

CORE: A Combinatorial Game-theoretic Framework for COexistence REndezvous in DSA Networks

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Abstract—Rendezvous is a vital process for connection establishment and recovery in dynamic spectrum access (DSA) networks. Frequency hopping (FH) is an effective rendezvous method that does not rely on a predetermined control channel. Recently, quorum-based FH approaches have been proposed for enabling asynchronous rendezvous between two or more secondary users (SUs). In this paper, we consider two colocated secondary networks, each represented by a pair of SUs. Both networks try to rendezvous concurrently, each aiming at maximizing its rendezvous performance, as measured by the average time-to-rendezvous and the number of rendezvous opportunities. To study this form of coexistence rendezvous, we follow a non-cooperative combinatorial game-theoretic framework, which we refer to as CORE. In this framework, SUs have different preferences towards various available licensed channels. Assuming first that SUs are time-synchronized, we formulate the interactions between the two networks as a two-player symmetric combinatorial game. We show the existence and uniqueness of a *finite-population evolutionary stable strategy* for this game. Furthermore, we conjecture that the game attains a pure-strategy Nash equilibrium (NE) for a wide range of design parameters. We also show that when SU pairs have the same preference towards all available channels, our game is an exact potential game, and hence the sequential best-response update is guaranteed to converge to a pure-strategy NE. We then study the time-asynchronous rendezvous game when SU pairs have the same preference towards all available channels. In this case, the game is also shown to be an exact potential game.

I. INTRODUCTION

To achieve efficient utilization of the licensed spectrum, significant research has been conducted towards enabling dynamic spectrum access (DSA) networks. The communicating entities in these networks, called secondary users (SUs), can utilize the available spectrum in a dynamic and opportunistic fashion without interfering with co-located primary users (PUs). Enabling opportunistic operation requires addressing various challenges, including channel access and device coordination.

Establishing a link between SU devices requires them to rendezvous, i.e., meet on a common frequency channel at the same time, and exchange control messages needed for connection establishment. In the absence of centralized control, the rendezvous problem is quite challenging because of the spatiotemporal variations in channel availability. Further challenges arise in the absence of node synchronization. To address the rendezvous problem, many existing MAC protocols for DSA networks rely on a dedicated control channel (e.g., [1], [2], [3]). While presuming a common control channel (CCC) simplifies the rendezvous process, it comes with two main drawbacks. First, a CCC can easily become a network

bottleneck and a prime target for selective jamming attacks [4]. Second, PU dynamics and spectrum heterogeneity make it difficult to always maintain a dedicated CCC [5].

Frequency hopping (FH) provides an alternative method for rendezvousing without relying on a predetermined CCC. One systematic way of constructing FH sequences is to use quorum systems [6]. Quorum-based FH designs have two key advantages. First, they deterministically guarantee that two FH sequences will overlap within a certain duration of time. Second, they are robust to synchronization errors [7]. Several quorum-based FH schemes have been recently proposed to enable rendezvous between SUs that belong to the same network (see, for example, [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]). The authors in [18] proposed the first game-theoretic framework for quorum-based anti-jamming rendezvous in DSA networks. They considered a pair of SUs that attempt to rendezvous in the presence of a jammer, whose objective is to hinder the rendezvous process. The interactions between the SUs and the jammer were formulated as a three-player game, but the treatment was restricted to a single rendezvous channel.

In this paper, we consider two coexisting secondary networks, each represented by a link (see Figure 1(a))¹. Both SU links try to rendezvous concurrently, each aiming to maximize its rendezvous performance. The rendezvous performance is measured by the average time-to-rendezvous (TTR), defined as the first time until the two SUs meet on a common channel, and the rate of rendezvous occurrence. To study this *coexistence rendezvous* problem, we propose a non-cooperative combinatorial game-theoretic framework, which we refer to as CORE. Figure 1 shows two such SU pairs, A_1-A_2 and B_1-B_2 . To rendezvous, they rely, for example, on a grid-quorum-based FH approach. Each FH sequence is divided into frames (in Figure 1(b), the frame length is nine slots). The slots of a frame are arranged into a square grid (a 3×3 grid in our example). Each SU selects a column and a row from the grid. The slots that correspond to the selected column and row are assigned a channel called the *outer rendezvous channel* (channel f_1 in Figure 1), and the remaining slots are assigned another channel called the *inner rendezvous channel* (channel f_2 in Figure 1). An SU pair, say A_1-A_2 , successfully rendezvous if both A_1 and A_2 are tuned to the same channel while both B_1 and B_2 are on a different channel. Because we consider a two-channel system in Figure 1, successful rendezvous of A_1-A_2 on one channel yields successful rendezvous of B_1-

¹Secondary networks typically execute the rendezvous process in a sequential way (i.e., one link at a time). Therefore, we represent each secondary network with a link.

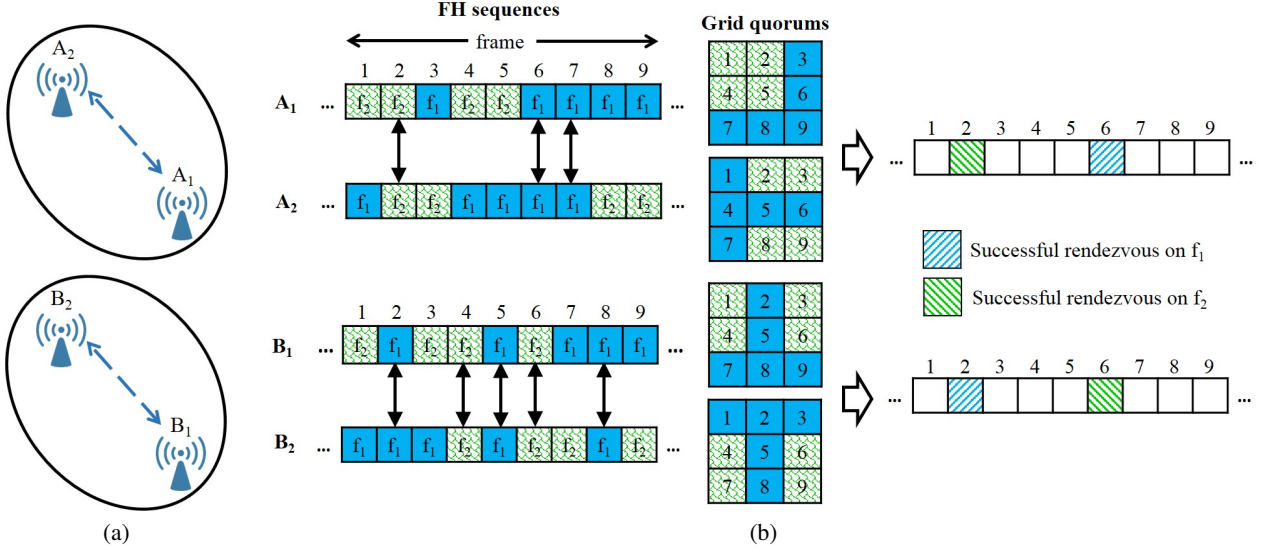


Fig. 1. (a) Two coexisting SU pairs, A_1 - A_2 and B_1 - B_2 , attempt to concurrently rendezvous in a DSA network, (b) SUs rely on a grid-quorum-based FH approach, with a frame length of nine slots.

