# Stochastic Resource Allocation in Opportunistic LTE-A Networks with Heterogeneous Self-interference Cancellation Capabilities

Mohammad J. Abdel-Rahman Wireless @ Virginia Tech mo7ammad@vt.edu Mohamed AbdelRaheem Wireless @ Virginia Tech mohmedht@vt.edu

Abstract-Opportunistic spectrum access (OSA) and selfinterference cancellation (SIC) are two emerging solutions for enhancing spectrum utilization, which are expected to impact the design of 5G networks. In this paper, we consider the problem of composing an opportunistic LTE-A network using a set of existing base stations (BSs) with heterogeneous SIC capabilities and costs. The objective is to design the cheapest network that can support the probabilistic rate demands of multiple users. Towards achieving this goal, we propose novel stochastic joint channel and BS allocation schemes that account for uncertainty in channel availability. First, we develop two static (proactive) joint allocation models. We refer to these models as Het-SMKP<sub>1</sub> and Hom-SMKP. In these models, the allocation is done once such that user demands are probabilistically met. In Het-SMKP<sub>1</sub>, a user can request different probabilistic rates for different small cells, whereas in Hom-SMKP each user requests the same probabilistic rate for the entire network. Second, we propose an *adaptive* (proactive and reactive), two-stage allocation model for heterogeneous rate demands, which we refer to as Het-SMKP<sub>2</sub>. The adaptive model allows for correcting the initial resource allocation once the channel availability uncertainties are partially resolved. We numerically evaluate the performance of the static and adaptive allocation schemes under various system parameters. Despite its computational complexity, the adaptive scheme improves the probability of rate demand satisfaction considerably compared to the static scheme.

Keywords—Opportunistic spectrum access, self-interference cancellation, LTE-A, resource allocation, stochastic optimization, multiple knapsack problem.

### I. INTRODUCTION

The massive growth in wireless devices and mobile traffic has motivated research and development on the next generation (i.e., 5G) cellular networks. 5G cellular networks are intended to support higher data rates, higher spectrum and energy efficiencies, and lower latency. Among the emerging solutions for enhancing spectrum utilization are opportunistic spectrum access (OSA), small cells [1]–[3], and in-band full-duplex (FD) wireless – more generally, self-interference cancellation (SIC) [4]. These solutions are expected to have a tremendous impact on 5G networks and beyond.

Traditionally, much of the spectrum is statically licensed for a given use in a given geographic area. Exceptions to this norm include the ISM bands. However, these bands are reaching their capacity limit, as more traffic is being pushed through them. OSA tries to address the rising demand by allowing spectrum-agile devices with cognitive radio capabilities



Allen B. MacKenzie

Wireless @ Virginia Tech mackenab@vt.edu

Fig. 1: With no changes to the macro eNB or to the handset, an FD-enabled small cell can provide its own backhaul, eliminating the need for additional out-of-band resources for backhaul [4].

to operate opportunistically as secondary users over certain licensed bands. Hence, improving the spectrum utilization considerably.

Parallel to OSA, in-band FD wireless is expected to have a significant impact on enabling 5G networks. Several studies [5]–[8] have successfully demonstrated the feasibility of FD communications using SIC techniques. As proposed in [4], SIC can allow LTE-A small cells to leverage the same radio spectrum for simultaneously communicating with the user equipments (UEs) and the macro eNB (see Figure 1), eliminating the need for additional out-of-band channels for backhaul, which significantly improves the spectrum utilization. As illustrated in Figure 1, in the downlink channel, the small cell receives data from the macro eNB while simultaneously transmitting to the UE. In the uplink channel, the small cell receives data from the UE while simultaneously transmitting to the macro eNB.

