UAV Search : Maximising Target Acquisition

Author: Hussain Al-Helal

239 N Grande Ave. Unit B

Tucson, Arizona, 85745

Branch Counselor: Brenda Huettner

IEEE Member #90594047

April 10, 2009
UAV Search: Maximising Target Acquisition
## Contents

1. Introduction ................................................. 1
2. Quadrotor Helicopter Dynamics .................... 2
3. Optimal Control ............................................ 5
4. Search and Rescue Scenario ......................... 7
5. Maximising Target Acquisition .................... 8
6. Analysis of Controller .................................. 11
7. Conclusion and Future Work ......................... 13
UAV Search : Maximising Target Acquisition

Abstract

This paper is an academic experience report describing analysis of optimal control techniques for simulated quad-rotor unmanned aerial vehicles (UAVs) performing search and rescue missions. Analysis of the controller and guidance laws governing the UAV are described in detail culminating in a closed form expression describing the probability of detection over a certain field.

1 Introduction

The introduction of autonomous unmanned vehicles has posed an interesting challenge to engineers. After removing the human navigator from the loop, more robust dynamic automation systems must be implemented to ensure safe and efficient control. Simulated scenarios have been developed investigating the use of quadrotor helicopters for search and rescue missions. According to the authors of Sliding mode control of a quadrotor helicopter [6] a quadrotor helicopter is a four rotor helicopter that was first built in 1907 by the Breguet Brothers. The simulated UAV was based on the STARMAC II (Fig. 1) rotorcraft developed by researchers at Stanford University as mentioned in [4].

![STARMAC quadrotor helicopter developed at Stanford University](image)

Figure 1: STARMAC quadrotor helicopter developed at Stanford University [3]

The quadrotor helicopter is a nonlinear system, hence the dynamics will be altered using
sliding mode control to improve motion control. Sliding mode control implements high-frequency switching control; control switches from one smooth condition to another are inherent in its variable structure control. This allows for the system to ‘slide’ between the boundaries of the controller thus the trajectory of the system is aptly named sliding mode.

The current system contains a single quadrotor helicopter at a height of 150m, under the command from a central ground station. A spiral search pattern is initiated which the quadrotor follows until the target is acquired within the quadrotor’s cameras field of view. Once the quadrotor ‘sees’ the target, a target tracking controller is implemented which allows the quadrotor helicopter to follow the target at a safe distance.

For the current system, target acquisition is not guaranteed. Many simulations provide evidence that target acquisition can be a slow procedure. Utilising a fixed height for searching also increases the chances of the target evading the quadrotor helicopter. The aim of this paper is to introduce a closed form solution to determining a search height which maximises the probability of detection of a target over a certain field.

Once a closed form expression has been achieved, a scenario will be introduced to analyse the performance of the proposed solution. The search height of a single quadrotor helicopter will be calculated to maximise the probability of detection of a ground vehicle starting 2.2km away. The area describing possible locations of the ground vehicle will be introduced and absolute certainty of target acquisition will be defined as the field of view of the camera totally encompassing the area where the ground vehicle could possibly exist.

2 Quadrotor Helicopter Dynamics

Accurate simulation requires well-defined dynamics. By studying the free body diagram (Fig. 2a) of a typical quadrotor helicopter, it is possible to derive well-defined dynamics for the system. According to Jason Hansen, [2], a proposed derivation of the nonlinear dynamics can be performed in North-East-Down (NED) inertial and body fixed coordinates as detailed
Moments generated about the centre of mass can be described as follows in [3].

\[
M_y = I_{yy} \ddot{\theta} = (T_3 r) - (T_1 r)
\] (1)

Figure 2: (a) Free body diagram of a typical quadrotor [2]. (b) Alternative free body diagram of quadrotor helicopter [3].