B_2 on the other channel (this is not generally the case if the system has more than two channels). As Figure 1 shows, A_1 and A_2 successfully rendezvous on f_2 during the second time slot when B_1 and B_2 successfully rendezvous on f_1 (similarly, A_1 and A_2 successfully rendezvous on f_1 during the sixth slot when B_1 and B_2 successfully rendezvous on f_2). Although A_1 and A_2 are both tuned to f_1 during the seventh time slot, they cannot successfully rendezvous because B_1 is also on f_1 (similarly, B_1 and B_2 cannot successfully rendezvous on f_1 during the fifth slot because A_2 is also on f_1 during this slot).

Our Contributions—Assuming first that all SUs are time-synchronized, we formulate the interactions between these SUs as a non-cooperative four-player combinatorial game played on a common grid quorum system. We reformulate the four-player game as a two-player game between the two SU pairs. We show that this two-player non-zero-sum game is symmetric, and it has a unique finite-population evolutionary stable strategy (FESS). Furthermore, a pure-strategy Nash equilibrium (NE) exists if the frame length of the FH sequence and the SUs preference of the outer rendezvous channel compared to the inner rendezvous channel are set properly. Moreover, we show that when SU pairs do not differentiate between the outer and inner rendezvous channels, our game is an exact potential game, and hence a sequential best-response update is guaranteed to converge to a pure-strategy NE. Next, we study the rendezvous game in the absence of time synchronization. Specifically, we consider the case when the outer and inner rendezvous channels are treated equally by SUs. In this case, we show that the game is also an exact potential game, and hence a sequential best-response update is guaranteed to converge to a pure-strategy NE. Our numerical results show that if the design parameters are set properly, the rendezvous performance at the NE, as characterized by the price of anarchy (PoA), is very close to optimal (i.e., PoA is very close to 1).

Paper Organization—The rest of the paper is organized as follows. In Section II, we propose a grid-quorum-based FH scheme that guarantees rendezvous on two frequency

channels within a prespecified frame duration. We refer to this scheme as GQFH-2. We study the coexistence rendezvous problem applied to GQFH-2 through a non-cooperative game-theoretical framework in Section III. Our numerical results are presented in Section IV. Finally, Section V concludes the paper and provides directions for future research.

II. GRID-QUORUM-BASED FH RENDEZVOUS

Before explaining the proposed rendezvous scheme, we provide preliminary definitions related to quorum systems, which facilitates understanding of the rest of the paper.

A. Quorum Systems

Definition 1: Given the set $\mathbb{Z}_m = \{0, 1, \dots, m-1\}$, a quorum system Q under \mathbb{Z}_m is a collection of non-empty subsets of \mathbb{Z}_m , each called a quorum, such that:

$$\forall G, H \in Q : G \cap H \neq \emptyset. \quad (1)$$

In other words, any two quorums in Q overlap by at least one element.

Definition 2: Given a non-negative integer i and a quorum G in a quorum system Q under \mathbb{Z}_m , we define the operation $rotate(G, i) = \{(x+i) \bmod m, x \in G\}$ to denote a cyclic rotation of quorum G by i times.

Definition 3: A quorum system Q under \mathbb{Z}_m is said to satisfy the rotation closure property if:

$$\forall G, H \in Q, i \in \{0, 1, \dots, m-1\} : G \cap rotate(H, i) \neq \emptyset. \quad (2)$$

The rotation closure property is what makes quorum systems suitable for operating in asynchronous FH settings [7].

Definition 4: A grid quorum system arranges the elements of \mathbb{Z}_m as a $\sqrt{m} \times \sqrt{m}$ array, where m is the square of some positive integer. A quorum is formed from the elements of one column and one row of the grid (see Figure 1).

The grid quorum system satisfies the rotation closure property [7].

B. Grid-quorum-based FH Rendezvous Algorithm (GQFH-2)

In this section, we explain the two-channel rendezvous scheme that we adopt in this paper. Time is divided into equal-length frames, each containing m slots (m needs to be the square of a positive integer). The slots of each frame are formed as a $\sqrt{m} \times \sqrt{m}$ grid, from which the quorums are derived. For each FH sequence, a grid quorum (a column and a row) is randomly selected. Given a set of available channels, a channel is assigned to all quorum slots of that frame (henceforth, called the outer rendezvous channel). A second rendezvous channel (henceforth, called the inner rendezvous channel) is then assigned to each of the remaining slots in the frame. The example in Figure 1 illustrates the idea for $m = 9$. The procedure in Figure 1 is repeated for all the frames in the FH sequence.

Proposition 1: The complement of a grid quorum system Q under \mathbb{Z}_m , where $m \geq 9$, is also a quorum system.

Proof: Consider two quorums, G_1 and G_2 , from the grid quorum system Q . Then, $G_1 \cap G_2 \neq \emptyset$. The subsets $\mathbb{Z}_m \setminus G_1$ and $\mathbb{Z}_m \setminus G_2$ can be expressed as:

$$\mathbb{Z}_m \setminus G_1 = \{\mathbb{Z}_m \setminus \{G_1 \cup G_2\}\} \cup \{G_2 \setminus \{G_1 \cap G_2\}\} \quad (3)$$

$$\mathbb{Z}_m \setminus G_2 = \{\mathbb{Z}_m \setminus \{G_1 \cup G_2\}\} \cup \{G_1 \setminus \{G_1 \cap G_2\}\}. \quad (4)$$

Note that $G_2 \setminus \{G_1 \cap G_2\}$ and $G_1 \setminus \{G_1 \cap G_2\}$ are disjoint. Hence,

$$(\mathbb{Z}_m \setminus G_1) \cap (\mathbb{Z}_m \setminus G_2) = \mathbb{Z}_m \setminus \{G_1 \cup G_2\} \stackrel{(*)}{\neq} \emptyset \quad (5)$$

where $(*)$ is because:

- $|G_1| = |G_2| = 2\sqrt{m} - 1$ and $|G_1 \cap G_2| \geq 2$. Hence, $|G_1 \cup G_2| \leq 2(2\sqrt{m} - 1) - 2 = 4(\sqrt{m} - 1)$.
- $|\mathbb{Z}_m| = m$
- $m - 4(\sqrt{m} - 1) \geq 0, \forall m \geq 4$ (note that $m = 4$ is the smallest frame length used in GQFH-2).

