Multicast Rendezvous in Fast-varying DSA Networks

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Appendix A
Proof of Theorem 2

T_{AMQFH} takes values in $\{0, 1, \ldots, n-1\}$. In (3), the probability that T_{AMQFH} equals i is the probability that the first i slots are non-rendezvous slots (i.e., $\prod_{j=1}^{i}(1 - \Delta(\delta_j))$) and the $i+1$st slot is a rendezvous slot (i.e., $\Delta(\delta_{i+1})$). In AMQFH, SUs may rendezvous during a quorum slot or a randomly assigned slot. Accordingly, $\Delta(\delta_i)$, which represents the probability that slot i is a rendezvous slot, is computed by considering all combinations of i randomly assigned slots and $k+1-i$ quorum slots.

The probability that slot i is a quorum slot (δ_i) depends on i. δ_i is computed by conditioning on the type (quorum/non-quorum) of all slots j such that $j < i$. Because the quorum slots in the uniform k-arbiter quorum system are consecutive, there are only two cases to consider:

Case 1: All slots $j < i$ are quorum slots. This case occurs with probability $(n-i+1)/n$.

Case 2: All slots except one are quorum slots. This case occurs with probability $(i-1)/n$.

In Case 1, the probability that slot i is a quorum slot is equal to $\frac{\lfloor \frac{kn}{k+1} \rfloor - i + 2}{n-i+1}$, and in Case 2 this probability equals $\frac{\lfloor \frac{kn}{k+1} \rfloor - i + 3}{n-i+1}$.

Appendix B
Proof of Theorem 3

T_{CMQFH} takes values in $\{0, 1, \ldots, n-1\}$. Θ represents the probability that a given slot is a rendezvous slot. Note that in contrast to Δ (for AMQFH), Θ is independent of the slot index. Θ can be derived in a similar way of deriving Δ, after considering the two following observations:

- In CMQFH, the probability that a given slot is a quorum slot in the FH sequence that uses prime number p_i is $1/p_i$, and the probability that it is a randomly assigned slot is $1 - 1/p_i = (p_i - 1)/p_i$. Note that, in contrast to δ (for AMQFH), this probability is independent of the slot index. This is because the quorum slots are equally-spaced in CRT quorums.
- In a CMQFH frame of length $p_1 \times p_2 \ldots \times p_k$, there is only one time slot where all the k SUs are at a quorum slot. Therefore, consider a given slot, the probability that all the k SUs are at a quorum slot in this given slot is $1/(p_1 \times p_2 \times \ldots \times p_k)$.

Appendix C
Proof of Theorem 4

Recall the definition of the expected HD in Section 3.3.2. ϕ represents the number of randomly assigned slots in each frame. To obtain $\mathbb{E}[H_{\text{AMQFH}}]$, note that SUs select the same quorum with probability $1/\varphi$ and select different quorums with probability $1 - 1/\varphi = (\varphi - 1)/\varphi$. The expected HD in the first case is equal to $\frac{L-1}{L} \times (\phi + 1) \ / \ n$. The first term (i.e., $\frac{L-1}{L}$) represents the probability that a randomly assigned slot is assigned two different channels at the two FH sequences, the second term (i.e., $\phi + 1$) is the number of slots where at least one of the two FH sequences is at a randomly assigned slot, and the third term (i.e., n) is the frame length. In the second case, the expected HD is the same, except that the number of slots where at least one of the two FH sequences is at a randomly assigned slot is equal to ϕ instead of $\phi + 1$. To obtain H^*_{AMQFH}, note that in this case different SUs are assumed to pick different quorums, and their randomly assigned parts are assumed to be non-overlapping.

Appendix D
Proof of Theorem 5

To obtain $\mathbb{E}[H_{\text{CMQFH}}]$, note that in CMQFH, the number of common quorum slots between two FH sequences with prime numbers p_i and p_j is equal to $\frac{y}{p_i p_j}$, where y is the frame length. Hence, the number of different slots equals $y - \frac{y}{p_i p_j} = y\left(1 - \frac{1}{p_i p_j}\right)$. Also, the probability that two SUs pick a pair of prime numbers p_i and p_j is $1/k^2$ if p_i and p_j need to be different and $\binom{k}{2}$ if p_i and p_j can be the same. The probability that a randomly assigned slot is assigned two different channels at the two FH sequences equals $\frac{L-1}{L}$. Finally, because for each pair of p_i and p_j the HD is added twice, we divide by 2. To obtain H^*_{CMQFH}, note that in this case different SUs pick different quorums, and their randomly assigned parts are non-overlapping.