Assuming a linear thrust-to-torque relationship and identical force generation across each axis control of the attitude can be related to the following

\[
T_1 = T - \delta T
\]
\[
T_3 = T + \delta T
\] (2)

As \( T \) corresponds to nominal thrust then \( \delta T \) describes the deviation from nominal thrust. With \( \theta = 0 \), nominal thrust \( T \) relates to the required force output by each rotor to satisfy gravitational equilibrium. This is represented mathematically as

\[
T = \frac{mg}{4}
\] (3)

By equation (2), it follows that

\[
T_{total} = T_1 + T_3 = (T - \delta_T) + (T + \delta_T) = 2T
\] (4)

Note the resulting force is equal as previously mentioned. Combining (4) with (1) allows
for

\[ M_y = I_{yy} \ddot{\theta} = T_3 r - T_1 r \]
\[ M_y = I_{yy} \ddot{\theta} = (T + \delta T) r - (T - \delta T) r \]  \hspace{1cm} (5)
\[ M_y = I_{yy} \ddot{\theta} = 2\delta T r \]

By Laplace transform of (5) we achieve

\[ I_{yy} s^2 \theta(s) = 2\delta T(s) r \]  \hspace{1cm} (6)

Transposition of formulae of (6) results in an expression relating the quadrotor’s attitude and \( \delta T \)

\[ \frac{\theta(s)}{T(s)} = \frac{2r}{I_{yy} s^2} \]  \hspace{1cm} (7)

Combining the forces corresponding to the x and z components of the quadrotor allows for a derivation of the translational acceleration as follows

\[ F_z = m \ddot{z} = T_1 \cos \theta + T_2 \cos \theta + T_3 \cos \theta + T_4 \cos \theta - mg \]
\[ F_x = m \ddot{x} = T_1 \sin \theta + T_2 \sin \theta + T_3 \sin \theta + T_4 \sin \theta \]  \hspace{1cm} (8)

Combining (2) and (8) allows for

\[ F_z = m \ddot{z} = (T - \delta T) \cos \theta + T \cos \theta + (T + \delta T) \cos \theta + T \cos \theta - mg \]
\[ F_x = m \ddot{x} = (T - \delta T) \sin \theta + T \sin \theta + (T + \delta T) \sin \theta + T \sin \theta \]  \hspace{1cm} (9)
\[ F_z = m \ddot{z} = 4T \cos \theta - mg \]
\[ F_x = m \ddot{x} = 4T \sin \theta \]

Which can then be linearised about \( \theta \) as follows
\[ F_z = m\ddot{z} = 4T - mg \]  \hspace{1cm} (10)

\[ F_x = m\ddot{x} = 4T\theta \]

If one assumes \( mg \) corresponds to a disturbance in the system then a Laplace transform will yield the following

\[ F_z = ms^2Z(s) = 4T(s) \] \hspace{1cm} (11)

\[ F_x = ms^2X(s) = 4T\theta(s) \]

By transposition of formulae of (11) one yields a relationship describing the quadrotor’s angle and resulting position. The transfer function is as follows

\[ \frac{Z(s)}{T(s)} = \frac{4}{ms^2} \] \hspace{1cm} (12)

\[ \frac{X(s)}{\theta(s)} = \frac{T(s)}{ms^2} \]

Deriving the aforementioned transfer functions allow for a description of the translational states as a function of the delta thrust generated by the rotors according to the author of [2].

### 3 Optimal Control

As mentioned in [1], for a control system to be considered optimal the system parameters must be adjusted such that the performance index is a minimum value. The performance index is realised as follows \( J = \int g(x, u, t)dt \) where \( x \) denotes the state vector, \( u \) denotes the control vector and \( t_f \) denotes final time.

Inherent to search and rescue missions, search patterns must be utilised in order to track a potential target. A guidance controller should be developed in such a way permitting the UAV to navigate in a spiral pattern whilst searching for its potential target. An ideal search pattern would minimise the time taken to locate the target whilst maximising the probability
that the target is located. In designing a controller to ensure the UAV conducts the desired flight path, a form of sliding mode control can be applied.

Sliding Mode Control (SMC) can be adapted to provide a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision and disturbances [5]. By implementing Integral Sliding Mode (ISM) robust control throughout the flight envelope can be assured. ISM control occurs in two parts, a standard successive loop closure applied to the linear plant and ISM techniques to reject disturbances.