Because the intersection of $\mathbb{Z}_m \setminus G_1$ and $\mathbb{Z}_m \setminus G_2$ is non-empty for every $G_1, G_2 \in Q$, the complement of a grid quorum system of size $m \geq 9$ is indeed a quorum system. ■

Proposition 2: The complement of a grid quorum system of size $m \geq 9$ satisfies the rotation closure property.

Proof: Let Q be a grid quorum system. We want to show that:

$$\forall G_1, G_2 \in Q, i \in \{0, 1, \dots, m-1\} : \quad \mathbb{Z}_m \setminus G_1 \cap \text{rotate}(\mathbb{Z}_m \setminus G_2, i) \neq \emptyset. \quad (6)$$

Note that:

$$\begin{aligned} \text{rotate}(\mathbb{Z}_m \setminus G_2, i) &= \mathbb{Z}_m \setminus \text{rotate}(G_2, i), \\ \forall i \in \{0, 1, \dots, m-1\}, \forall G_2 \in Q. \end{aligned} \quad (7)$$

From (5),

$$\begin{aligned} &(\mathbb{Z}_m \setminus G_1) \cap (\mathbb{Z}_m \setminus \text{rotate}(G_2, i)) \\ &= \mathbb{Z}_m \setminus \{G_1 \cup \text{rotate}(G_2, i)\} \stackrel{(**)}{\neq} \emptyset. \end{aligned} \quad (8)$$

where $(**)$ is because $|G_1 \cup \text{rotate}(G_2, i)| \leq 4(\sqrt{m} - 1)$, $|\mathbb{Z}_m| = m$, and $m - 4(\sqrt{m} - 1) \geq 0, \forall m \geq 4$. ■

Theorem 1: Using GQFH-2, a pair of SUs are guaranteed to rendezvous on two different channels if $m \geq 9$, under any arbitrary time-misalignment.

Proof: From Propositions 1 and 2, the complement of a grid quorum system is a quorum system that satisfies the rotation closure property. Hence, the theorem holds. ■

III. COEXISTENCE RENDEZVOUS GAME

In this section, we formulate a non-cooperative rendezvous game between two SU pairs, and study the equilibrium strategies of the game. We first consider the case when SUs are time-synchronized, and then study the asynchronous case.

A. Game Formulation

A game is characterized by a set of players, a set of actions for each player, and a payoff (utility) function for each player. In the following, we define these components.

Players: The underlying game has four players: A_1 , A_2 , B_1 , and B_2 (as shown in Figure 1).

Actions: The actions that can be taken by each of the four players are the m different grid quorums. All players have the same strategy space, denoted by $S \triangleq \{1, 2, \dots, m\}$, which consists of all quorums (pure strategies). The i th strategy in S corresponds to row $\lfloor \frac{i-1}{\sqrt{m}} \rfloor + 1$ and column $i - \sqrt{m} \lfloor \frac{i-1}{\sqrt{m}} \rfloor$ in the $\sqrt{m} \times \sqrt{m}$ grid quorum system. We denote the actions (strategies) taken by A_1 , A_2 , B_1 , and B_2 by s_{A_1} , s_{A_2} , s_{B_1} , and s_{B_2} , respectively. Each strategy (i.e., quorum selection) results in a corresponding FH sequence. We refer to the FH sequences of A_1 , A_2 , B_1 , and B_2 by \mathcal{S}_{A_1} , \mathcal{S}_{A_2} , \mathcal{S}_{B_1} , and \mathcal{S}_{B_2} , respectively, where $\mathcal{S}_{A_1} \triangleq (s_{A_1}^{(1)}, \dots, s_{A_1}^{(m)})$ and $s_{A_1}^{(i)}$ is the frequency used during slot i . FH sequences \mathcal{S}_{A_2} , \mathcal{S}_{B_1} , and \mathcal{S}_{B_2} are defined in a similar way.

Payoff (Utility). One important metric for evaluating a rendezvous scheme is the TTR. Another metric is the rate of rendezvous instances (i.e., the number of successful rendezvous slots per frame). Considering only one of these metrics is not sufficient. For instance, if only the TTR is considered, a strategy that results in one rendezvous slot that comes early in the frame (say, the first slot) will be preferred over another strategy that yields many rendezvous slots, all located after the first slot. On the other hand, the TTR can be used to differentiate between two strategies that result in the same number of rendezvous slots per frame, so that the one with the smaller average TTR (over all rendezvous slots) is selected. Each SU pair aims to maximize the number of rendezvous slots per frame and, at the same time, minimize the average TTR. Accordingly, we define the utility that each player wants to maximize as the number of rendezvous slots per frame divided by the average TTR. Furthermore, our utility function differentiates between the rendezvous instances on f_1 and those on f_2 .

The utility of A_1 (also, A_2), denoted by U_A , is given by (9), where $q(f_1)$ ($q(f_2)$) reflects the SUs preference of the outer (inner) rendezvous channel and $1[\cdot]$ is the indicator function.

$$\begin{aligned}
U_A(s_{A_1}, s_{A_2}, s_{B_1}, s_{B_2}) &= q(f_1) \frac{\text{\# of rendezvous slots per frame over } f_1}{\text{Average TTR over } f_1} + q(f_2) \frac{\text{\# of rendezvous slots per frame over } f_2}{\text{Average TTR over } f_2} \\
&= q(f_1) \frac{\left(\sum_{i=1}^m \mathbf{1} [s_{A_1}^{(i)} = s_{A_2}^{(i)} = f_1, s_{B_1}^{(i)} = s_{B_2}^{(i)} = f_2] \right)^2}{\sum_{i=1}^m i \mathbf{1} [s_{A_1}^{(i)} = s_{A_2}^{(i)} = f_1, s_{B_1}^{(i)} = s_{B_2}^{(i)} = f_2]} + q(f_2) \frac{\left(\sum_{i=1}^m \mathbf{1} [s_{A_1}^{(i)} = s_{A_2}^{(i)} = f_2, s_{B_1}^{(i)} = s_{B_2}^{(i)} = f_1] \right)^2}{\sum_{i=1}^m i \mathbf{1} [s_{A_1}^{(i)} = s_{A_2}^{(i)} = f_2, s_{B_1}^{(i)} = s_{B_2}^{(i)} = f_1]} \quad (9)
\end{aligned}$$

The utility of B_1 is the same as the utility of B_2 and is denoted by U_B . U_B can be expressed similar to (9).