**Our Contributions**–In this paper, we consider the problem of composing an opportunistic LTE-A network using a set of existing base stations (BSs) with heterogeneous SIC capabilities and costs. The opportunistic LTE-A network is intended to support multiple users with different *probabilistic* rate demands. Our objective is to build the opportunistic LTE-A network that can support user demands with the least cost. To achieve this objective, we propose novel stochastic joint channel and BS allocation schemes that account for uncertainty in channel availability. Specifically,

- We develop two static 'chance-(proactive) constrained' joint channel and BS allocation models. We refer to these models as Het-SMKP<sub>1</sub> and Hom-SMKP. In these models, the allocation is done once such that the user demands are probabilistically met. The chance constraint is introduced to limit the probability of under-satisfying the user demand to a certain threshold. In Het-SMKP<sub>1</sub>, a user can request different *probabilistic* rates for different small cells, whereas in Hom-SMKP each user requests the same rate for the entire network. In our allocation schemes, a user, say l, requests a rate  $R_l$  (in Mbps) to be satisfied with probability at least  $\alpha_l \in (0, 1)$ .
- We propose an *adaptive* (proactive and reactive) twostage allocation model for heterogeneous rate demands, which we refer to as Het-SMKP<sub>2</sub>. The adaptive model allows for correcting the initial resource allocation once the channel availability uncertainties are partially resolved; channels are released from oversatisfied users (if any) and allocated to under-satisfied users (if any), reducing both user under-satisfaction as well as user over-satisfaction. Despite its computational complexity, Het-SMKP<sub>2</sub>, due to its recourse (adaptive) capability, (i) increases the probability of demand satisfaction considerably, and (ii) reduces the cost of composing an opportunistic LTE-A network by returning the additional resources (if any) after fulfilling user demands.

Recently, the authors in [9], [10] proposed stochastic channel allocation schemes for dynamic spectrum access networks. The schemes in [9], [10] neither consider BS allocation nor account for SIC. To the best of our knowledge, this is the first paper that applies stochastic programming techniques for joint allocation of channels and BSs in OSA networks, while accounting for heterogeneous SIC capabilities of the BSs.

Finally, our proposed stochastic formulations can be easily extended to model the resource allocation problem in other network settings, such as:

- LTE-U networks. Recently, Qualcomm and other companies have proposed extending 3GPP LTE-A to the unlicensed 5 GHz U-NII band (referred to as LTE-U) by exploiting supplemental downlink and carrier aggregation features in LTE-A systems [11], [12]. In such LTE-U networks, the ability to successfully access a channel in the 5 GHz band is only stochastic. Our proposed stochastic allocation schemes can be extended to optimally provide some probabilistic guarantees to LTE-U users.
- Spectrum-licensed networks that opportunistically access other licensed spectra. A spectrum-licensed network may enhance its throughput by opportunistically accessing other licensed spectra. In this case, some of the network resources (channels) are deterministic while others are stochastic. Our stochastic resource allocation formulations can also be applied in such a setting.

**Paper Organization**—The rest of the paper is organized as follows. We present the system model in Section II. The static channel and BS allocation schemes are formulated and solved in Section III. We develop the adaptive stochastic allocation scheme in Section IV. All proposed schemes are numerically evaluated in Section V. Finally, in Section VI we conclude the paper and provide directions for future research.

### II. SYSTEM MODEL

We consider a geographical area that is divided into a set  $\mathcal{N} \stackrel{\text{def}}{=} \{1, 2, \dots, N\}$  of small cells. A heterogeneous set of BSs, denoted by  $\mathcal{S} \stackrel{\text{def}}{=} \{1, 2, \dots, S\}$ , exists in each small cell; each BS has a different SIC capability and cost. The SIC capability of BS s in cell n is characterized by  $\eta_{sn} \in [0, 1]$ .  $\eta_{sn} = 0$  means complete SIC (i.e., perfect FD) and  $\eta_{sn} = 1$  represents the case when the BS does not have the SIC capability, i.e., half-duplex (HD). The cost of BS s in cell n is denoted by  $c_{sn}$  ( $c_{sn}$  decreases with  $\eta_{sn}$ ). While FD BSs are self-backhauled (i.e., a single channel can be used to simultaneously communicate with the UE and macro eNB, HD BSs require two channels to simultaneously communicate with the UE and macro eNB. Equivalently, the spectrum efficiency of a FD BS is twice that of a HD BS. We define  $f(\eta_{sn}) \in [\frac{1}{2}, 1]$  to represent the normalized effective per-channel rate that BS s in cell n can support.  $f(\eta_{sn})$  is a decreasing function in  $\eta_{sn}$ , with  $f(1) \stackrel{\text{def}}{=} \frac{1}{2}f(0)$  and  $f(0) \stackrel{\text{def}}{=} 1$ . As an example, in Section V we evaluate the proposed allocation schemes assuming  $f(\eta_{sn}) = 1 - \frac{1}{2}\eta_{sn}$ .