Linear approximations can be made regarding altitude error dynamics inherent to a hovering quadrotor helicopter. The approximation is described as follows

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u + \xi(g, x)
\end{align*}
\]  

Here \(x_1, x_2 = (r_z, des - r_z)\) relate to the altitude error states, the control input, \(u = \sum_{i=1}^{4} u_i\) and the bounded model of disturbances and dynamic uncertainty, \(\xi(\cdot)\) as mentioned in [5]. A proportional and derivative loop gain can be utilised to improve stability,

\[
\begin{align*}
u &= u_p + u_d \\
u_p &= -K_p x_1 - K_d x_2
\end{align*}
\]  

Disturbance rejection is achieved by designing the sliding surface, \(s\), as follows

\[
\begin{align*}
s &= s_0(x_1, x_2) + z \\
s_0 &= \alpha(x_2 + k x_1)
\end{align*}
\]  

This forces state trajectories towards the manifold \(s = 0\). A conventional sliding mode has been designed, \(s_0\), with integral control based on the parameter \(z, \alpha, k \in \mathbb{R}\) describe positive constants. Implementing \(V = \frac{1}{2}s^2\) Lyapunov function, convergence of the sliding manifold can be guaranteed by determining a control component \(u_d\) such that \(V < 0\).
\[ \dot{V} = s \dot{s} = s[\alpha \dot{x}_2 + k \dot{x}_1] + \dot{z} \]  
\[ \dot{V} = s[\alpha (u_p + u_d + \xi(g, x)) + kx_2 + \dot{z}] < 0 \]  

If \( \dot{z} = -\alpha (u_p + kx_2) \) then the above condition holds true hence \( u_d \) is guaranteed to satisfy the following

\[ s[u_d + \xi(g, x)] < 0, \alpha > 0 \]  

Disturbances, \( \xi(g, x) \), are bounded by \( \gamma \) therefore one must represent the control component as \( u_d = -\lambda s \) with \( \lambda \in \mathbb{R} \). Applying this knowledge to (17) gives

\[ \dot{V} = s[\alpha (-\lambda s + \xi(g, x))] \]
\[ \dot{V} \leq \alpha [-\lambda |s|^2 + \gamma |s|] < 0 \]  

From (18) it is obvious, \( \lambda |s| - \gamma > 0 \) thus, the sliding mode condition holds when

\[ \lambda |s| > \frac{\gamma}{\lambda} \]  

Using the derived input, the dynamics are guaranteed to evolve such that \( s \) decays to within the boundary layer of the sliding manifold. A spiral guidance controller was chosen with equation \( r = m \theta \) shown in Fig. 3a. The separation between the UAV and ground vehicle displayed in Fig. 3a can be calculated as follows

\[ R_i = \sqrt{(x_{UAV} - x_{gv})^2 + (y_{UAV} - y_{gv})^2} \]  

4 Search and Rescue Scenario

In order to analyse the search and rescue capabilities of the UAV, a test scenario was formulated. In this search and rescue scenario the UAV is supposed to be tracking a specific
ground vehicle, and communication exists between the UAV and a central command station. Initially, the location of the ground vehicle is not detected, so the UAV conducts a spiral search pattern at a fixed height until the ground vehicle is located.

Visualisation of objects is simulated through a camera which presents the ground vehicle as a pixel location on the map. The reported pixel location can be mapped to a world location which in turn allows for UAV waypoint navigation tracking of the ground vehicle. The controller of the UAV tracks the ground vehicle by ensuring that this object exists within the field of view of the camera.