B. Reducing the Size of the Game

The four-player game formulated in Section III-A is an $m \times m \times m \times m$ game. In this section, we reformulate this game into a two-player game, in which each SU pair constitutes one player. The resulted two-player game is an $m \times m$ game.

First, treating each SU pair as one player (i.e., A_1 – A_2 is one player and B_1 – B_2 is the second player), our game can be reformulated as an $m^2 \times m^2$ two-player game. Next, note that for any fixed strategies of B_1 and B_2 , there is no incentive for A_1 and A_2 to select different strategies (i.e., quorums). In other words, it can be easily shown that:

$$\begin{aligned}
U_A(s_{A_1}, s_{A_1}, s_{B_1}, s_{B_2}) &\geq U_A(s_{A_1}, s_{A_2} \neq s_{A_1}, s_{B_1}, s_{B_2}), \\
&\quad \forall s_{A_1}, s_{A_2}, s_{B_1}, s_{B_2} \in S.
\end{aligned}$$

Similarly, it can be shown that:

$$\begin{aligned}
U_B(s_{A_1}, s_{A_2}, s_{B_1}, s_{B_1}) &\geq U_B(s_{A_1}, s_{A_2}, s_{B_1}, s_{B_2} \neq s_{B_1}), \\
&\quad \forall s_{A_1}, s_{A_2}, s_{B_1}, s_{B_2} \in S.
\end{aligned}$$

Accordingly, considering link A , any strategy (s_{A_1}, s_{A_2}) with $s_{A_2} \neq s_{A_1}$ is weakly-dominated by each of the strategies (s_{A_1}, s_{A_1}) , $\forall s_{A_1} \in S$. Therefore, starting with the $m^2 \times m^2$ two-player game, iterative elimination of weakly-dominated strategies yields an $m \times m$ two-player game. Although iterative elimination of weakly-dominated strategies might be order dependent [19], in here elimination is done offline before the game starts. The two SU pairs start playing the $m \times m$ game from the beginning.

The reduced game is a non-zero-sum two-player game. The strategy space of each player (SU pair) is $S = \{1, 2, \dots, m\}$, where strategy $i \in S$ means that both SUs of the considered pair (player) are using grid quorum i . The utility function in (9) can be written in a simpler way. Let $s_{A_1} = s_{A_2} \triangleq s_A$ and $s_{B_1} = s_{B_2} \triangleq s_B$ (also, let $S_{A_1} = S_{A_2} \triangleq S_A$ and $S_{B_1} = S_{B_2} \triangleq S_B$). Then, U_A can be written as follows:

$$\begin{aligned}
U_A(s_A, s_B) &= q(f_1) \frac{\left(\sum_{i=1}^m \mathbf{1} [s_A^{(i)} = f_1, s_B^{(i)} = f_2] \right)^2}{\sum_{i=1}^m i \mathbf{1} [s_A^{(i)} = f_1, s_B^{(i)} = f_2]} \\
&\quad + q(f_2) \frac{\left(\sum_{i=1}^m \mathbf{1} [s_A^{(i)} = f_2, s_B^{(i)} = f_1] \right)^2}{\sum_{i=1}^m i \mathbf{1} [s_A^{(i)} = f_2, s_B^{(i)} = f_1]}. \quad (10)
\end{aligned}$$

A's Utility		SU pair B			
		1	2	3	4
SU pair A	1	0	0.75	1	1.75
	2	0.7083	0	1.0833	1.8333
	3	0.875	1	0	2
	4	1.375	1.5	1.75	0

(a) U_A

B's Utility		SU pair B			
		1	2	3	4
SU pair A	1	0	0.7083	0.875	1.375
	2	0.75	0	1	1.5
	3	1	1.0833	0	1.75
	4	1.75	1.8333	2	0

(b) U_B

Fig. 2. U_A and U_B for $m = 4$ and $q(f_1) = 1.5q(f_2)$.

Figure 2 shows U_A and U_B for $m = 4$ and $q(f_1) = 1.5q(f_2)$. Next, we show that our non-zero-sum two-player $m \times m$ game is symmetric.

Definition 5: Consider a two-player game. Let S_A and S_B be the strategy spaces of players A and B , respectively. Let U_A and U_B be the utilities of players A and B , respectively. Then, the game (S_A, S_B, U_A, U_B) is symmetric if the following two conditions are satisfied:

- $S_A = S_B$.
- $U_A(s_A, s_B) = U_B(s_B, s_A), \forall s_A \in S_A, \forall s_B \in S_B$.

If the game (S_A, S_B, U_A, U_B) is symmetric, we refer to it as (S, U) where $S = S_A = S_B$ and $U = U_A$.

Proposition 3: Our two-player $m \times m$ game is symmetric.

Proof: First, both players in our game have a common strategy space which is $S = \{1, 2, \dots, m\}$. Second, from (10),

$$U_A(s_A, s_B) = U_B(s_B, s_A), \forall s_A \in S_A, \forall s_B \in S_B. \quad (11)$$

Hence, the game is symmetric. Equation 11 says that if one SU pair follows strategy s_A while the other SU pair follows strategy s_B , then the utility of the player who plays s_A , for instance, is the same whether it is player A or player B . ■

C. Equilibrium Analysis

1) Finite Population Evolutionary Stable Strategy (FESS):

In symmetric two-player games, the notion of a finite population evolutionary stable strategy (FESS), introduced by Schaffer [20], [21], is typically considered. Schaffer observed that a FESS of an arbitrary symmetric game coincides with the NE of its zero-sum relative payoff game.

Definition 6: A strategy $s^* \in S$ is an FESS of the two-player symmetric game (S, U) if:

$$U(s^*, s) \geq U(s, s^*), \forall s \in S. \quad (12)$$

Definition 7: Given a symmetric two-player game (S, U) . The associated relative payoff game of (S, U) is denoted by (S, \bar{U}) , where \bar{U} is the relative payoff function (defined as the difference between the player's payoff and the payoff of its opponent). \bar{U} can be expressed as:

$$\bar{U}(s_A, s_B) = U(s_A, s_B) - U(s_B, s_A), \forall s_A, s_B \in S. \quad (13)$$

Schaffer observed that s^* is an FESS of the symmetric game (S, U) if and only if (s^*, s^*) is a pure-strategy NE of the associated relative payoff game. In the following, we will show that the associated relative payoff game with our two-player game has a pure-strategy NE.