We assume that there are M users, with  $\mathcal{M}$  $\{1, 2, \ldots, M\}$  representing the set of users. Each user, say m, requests a certain probabilistic rate, i.e., a rate (denoted by  $R_m$ ) to be satisfied with a minimum prespecified probability. In this paper, we consider two models for probabilistic rate demands: Heterogeneous and homogeneous. In the heterogeneous model, a user, say  $m \in \mathcal{M}$ , requests (in general) different probabilities, denoted by  $\alpha_{nm} \in (0, 1]$ , for different small cells  $n \in \mathcal{N}$ . In contrast, in the homogeneous model, a user, say m, requests one probability, denoted by  $\alpha_m$ , for the entire network. In both models,  $R_m$  is the same across all small cells<sup>1</sup>. The set of channels that can be used by the opportunistic LTE-A network (i.e., when they are available) is denoted by  $\mathcal{K} \stackrel{\text{def}}{=} \{1, 2, \dots, K\}$ . The opportunistic LTE-A network adopts frequency reuse with factor one (i.e., reuse-1), in which the entire set of channels  $\mathcal{K}$  can be used (when they are available) in all small cells. We associate with each channel  $k \in \mathcal{K}$  a cell-dependent binary random variable  $\tilde{w}_{kn}, k \in \mathcal{K}, n \in \mathcal{N}$ , which describes its availability. Let  $p_{kn}$  be the probability that channel k is available in small cell n. Then,  $\tilde{w}_{kn}$  equals one with probability  $p_{kn}$  and zero otherwise.

<sup>&</sup>lt;sup>1</sup>There are several scenarios under which the objective may be providing a given user with a constant rate across all cells, but with different (celldependent) probabilistic guarantees. One such scenario is when a user, say m, is using an app that requires a particular rate,  $R_m$ , and wants to limit outage probability. If the user's mobility pattern is well known, we may want to particularly guarantee performance in the cells where the user is most likely to be (e.g., achieving an outage probability of below 5% in a handful of cells and 20% in the rest may be much cheaper than achieving an outage probability of below 7% in all cells).

Next, we propose two static stochastic joint channel and BS allocation schemes for heterogeneous and homogeneous demands, which we refer to as Het-SMKP<sub>1</sub> and Hom-SMKP, respectively. Then, we develop Het-SMKP<sub>2</sub>, a two-stage adaptive channel and BS allocation scheme for heterogeneous demands.

### III. STATIC JOINT CHANNEL AND BS ALLOCATION

In this section, we formulate the joint channel and BS allocation problem, following the static (non-corrective) stochastic optimization model. In this model, the allocation is performed once, and cannot be corrected after observing the availability of the assigned channels. The objective of the allocation problem is to minimize the cost of composing an opportunistic LTE-A network that probabilistically fulfills all user demands.

### A. Heterogeneous (Per-cell) Allocation

Because in the heterogeneous case, a user requests (in general) different probabilistic rates for different small cells, we formulate the joint allocation problem in this case considering a single cell.

If each channel's availability is assumed to be known, the heterogeneous (per-cell) joint channel and BS allocation problem can be formulated as a multiple knapsack problem (MKP) [13] with an additional constraint, as follows. Each (channel, BS) pair, say (k, s), represents an 'item' in MKP. The 'cost' of item (k, s) is  $c_{ks}$  and its 'weight' is  $\tilde{w}_k \times f(\eta_s)$ . The 'capacity' of the *m*th knapsack is  $R_m$ . The constraint that the joint allocation problem adds to MKP is that each channel (item) is prevented not only from being assigned to multiple users (knapsacks) simultaneously, but also from being simultaneously assigned to multiple BSs.