5 Maximising Target Acquisition

Many parameters are involved in maximising the probability of finding the ground vehicle. One such parameter includes the length of the spiral between the UAV and ground vehicle at time $T = 0$. Knowing the description of the spiral one can derive the spiral length.

\[ r = m\theta \]  

(21)

Taking the derivative of (22) gives
\[
\frac{dr}{d\theta} = m
\]  

(22)

Hence the length of the spiral can be found as follows

\[
L = \int_{\theta_0}^{\theta_{max}} m \sqrt{1 + \theta^2} d\theta
\]

\[
L = \left[ \frac{m \theta_{max}}{2} \sqrt{1 + \theta_{max}^2} + \frac{1}{2} \ln |\theta_{max} + \sqrt{1 + \theta_{max}^2}| \right]_{\theta_0}^{\theta_{max}}
\]

(23)

With (23) and the UAV’s average speed it is now possible to determine the time it takes for the UAV to navigate to the start point of the ground vehicle.

\[
t = \frac{L}{V_{UAV}}
\]

(24)

The location of the ground vehicle after the elapsed time needs to be predicted. One method for mapping the possible location includes describing circles with radius \( R_{gv} \) about the location of the ground vehicle at time \( T = 0 \). The radius of the circle is simply calculated as follows

\[
R_{gv} = V_{gv_{max}} t
\]

(25)

Once the area the ground vehicle may be located in is described, one may determine a suitable height such that the field of view of the UAV camera encapsulates the entire area. If the entire area is encapsulated by the UAV’s camera then one may have total confidence that the ground vehicle has been detected if one assumes no visual obstructions exist. A cameras field of view is related to the focal length, \( f \), of the lens implemented. The focal length can be determined as follows

\[
f = \frac{ccd_y}{\tan(\frac{\pi}{7})}
\]

(26)
From (26) the angle of view of the camera can be defined as

\[
\begin{align*}
\text{aov}_y &= 2 \arctan \left( \frac{\text{c}\text{d}_y}{f} \right) \\
\text{aov}_x &= 2 \arctan \left( \frac{\text{c}\text{d}_x}{f} \right)
\end{align*}
\]  

(27)

Now the field of view can be described as

\[
\begin{align*}
\text{fov}_y &= h \tan(\text{aov}_y) \\
\text{fov}_x &= h \tan(\text{aov}_x)
\end{align*}
\]  

(28)

Figure 4: (a) Angle of View of UAV Camera. (b) Field of View of UAV Camera.

Figure 5: (a) Field of View of UAV Camera with absolute guarantee of locating ground vehicle. (b) Varying the height inherently varies the field of view of the UAV hence altering the probability of visualising the ground vehicle.
The maximising factor to determine is the height of the UAV. Transposing (28) results in an equation for the height of the UAV

\[
h = \max\left( \frac{f_{ov_y}}{\tan(aov_y)}, \frac{f_{ov_x}}{\tan(aov_x)} \right)
\] (29)

Maximising the probability of capturing the ground vehicle implies that the field of view must be greater or equal to the circular area describing the possible location of the ground vehicle. From (28) and (25), one can infer that the following relationship assures the visualisation of the ground vehicle

\[
f_{ov_y} = f_{ov_x} \geq 2R_{gv}
\] (30)

Transposing (30) results in a more refined relationship between the height of the UAV and the encapsulation of the possible ground vehicle locations within the field of view of the camera. This relationship is defined below

\[
h \geq \max\left( \frac{2R_{gv}}{\tan(aov_y)}, \frac{2R_{gv}}{\tan(aov_x)} \right)
\] (31)

In order to guarantee target acquisition, the maximum value of the two equations present in (31). Guaranteeing target acquisition in this case assumes that no visual obstructions exist such as tunnels or bridges. In reality this will probably not be the case.

6 Analysis of Controller

The following search and rescue mission is presented for analysis. Imagine a ground vehicle whose average speed is $15m/s^{-1}$ and at time $T = 0$ is approximately $2.2km$ away from the UAV as shown in Fig. 6. The UAV has an average speed of $25m/s^{-1}$, is it possible to derive a height for the UAV to have absolute confidence the target will be acquired? Based off equations presented prior to this section one can begin to investigate the possibility.
The time taken to reach the ground vehicles initial position based on (24) and (23) for the test scenario can be calculated as follows

\[
L = \frac{200}{2} \sqrt{1 + \left(\frac{7\pi}{2}\right)^2} + \frac{1}{2} \ln \left| \frac{7\pi}{2} + \sqrt{1 + \left(\frac{7\pi}{2}\right)^2} \right| \\
L = 12.4494km \tag{32}
\]

\[
t = \frac{12.4494}{25} = 497.977s
\]