2) Generalized Rock-Paper-Scissors Matrix:

The generalized rock-paper-scissors (gRPS) matrix plays an important role in determining the existence of a pure-strategy NE in symmetric games.

Definition 8: A symmetric two-player zero-sum game (S, U) is a gRPS matrix if in each column there exists a row with a strictly positive payoff to the row player, i.e., if $\forall s_B \in S, \exists s_A \in S$ such that $U(s_A, s_B) > 0$.

Next, we state a theorem taken from [22] about the existence of a pure-strategy NE in symmetric games.

Theorem 2: A symmetric two-player zero-sum game (S, U) possesses a pure-strategy NE if and only if it is not a gRPS matrix [22].

Proof: See [22]. ■

3) Existence of an FESS:

Proposition 4: The relative payoff game associated with our two-player $m \times m$ game is symmetric.

Proof: First, note that in the relative payoff game, both players have a common strategy space. Second,

$$\begin{aligned} \bar{U}_B(s_B, s_A) &= U_B(s_B, s_A) - U_B(s_A, s_B) \\ &= U_A(s_A, s_B) - U_A(s_B, s_A) \\ &= \bar{U}_A(s_A, s_B). \end{aligned} \quad (14)$$

Hence, the relative payoff game is symmetric. ■

Proposition 5: The relative payoff game associated with our two-player game is not a gRPS.

Proof: From Definition 8, in order to prove that the game is not a gRPS, we want to find a strategy of the column player that yields a non-positive payoff to the row player irrespective of the strategy played by the row player. In other words, we want to find an $s_B^* \in S$ such that:

$$\begin{aligned} \bar{U}_B(s, s_B^*) &\leq 0, \forall s \in S \\ \Rightarrow U(s, s_B^*) - U(s_B^*, s) &\leq 0, \forall s \in S \\ \Rightarrow U(s_B^*, s) &\geq U(s, s_B^*), \forall s \in S. \end{aligned} \quad (15)$$

The value of $s_B^* \in S$ that satisfies (15) depends on $q(f_1)/q(f_2)$. Specifically, we will show that:

$$s_B^* = \begin{cases} 1, & \text{if } q(f_1)/q(f_2) > 1 \\ m, & \text{if } q(f_1)/q(f_2) < 1. \end{cases} \quad (16)$$

To prove (16), let us simplify the expressions of U_A and U_B in (10) as follows. Let $U_A \triangleq q(f_1) \frac{\alpha_1^{(A)}}{\beta_1^{(A)}} + q(f_2) \frac{\alpha_2^{(A)}}{\beta_2^{(A)}}$ and $U_B \triangleq q(f_1) \frac{\alpha_1^{(B)}}{\beta_1^{(B)}} + q(f_2) \frac{\alpha_2^{(B)}}{\beta_2^{(B)}}$. Recall that a successful rendezvous instance of SU pair A on channel f_1 is also a successful rendezvous instance of SU pair B on channel f_2 . Similarly, a successful rendezvous instance of SU pair A on channel f_2 is also a successful rendezvous instance of SU pair B on channel f_1 . Hence,

$$\begin{aligned} \frac{\alpha_1^{(A)}}{\beta_1^{(A)}} &= \frac{\alpha_2^{(B)}}{\beta_2^{(B)}} \\ \frac{\alpha_2^{(A)}}{\beta_2^{(A)}} &= \frac{\alpha_1^{(B)}}{\beta_1^{(B)}}. \end{aligned} \quad (17)$$

Therefore, $\frac{\alpha_1^{(A)}}{\beta_1^{(A)}} + \frac{\alpha_2^{(A)}}{\beta_2^{(A)}} = \frac{\alpha_1^{(B)}}{\beta_1^{(B)}} + \frac{\alpha_2^{(B)}}{\beta_2^{(B)}}$. Hence, if $q(f_1) = q(f_2)$, $U_A = U_B$. Furthermore, note from (10) that $\alpha_1^{(A)} = \alpha_1^{(B)}$ and $\alpha_2^{(A)} = \alpha_2^{(B)}$.

Now, if $s_A = 1$, then the average TTR of SU pair A over channel f_1 is less than or equal to the average TTR of SU pair B over channel f_1 irrespective of s_B , i.e., $\beta_1^{(A)} \leq \beta_1^{(B)}$. Because $\alpha_1^{(A)} = \alpha_1^{(B)}$, we have:

$$\frac{\alpha_1^{(A)}}{\beta_1^{(A)}} \geq \frac{\alpha_1^{(B)}}{\beta_1^{(B)}} = \frac{\alpha_2^{(A)}}{\beta_2^{(A)}}. \quad (18)$$

Accordingly, if $q(f_1) > q(f_2)$, $U(1, s) \geq U(s, 1), \forall s \in S$. The second case in (16) can be shown in a similar way. ■

Theorem 3: The relative payoff game associated with our two-player game has a pure-strategy NE. Furthermore, the pure-strategy NE is unique if $q(f_1) \neq q(f_2)$ and is given by:

$$(s_A^*, s_B^*) = \begin{cases} (1, 1), & \text{if } q(f_1)/q(f_2) > 1 \\ (m, m), & \text{if } q(f_1)/q(f_2) < 1. \end{cases} \quad (19)$$

Proof: The relative payoff game is symmetric, zero-sum, and is not a gRPS matrix. Hence, it has a pure-strategy NE. If $q(f_1) \neq q(f_2)$, there will be exactly one strategy of the column player, s_B^* , such that:

$$\begin{aligned} \bar{U}(s_A, s_B^*) &< 0, \forall s_A \in S \setminus s_B^* \\ \bar{U}(s_B^*, s_B^*) &= 0. \end{aligned}$$

In this case, (s_B^*, s_B^*) is the pure-strategy NE. If the row player deviates from following the strategy s_B^* , it will lose; because $\bar{U}(s_A, s_B^*) < 0, \forall s_A \in S \setminus s_B^*$. Moreover, because the relative payoff game is symmetric and zero-sum, $\bar{U}(s_B^*, s_B) > 0, \forall s_B \in S \setminus s_B^*$, and the column player will lose if it deviates from s_B^* (recall that the utility of the column player in the relative payoff game is $-\bar{U}$). From Proposition 5, $s_B^* = 1$ if $q(f_1) > q(f_2)$ and $s_B^* = m$ if $q(f_1) < q(f_2)$. ■

Corollary 1: Our two-player symmetric game has a unique FESS, s_B^* , if $q(f_1) \neq q(f_2)$. s_B^* is given by (16).