Adding the uncertainty in the channel availability to the allocation problem causes the feasibility region of the problem to be uncertain. Different stochastic optimization approaches have been proposed in the literature to deal with the uncertainty of the feasibility region of an optimization problem [14]. In this section, we adopt a 'chance constraint approach.' In the following, we develop two chance-constrained stochastic MKP formulations to model the allocation problem under uncertainty; one (in this subsection) for the heterogeneous case (which we refer to as Het-SMKP<sub>1</sub>) and the other (in the next subsection) for the homogeneous case (which we refer to as Hom-SMKP).

### 1) Problem Formulation:

Let  $x_{ksm}, k \in \mathcal{K}, s \in \mathcal{S}, m \in \mathcal{M}$ , be a binary decision variable that is defined as follows:

$$x_{ksm} = \begin{cases} 1, & \text{if channel } k \text{ will be used in BS } s \text{ to serve} \\ & \text{user } m \\ 0, & \text{otherwise.} \end{cases}$$



where  $\tilde{w}_k$ ,  $\eta_s$ , and  $\alpha_m$  are as defined in Section II, after dropping the small-cell index. The objective (1) is to minimize the cost of the opportunistic LTE-A network, and the chance constraint (2) enforces satisfying the demand of user *m* with probability  $\geq \alpha_m$ . While the chance constraint probabilistically accounts for user under-satisfaction, it does not hedge against the problem of user over-satisfaction<sup>2</sup>. Oversatisfying one user may result in under-satisfying other users.

In Section V, we implement a variant of Het-SMKP<sub>1</sub>, which we call Het-SMKP<sub>1</sub><sup>\*</sup>. In Het-SMKP<sub>1</sub><sup>\*</sup>, we replace the objective function in (1) with the following objective function:

$$\min_{\{x_{ksm}, k \in \mathcal{K}, s \in \mathcal{S}, m \in \mathcal{M}\}} \left\{ \sum_{s=1}^{S} c_s \, \mathbb{1}_{\{\sum_{k=1}^{K} \sum_{m=1}^{M} x_{ksm} \ge 1\}} \right\}$$
(5)

which can be equivalently represented by the following linear formulation:

$$\min_{\{x_{ksm}, k \in \mathcal{K}, s \in \mathcal{S}, m \in \mathcal{M}\}} \left\{ \sum_{s=1}^{S} c_s \, \delta_s \right\}$$
(6)

subject to:

$$\delta_s \le \sum_{k=1}^K \sum_{m=1}^M x_{ksm} \le \max\{M, K\} \ \delta_s, \forall s \in \mathcal{S}.$$
(7)

In Het-SMKP<sub>1</sub><sup>\*</sup>, the cost of using a certain BS does not increase with the number of channels assigned to this BS. Accordingly, Het-SMKP<sub>1</sub><sup>\*</sup> tends to select cheaper BSs and use more channels compared to Het-SMKP<sub>1</sub>. In Section V, we numerically compare between Het-SMKP<sub>1</sub> and Het-SMKP<sub>1</sub><sup>\*</sup>.

### 2) Problem Reformulation and Solution Approach:

Our approach to solving the proposed stochastic optimization problems is to derive their deterministic equivalent programs (DEPs). The DEP is an equivalent reformulation of the original stochastic program, but contains only deterministic variables [14].

To obtain the DEP of Het-SMKP<sub>1</sub>, we need to reformulate the chance constraint (2), so that it does not include the probability term or the random variables:  $\tilde{w}_k, k \in \mathcal{K}$ . Let  $p^{(\omega)}$ be the probability of scenario  $\omega \in \Omega$ , where  $\Omega$  is the set of "scenarios," various realizations of the channels availability. To

Then, the Het-SMKP<sub>1</sub> formulation is given by:

<sup>&</sup>lt;sup>2</sup>In Het-SMKP<sub>1</sub>, under certain scenarios (i.e., realizations of the channels availability), a user, say m, may receive more than its demand,  $R_m$ .