Once the time has been determined, a circle describing the possible locations of the ground vehicle must be determined. This circle’s centre will appear at the beginning location of the ground vehicle, the radius can be calculated based on (25) as follows

\[
R_{gv} = (15)(497.977) = 7.46966km \tag{33}
\]

The field of view of the camera has to be manipulated such that it encapsulates the entire circle with radius \(R_{gv}\) described in (33). With a fixed focal length camera one parameter that influences the field of view is the height of the UAV. A flying height providing the required field of view can be calculated once the focal length and angle of view are determined. Determining the focal length based on (26) can be done as follows

\[
f = \frac{0.024}{2 \tan\left(\frac{\pi}{7}\right)} = 24.918mm \tag{34}
\]
Angles of view can now be calculated based on (34) and (27)

\[
aov_y = 2 \arctan\left(\frac{0.024}{0.024918}\right) = 0.482 \text{rads}
\]

\[
aov_x = 2 \arctan\left(\frac{0.036}{0.024918}\right) = 0.722 \text{rads}
\]

(35)

The height can finally be determined based on (31) the computation is as follows

\[
h \geq \max\left(\frac{(2)(7.46966k)}{\tan(0.482)}, \frac{(2)(7.46966k)}{\tan(0.722)}\right)
\]

\[
h \geq 2.485 km
\]

(36)

It has been determined that in order to guarantee target acquisition of a ground vehicle starting 2.2 km with \(\theta = \frac{\pi}{7}\) from the UAV, the UAV must be flying at a height of 2.485 km. The controller provides guarantee of target acquisition in this case in under 500 seconds. Providing a method for determining boundaries for target acquisition can allow for a better understanding of search and rescue missions. The success of the mission becomes measurable and further analysis can be conducted to optimise search methods.

Figure 7: Diagram representing the computed height required to acquire the target

7 Conclusion and Future Work

The work presented in this paper introduces a method for analysing the effectiveness of controllers used for UAV search and rescue missions. From the calculations derived, one is able to compute parameters required to maximise the probability that a search and rescue mission will be successful. A search and rescue mission was presented for analysis. The ground vehicle had an average speed of 15 ms\(^{-1}\) and at time \(T = 0\) was approximately 2.2 km
away from the UAV. The UAV had an average speed of $25m/s$, it was determined that the height of the UAV to have absolute confidence the target will be acquired was $2.485km$. The computed height does indeed guarantee target acquisition within $497.977$ s. One now has the ability to determine the fastest moving target they can acquire if the required flight altitude is $1.5km$ and the target must be acquired within $500$ seconds with UAV maximum speed of $25m/s$.

The work presented in this paper relied on the following assumptions; no visual obstructions existed between the UAV and ground vehicle, the ground vehicle started somewhere on the spiral and the UAV’s camera was always perpendicular to the surface of the earth. Future work would include the expansion of the aforementioned exceptions and deriving an optimal control scheme mitigating the problems introduced by these assumptions. Such mitigations could include the introduction of a control scheme if the target disappears from the field of view due to visual obstructions such as tunnels. One such scheme could take advantage of the hovering ability inherent to helicopters, the UAV might increase its height until all exits of the tunnel are present in its field of view and then hover at this height until the ground vehicle appears back in the field of view or until a certain amount of time has elapsed.

Another interesting topic to investigate in the future would be the deployment of multiple UAV’s conducting search patterns. Analysis into the speedup of this implementation and efficiency of the spiral search pattern used in this paper against other accepted search patterns would provide useful results for those researching this area.

**Acknowledgements**

This work would not have been possible without the efforts of Gyorgy Balogh, Himanshu Neema, Harmon Nine, Gabor Karsai, and Janos Sztipanovits of the Institute for Software Integrated Systems. Special thanks are due to Gabe Hoffman, Claire Tomlin, and Hal Tharp.
for their generous advice in the development of the quad-rotor controllers.

References