Proof: Since the relative payoff game associated with our symmetric game has a unique pure-strategy NE (s_B^*, s_B^*) , our game has a unique FESS, s_B^* . ■

Although our game has a unique FESS, the utility of both players at the FESS strategy is zero. Accordingly, both players have an incentive to deviate from the FESS strategy.

4) Existence of a Pure-strategy NE:

The existence of a pure-strategy NE for our symmetric game, and the convergence of the sequential best-response update to a pure-strategy NE depends on: (i) the frame length m and (ii) $q(f_1)/q(f_2)$.

Result 1: For certain values of m and $q(f_1)/q(f_2)$, starting from the unique FESS strategy (given by Corollary 1), if each SU pair plays a best-response strategy to the other pair's strategy in a sequential way, both SU pairs converge to a pure-strategy NE.

Note that m is a design parameter. Furthermore, $q(f_1)$ and $q(f_2)$, which represent the SUs' preferences of f_1 and f_2 , can be considered as design parameters. If m and $q(f_1)/q(f_2)$ are set properly, a sequential best-response update converges to a pure-strategy NE. In Section IV, we study the rendezvous performance at the NE for different values of m and $q(f_1)/q(f_2)$.

Proposition 6: If $q(f_1) = q(f_2)$ (i.e., SU pairs do not differentiate between rendezvousing on f_1 or f_2), then a sequential best-response update is guaranteed to converge to a pure-strategy NE.

Proof: If $q(f_1) = q(f_2)$, then both SU pairs have the same utility, as can be seen from (10). In this case, our symmetric game is also an *exact potential game*, and the potential function is equal to the utility of SU pair A (which is the same as the utility of SU pair B). In this case, a sequential best-response update is guaranteed to converge to a pure-strategy NE that maximizes the potential function [23]. ■

D. Asynchronous Rendezvous

Maintaining time synchronization between SUs in an opportunistic ad-hoc network is challenging. In this section, we study the rendezvous game in the absence of time-synchronization between SUs. One way to analyze this game is to treat it as a four-player game, in which the utility of each SU depends on the time-misalignment between this SU and each of the three other SUs. Another way of analyzing this game is to treat it as a two-player game, similar to the synchronous case, where each player represents an SU pair. In this paper, we follow the latter approach. We assume that the two SUs

in each pair are time-synchronized. They can achieve this time synchronization by exchanging their timing information during the first rendezvous instance. Note that the rendezvous process is intended for establishing new communication links as well as recovering disrupted communications (e.g., due to the sudden appearance of a PU). Therefore, rendezvous is not a one-time process and it might be needed any time during the network operation.

The asynchronous two-player game formulation is similar to the synchronous case, except that in the asynchronous case the utility is computed by taking the expectation over all time-shifts between the two SU pairs. Similar to [18], we assume that a half time slot is enough for exchanging a pair of rendezvous messages. Therefore, if two SUs met on a half (or more) slot it is treated as if they met on the whole slot. On the other hand, if they met on less than a half slot then it is the same as if they did not meet. This way, the misalignment between the two SU pairs takes integer values only. Accordingly, the utility of SU pair A in the asynchronous game is given by (similarly for SU pair B):

$$\mathcal{U}_A(s_A, s_B) = \sum_{i=-m}^m p(i) U_A(s_A, \text{rotate}(s_B, i)) \quad (20)$$

where $p(i)$ is the probability that SU pair A 's frame starts i slots before SU pair B 's frame and $\text{rotate}(s_B, i)$ is the cyclic rotation of SU pair B 's frame by i , as defined in Definition 2.

Proposition 7: If $q(f_1) = q(f_2)$, then a sequential best-response update of the asynchronous rendezvous game is guaranteed to converge to a pure-strategy NE.

Proof: When $q(f_1) = q(f_2)$, it can be easily shown that $\mathcal{U}_A = \mathcal{U}_B$ and the game is an exact potential game where the potential function is equal to $\mathcal{U}_A = \mathcal{U}_B$. Hence, a sequential best-response update is guaranteed to converge to a pure-strategy NE that maximizes the potential function [23]. ■

Remark 1: When $q(f_1) \neq q(f_2)$, the equilibrium analysis of the asynchronous game depends on the misalignment distribution (i.e., $p(i), i \in \{-m, \dots, -1, 0, 1, \dots, m\}$). In this paper, we consider the asynchronous rendezvous game only when $q(f_1) = q(f_2)$.

E. Deducing the Strategy of the Other Player

In order to perform a sequential best-response update, an SU pair needs to deduce the strategy played by its opponent. In Algorithm 1, we present a simple procedure that SU pair A follows to deduce the strategy followed by SU pair B .

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed rendezvous games. We implement our games in MATLAB.

A. Synchronous Rendezvous Game

Figure 3(a) shows the utility of SU pairs A and B at the NE vs. the frame length when $q(f_1) = 0.5q(f_2)$. In this case,

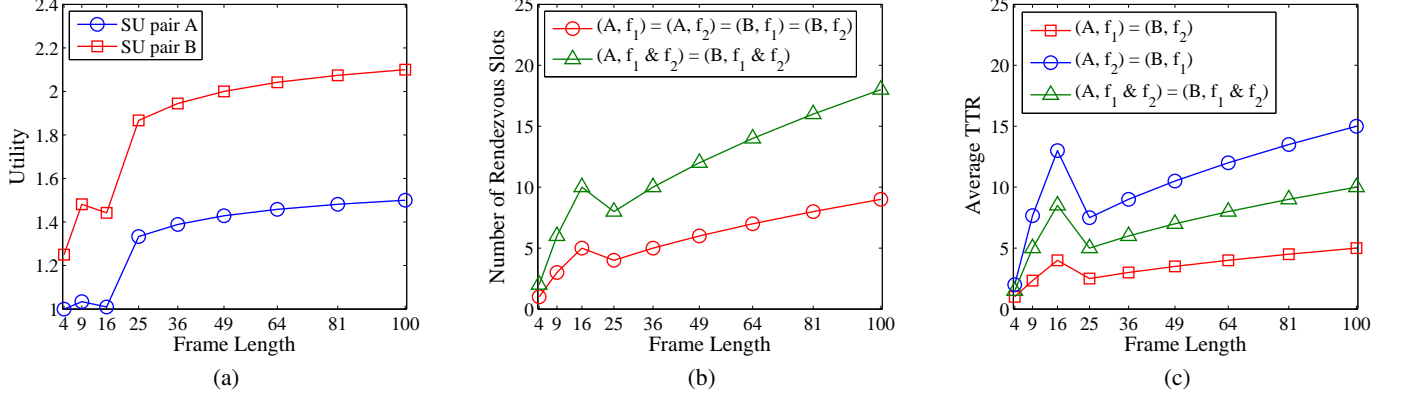


Fig. 3. (a) The utilities, (b) number of rendezvous slots, and (c) average TTR of SU pairs A and B vs. the frame length when $q(f_1) = 0.5q(f_2)$ (SUs are time-synchronized).