Fig. 2: A two-cell network, with one FD BS in each cell. Each BS is assigned one channel that is available with probability 0.7.

reformulate the chance constraint, we will introduce a binary variable  $u_m^{(\omega)}$  for each user  $m \in \mathcal{M}$  and each scenario  $\omega \in \Omega$ .  $u_m^{(\omega)} = 0$  only if the joint channel and BS allocation satisfies the demand  $R_m$  under scenario  $\omega$ . Then, (2) is equivalent to constraints (9) and (10). The DEP of Het-SMKP<sub>1</sub> is given by:

Het-SMKP<sub>1</sub> (DEP):  

$$\begin{array}{l} \underset{u_{m}^{(\omega)}, \omega \in \Omega}{\mininize} \\ \left\{ \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ks} x_{ksm} \right\} (8) \\ \text{subject to:} \\ \sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksm} w_{k}^{(\omega)} f(\eta_{s}) \geq R_{m} \left(1 - u_{m}^{(\omega)}\right), \\ \forall m \in \mathcal{M}, \forall \omega \in \Omega \qquad (9) \\ \sum_{\omega \in \Omega} p^{(\omega)} \left(1 - u_{m}^{(\omega)}\right) \geq \alpha_{m}, \forall m \in \mathcal{M} \qquad (10) \\ \end{array}$$

$$\sum_{s=1}^{S} \sum_{m=1}^{M} x_{ksm} \le 1, \forall k \in \mathcal{K}$$

$$\tag{11}$$

$$x_{ksm} \in \{0,1\}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}$$
(12)

$$u_m^{(\omega)} \in \{0, 1\}, \forall m \in \mathcal{M}, \forall \omega \in \Omega.$$
(13)

### B. Homogeneous (Multi-cell) Allocation

In this section, we consider the homogeneous (multi-cell) allocation problem. This problem cannot be formulated by simply repeating constraint (2) in Problem 1 for each small cell. To illustrate this, consider the simple example in Figure 2. In this example, our objective is to compose a two-cell network that supports a rate of R Mbps with probability  $\geq 0.7$ . There exists one FD BS in each cell (i.e.,  $f(\eta) = 1$ ) and two channels. Each channel can support a rate of R Mbps (when it is available), and it is available with probability 0.7. If each BS is assigned one of these channels, then although each cell can support the requested rate with probability 0.7, the entire two-cell network supports the demand R only with probability 0.49.

### 1) Problem Formulation:

To formulate the homogeneous multi-cell allocation problem, we introduce the following binary variables,  $\tilde{d}_{nm}, n \in \mathcal{N}, m \in \mathcal{M}$ :

$$\tilde{d}_{nm} = \begin{cases} 1, & \text{if } \sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \ \tilde{w}_{kn} \ f(\eta_{sn}) \ge R_m \\ 0, & \text{otherwise} \end{cases}$$
(14)

where  $x_{ksmn}$  is as defined in Problem 1, after adding the cell index. Then, the multi-cell allocation problem is formulated

by replacing (2) with the following constraint:

$$\Pr\left\{ \left[ \tilde{D}_m \stackrel{\text{def}}{=} \tilde{d}_{1m} \text{ AND } \dots \text{ AND } \tilde{d}_{Nm} \right] \ge 1 \right\} \ge \alpha_m, \quad (15)$$
$$\forall m \in \mathcal{M}.$$

Next, we derive equivalent linear formulations for the indicator function (14) and the AND operation (15). Equation (14) can be reformulated as follows:

First,  $\tilde{d}_{nm} = 1 \Rightarrow \sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \ \tilde{w}_{kn} \ f(\eta_{sn}) \ge R_m$  can be reformulated as:

$$\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \, \tilde{w}_{kn} \, f(\eta_{sn}) + m \, \tilde{d}_{nm} \ge m + R_m \qquad (16)$$

where m is a lower bound of  $\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \tilde{w}_{kn} f(\eta_{sn}) - R_m$ . Selecting m to be  $-R_m$ , (16) reduces to  $\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \tilde{w}_{kn} f(\eta_{sn}) \geq R_m \tilde{d}_{nm}$ .

The second part of (14),  $\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \quad \tilde{w}_{kn} \quad f(\eta_{sn}) \geq R_m \Rightarrow \tilde{d}_{nm} = 1,$ can be reformulated as:

$$\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \, \tilde{w}_{kn} \, f(\eta_{sn}) - M \, \tilde{d}_{nm} \le R_m \qquad (17)$$

where M is an upper bound of  $\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \tilde{w}_{kn} f(\eta_{sn}) - R_m$ . Selecting M to be  $K - R_m$ , (17) reduces to  $\sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \tilde{w}_{kn} f(\eta_{sn}) \leq (K - R_m) \tilde{d}_{nm} + R_m$ .

