -C_L \\
-C_L & C_D
\end{bmatrix} \begin{bmatrix}
\dot{x}_E \\
\dot{z}_E
\end{bmatrix} + R(\theta) \begin{bmatrix}
\frac{T}{m} \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
g
\end{bmatrix}
\]
\[ L = C_L(\alpha, T)\left(\frac{\rho V^2}{2} S\right) \]
The dynamic model of the MAV is highly nonlinear, it is not feasible to use an exact analytical approach to determine steady state points.

An iterative numerical method was utilized to search the five parameters: $V$, $\alpha$ (angle of attack), $\delta_t$ (throttle-setting), $\delta_e$ (elevon deflection) and $\theta$ (pitching-angle).

For $\theta=0$: $\pi/1800$: $\pi/2$
  For $\alpha=0$: $\pi/1800$: $\pi/2$
    For $\delta_t=0$: $0.01$: $1.00$
      For $V=1$: $1$: $15$
        (verify horizontal and vertical equilibrium)...

If First two equations...
  For $\delta_e=-\pi/6$: $\pi/18000$: $\pi/6$
    (verify angular equilibrium)...

...
The red-dashed box on each of the plots encloses the points which are within the domain of the empirical data (which was used to develop the dynamic model).
MV2 Steady State Parameter Values

- theta = 0°
- theta = 10°
- theta = 20°
- theta = 30°
- theta = 40°
- theta = 50°
- theta = 60°
- theta = 70°
- theta = 80°
- theta = 90°

Velocity (m/s) vs. Parameter Values
MV2 Steady State Parameter Values

- $\theta = 90^\circ$ deg
- $\theta = 70^\circ$ deg
- $\theta = 80^\circ$ deg
- $\theta = 60^\circ$ deg
- $\theta = 50^\circ$ deg
- $\theta = 40^\circ$ deg
- $\theta = 30^\circ$ deg
- $\theta = 20^\circ$ deg
- $\theta = 10^\circ$ deg
- $\theta = 0^\circ$ deg

Delta values for different angles are plotted against each other.
The highest simulated pitch angle that MV2 can achieve without descending is 75 degrees.
$L = C_L(\alpha, T)(\frac{\rho V^2}{2} S)$

$C_L$: Nonlinear relationships with $V$, $\delta_t$

- $V=5, \delta_t=0.55$
- $V=10, \delta_t=0.55$
- $V=15, \delta_t=0.55$
- $V=5, \delta_t=0.60$
- $V=10, \delta_t=0.60$
- $V=15, \delta_t=0.60$
- $V=5, \delta_t=0.65$
- $V=10, \delta_t=0.65$
- $V=15, \delta_t=0.65$
- $V=5, \delta_t=0.70$
- $V=10, \delta_t=0.70$
- $V=15, \delta_t=0.70$
High-rate transition simulation

Low-rate transition simulation
Piecewise Hybrid Controller

A proportional controller for pitch, where the elevon angle \( \delta_e = K_e (\theta_f - \theta) \), where \( \theta_f \) represents the desired pitch.

\[
K_e = \begin{cases} 
K_{e1} & 0 \leq \alpha \leq \alpha_1 \\
K_{e2} & \alpha_1 \leq \alpha \leq \alpha_2 \\
K_{e3} & \alpha_2 \leq \alpha \leq \frac{\pi}{2} 
\end{cases}
\]

Open-loop controller

Require less tuning than simple closed loop designs, but not appropriate for this model.

Feed-forward Controller

A useful technique to mitigate measureable disturbances through a model, rather than control based solely on measured difference from a desired set point.
• Developed simulator based on wind tunnel data
• Validated simulator with steady-state flight analysis
• Validated controllers with flight demonstrations
• The simulator results may be used for future autopilot design