Algorithm 1 Strategy Deduction Procedure

Input: $S_A = (s_A^{(1)}, \dots, s_A^{(m)})$
Output: $S_B = (s_B^{(1)}, \dots, s_B^{(m)})$

```

1: for  $i = 1 : m$  do
2:   if rendezvous is successful during slot  $i$  then
3:     if  $s_A^{(i)} == f_1$  then  $s_B^{(i)} = f_2$ 
4:     else  $s_B^{(i)} = f_1$ 
5:     end if
6:   else
7:     if  $s_A^{(i)} == f_1$  then  $s_B^{(i)} = f_1$ 
8:     else  $s_B^{(i)} = f_2$ 
9:     end if
10:  end if
11: end for

```

starting at the FESS strategy, which is (m, m) , the SU pairs converge to the pure-strategy NE (s_A^*, s_B^*) given by:

$$(s_A^*, s_B^*) = \begin{cases} (\sqrt{m}, 2\sqrt{m}), & \text{if } 25 \leq m \leq 100 \\ (1, m), & \text{if } 9 \leq m \leq 16 \\ (3, 4), & \text{if } m = 4. \end{cases} \quad (21)$$

In addition to the utility, we show in Figure 3(b) the number of rendezvous slots of each pair over channels f_1 and f_2 as a function of the frame length. Furthermore, the average TTR of both SU pairs is depicted in Figure 3(c). In the legend of Figure 3, (I, f_j) , $I = A, B, j = 1, 2$, means that only the rendezvous instances of SU pair I over channel f_j are considered. Note that although the average TTR increases with the frame length, the number of rendezvous slots also increases and the utilities of both SU pairs increase with the frame length, except when the frame length increases from 9 to 16.

Figure 4 shows the same metrics as Figure 3, but with $q(f_1) = 1.5q(f_2)$. In this case, starting at the FESS strategy, which is $(1, 1)$, the SU pairs converge to the pure-strategy NE

(s_A^*, s_B^*) given by:

$$(s_A^*, s_B^*) = \begin{cases} (\sqrt{m} + 1, 1), & \text{if } 25 \leq m \leq 100 \\ (8, 4), & \text{if } m = 16 \\ (9, 1), & \text{if } m = 9 \\ (4, 3), & \text{if } m = 4. \end{cases} \quad (22)$$

To quantify the quality of the pure-strategy NE, we calculate the PoA, defined as the maximum sum-utility that the two SU pairs can achieve divided by the sum of their utilities at the NE. Figure 5 shows the PoA for the two above cases: (i) $q(f_1) = 0.5q(f_2)$ and (ii) $q(f_1) = 1.5q(f_2)$. Note that for most of the frame lengths, the PoA when $q(f_1) < q(f_2)$ is significantly lower than that when $q(f_1) > q(f_2)$. To understand the reason behind this, consider Figure 6. In Figure 6, we plot the percentage of time slots in a frame that are assigned channels f_1 and f_2 according to GQFH-2. As shown in Figure 6, when $m \geq 16$ the number of slots assigned f_2 is significantly larger than the number of slots assigned f_1 . Therefore, it is preferable to have $q(f_2) > q(f_1)$ (i.e., assign the better quality channel to the non-grid-quorum slots in the frame and assign the lower quality channel to the grid-quorum slots).

B. Asynchronous Rendezvous Game

Assuming that the time shift between the two SU pairs follows a uniform distribution (i.e., $p(i) = \frac{1}{2m+1}$ in (20)), we plot in Figure 7 the utility, number of rendezvous slots, and average TTR vs. the frame length when $q(f_1) = q(f_2)$. Recall that in this case both links have the same utility and the game is an exact potential game. Figure 7(a) shows that the utility under the uniform distribution increases with the frame length up to a certain value, beyond which it starts decreasing. The maximum utility is attained when $m = 25$. The pure-strategy NE of the potential game, (s_A^*, s_B^*) , is given by:

$$(s_A^*, s_B^*) = \begin{cases} (2, 1), & \text{if } m = 16, 64 \leq m \leq 100 \\ (2, 43), & \text{if } m = 49 \\ (2, 13), & \text{if } m = 36 \\ (2, 6), & \text{if } m = 25 \\ (9, 8), & \text{if } m = 9 \\ (4, 1), & \text{if } m = 4. \end{cases} \quad (23)$$

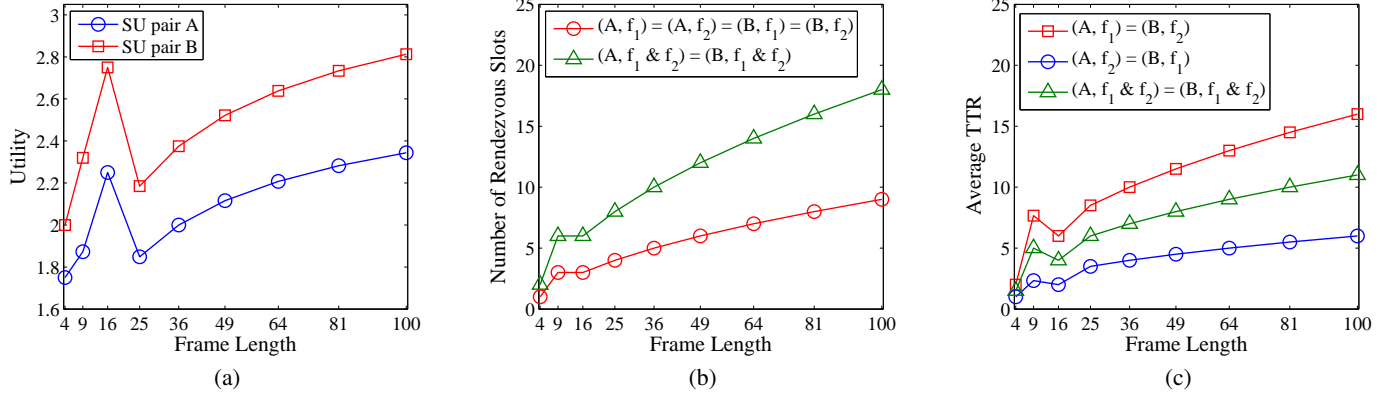


Fig. 4. (a) The utilities, (b) number of rendezvous slots, and (c) average TTR of SU pairs A and B vs. the frame length when $q(f_1) = 1.5q(f_2)$ (SUs are time-synchronized).