Therefore, (14) can be equivalently written as:

$$R_m \, \tilde{d}_{nm} \le \sum_{k=1}^K \sum_{s=1}^S x_{ksmn} \, \tilde{w}_{kn} \, f(\eta_{sn}) \le (K - R_m) \, \tilde{d}_{nm} + R_m$$
(18)

The equivalent linear representation of  $D_m$  in (15) is the following set of inequalities:

$$\tilde{D}_m \leq \tilde{d}_{nm}, \forall n \in \mathcal{N} 
\tilde{D}_m \geq \sum_{n=1}^N \tilde{d}_{nm} - (N-1) 
\tilde{D}_m \geq 0.$$
(19)

Then, the Hom-SMKP formulation is given by:

Problem 2 (Hom-SMKP):  

$$\underset{\substack{x_{ksmn}, \tilde{d}_{nm}, \\ \tilde{D}_{m}, \\ k \in \mathcal{K}, s \in \mathcal{S}, \\ m \in \mathcal{M}, m \in \mathcal{N}}}{\text{minimize}} \left\{ \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ksn} x_{ksmn} \right\} (20)$$

subject to:

$$\Pr\left\{\tilde{D}_m \ge 1\right\} \ge \alpha_m, \forall m \in \mathcal{M}$$
(21)

$$D_m \le d_{nm}, \forall n \in \mathcal{N}, \forall m \in \mathcal{M}$$
(22)

$$\tilde{D}_m \ge \sum_{n=1}^{\infty} \tilde{d}_{nm} - (N-1), \forall m \in \mathcal{M}$$
(23)

$$\tilde{D}_m \ge 0, \forall m \in \mathcal{M} \tag{24}$$

$$R_{m} \tilde{d}_{nm} \leq \sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} \tilde{w}_{kn} f(\eta_{sn})$$
$$\leq (K - R_{m}) \tilde{d}_{nm} + R_{m},$$
$$\forall n \in \mathcal{N}, \forall m \in \mathcal{M} \quad (25)$$

$$\sum_{s=1}^{S} \sum_{m=1}^{M} x_{ksmn} \le 1, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$$
(26)

 $x_{ksmn} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \\ \forall m \in \mathcal{M}, \forall n \in \mathcal{N}$ (27)

$$\tilde{d}_{nm}, \tilde{D}_m \in \{0, 1\}, \forall n \in \mathcal{N}, \forall m \in \mathcal{M}.$$
(28)

## 2) Problem Reformulation and Solution Approach:

Similar to Het-SMKP<sub>1</sub>, we solve Hom-SMKP by deriving its DEP. Similar to (9) and (10), constraint (21) is equivalent to constraints (30) and (31). Furthermore, in the DEP, constraints (22)-(25) are defined for each scenario  $\omega \in \Omega$ , as in (32)-(35). The DEP of Hom-SMKP is given below.

### IV. ADAPTIVE JOINT CHANNEL AND BS ALLOCATION

In this section, we formulate the adaptive (corrective) joint channel and BS allocation problem. The channels and BSs are initially allocated such that the chance constraint is satisfied. After observing the actual channel availability, additional channels (if any) are released from over-satisfied users, and added to under-satisfied users (if any). In this section, we only formulate the heterogeneous (per-cell) adaptive allocation problem. The homogeneous multi-cell adaptive allocation problem is left for future research.