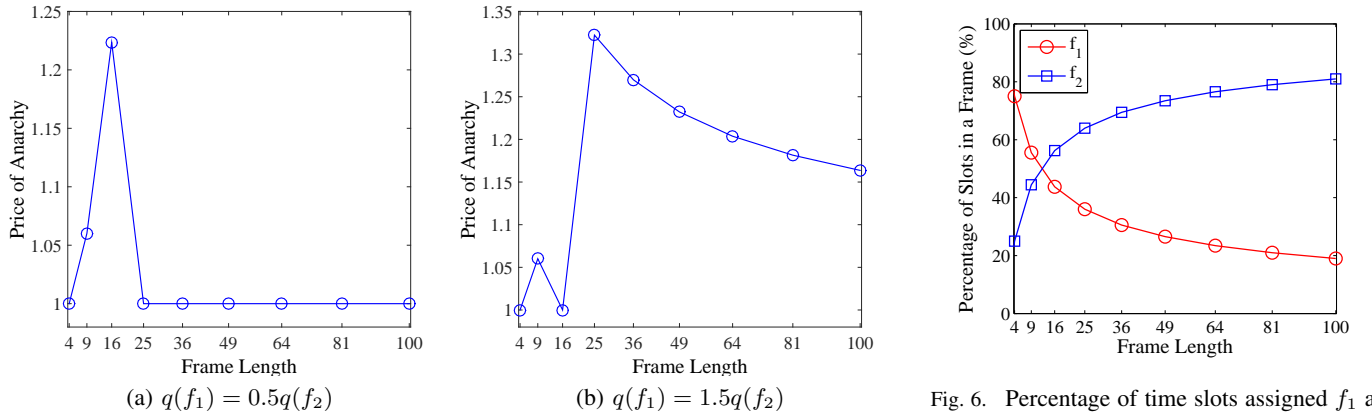


Fig. 5. PoA vs. the frame length (SUs are time-synchronized).

Fig. 6. Percentage of time slots assigned f_1 and f_2 in a frame, according to GQFH-2.

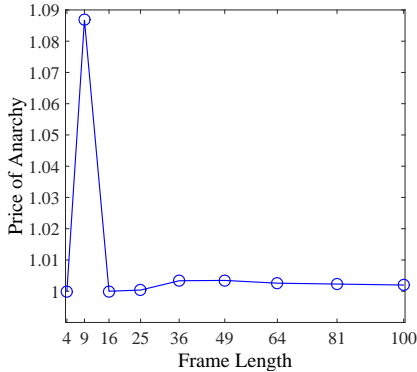


Fig. 8. PoA vs. the frame length (misalignment between SU pairs is uniform).

Figure 8 shows the PoA for the asynchronous game under the uniform distribution. As shown in the figure, the PoA is very close to 1 (< 1.09).

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we investigated the non-cooperative coexistence rendezvous problem, in which two SU pairs that belong to two collocated secondary networks try to rendezvous

concurrently. We studied this problem through a combinatorial game-theoretic framework. We considered both cases, when SUs are time-synchronized and when there is a time misalignment between the SU pairs. Our key observations are: (i) When SUs are time-synchronized, large frame lengths (in general) result in better rendezvous performance, (ii) in the presence of a uniform time-misalignment between the SU pairs the rendezvous performance is a concave function with the frame length, (iii) assigning the better quality channel to the non-grid-quorum slots in the frame leads to a better performance at the NE, as characterized by the price of anarchy.

As a future research, the developed coexistence rendezvous framework can be extended to the multicast case, where each secondary network consists of a multicast group of SUs. The rendezvous schemes proposed in [12] can be used as a bases for this game formulation. Another direction for future research is to study a cooperative form of unicast as well as multicast coexistence rendezvous. In cooperative coexistence rendezvous, each SU pair/group aims to maximize the overall rendezvous performance of all coexisting secondary networks.

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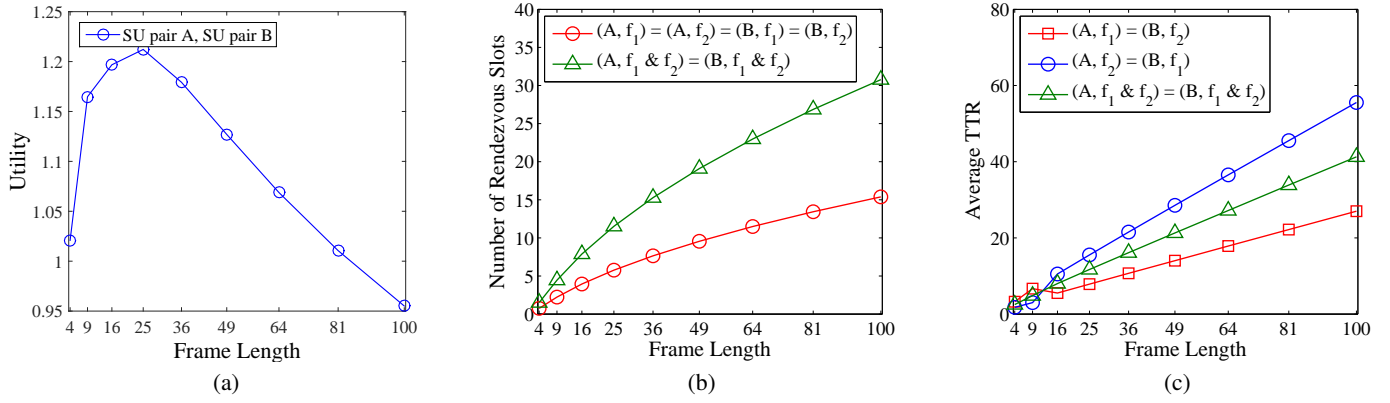


Fig. 7. (a) The utilities, (b) number of rendezvous slots, and (c) average TTR of SU pairs A and B vs. the frame length when $q(f_1) = q(f_2)$ (misalignment between SU pairs is uniform).

conclusions, or recommendations expressed in this paper are those of the author(s) and do not necessarily reflect the views of NSF or ARO.

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