# Hom-SMKP (DEP): $\left\{\begin{array}{l} \underset{\substack{x_{ksmn}, d_{nm}^{(\omega)}, \\ D_{m}^{(\omega)}, u_{m}^{(\omega)}, \\ m \in \mathcal{M}, n \in \mathcal{M}}\end{array}}{\min \left\{\sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ksn} x_{ksmn}\right\}} (29)$ $m {\in} \mathcal{M}, n {\in} \mathcal{N},$ $\omega \in \Omega$ subject to: $D_m^{(\omega)} \ge 1 - u_m^{(\omega)}, \forall \omega \in \Omega, \forall m \in \mathcal{M}$ (30) $\sum_{\omega \in \Omega} p^{(\omega)} \left( 1 - u_m^{(\omega)} \right) \ge \alpha_m, \forall m \in \mathcal{M}$ (31) $D_{m}^{(\omega)} \leq d_{nm}^{(\omega)}, \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$ (32) $D_m^{(\omega)} \ge \sum_{n=1}^{N} d_{nm}^{(\omega)} - (N-1), \forall m \in \mathcal{M}, \forall \omega \in \Omega$ (33) $D_m^{(\omega)} \ge 0, \forall m \in \mathcal{M}, \forall \omega \in \Omega$ (34) $R_{m} d_{nm}^{(\omega)} \leq \sum_{k=1}^{K} \sum_{s=1}^{S} x_{ksmn} w_{kn}^{(\omega)} f(\eta_{sn})$ $\leq (K - R_m) d_{nm}^{(\omega)} + R_m,$ $\forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$ (35) $\sum_{s=1}^{S} \sum_{m=1}^{M} x_{ksmn} \le 1, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$ (36) $x_{ksmn} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S},$ $\forall m \in \mathcal{M}, \forall n \in \mathcal{N}$ (37) $d_{nm}^{(\omega)}, D_m^{(\omega)}, u_m^{(\omega)} \in \{0, 1\}, \forall n \in \mathcal{N},$ $\forall m \in \mathcal{M}, \forall \omega \in \Omega.$ (38)

### A. Problem Formulation

The adaptive heterogeneous (per-cell) allocation problem is formulated as a two-stage stochastic MKP with recourse, which we refer to as Het-SMKP<sub>2</sub>. The first stage is similar to Het-SMKP<sub>1</sub>. The objective of the second stage is to maximize the number of extra channels/BSs that can be taken from oversatisfied users. These extra resources will be added to undersatisfied users (if any), or released (otherwise). By doing so, we (i) maximize the probability of user demands satisfaction, and (ii) minimize the cost of composing an LTE-A network (by releasing the extra resources). Let  $y_{ksm}$  and  $z_{ksm}$ ,  $k \in$  $\mathcal{K}, s \in \mathcal{S}, m \in \mathcal{M}$ , be binary variables;  $y_{ksm} = 1$  if channel k operating in BS s is released from user m, and zero otherwise, and  $z_{ksm} = 1$  if channel k operating in BS s is added to user m, and zero otherwise. Then, the objective function of the second stage of Het-SMKP $_2$  can be expressed as in (45), where  $\gamma_{ks} \in [0,1)$  is a discount factor. We assume that the value of the resource at the second-stage (i.e., when it is released after it was previously assigned) is strictly smaller than its first-stage value, i.e.,  $\gamma_{ks} < 1$ . This way, we avoid having an allocation where all resources will be allocated in the first stage (when  $\gamma_{ks} = 1).$ 

The constraints of the second-stage problem of  $\text{Het-SMKP}_2$  can be summarized as follows:

1. A channel can be released only if it has been already

assigned in the first stage.

- 2. A channel can be taken only from over-satisfied users.
- 3. A channel can be assigned only to under-satisfied users.
- 4. A channel can be assigned to an under-satisfied user only if it can be released from an over-satisfied user.
- 5. A released channel can be assigned only to one undersatisfied user.

Constraint 1 is enforced by adding:

$$y_{ksm} \le x_{ksm}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}.$$
 (39)

Constraint 2 is enforced by adding:

$$\sum_{i=1}^{K} \sum_{j=1}^{S} (x_{ijm} - y_{ijm}) \quad \tilde{w}_i \ f(\eta_j) \ge R_m \ y_{ksm}, \qquad (40)$$
$$\forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}.$$

Constraint 3 is ensured by adding:

$$\sum_{i=1}^{K} \sum_{j=1}^{S} (x_{ijm} - y_{ijm} + z_{ijm}) \quad \tilde{w}_i f(\eta_j) < (K - R_m - f(\eta_s)) (1 - z_{ksm}) + R_m + f(\eta_s), \quad (41)$$
$$\forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}.$$

Constraint 4 is ensured by adding:

$$z_{ksm} \le \sum_{i=1}^{M} y_{ksi}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}.$$
(42)

Finally, constraint 5 is ensured by adding:

$$\sum_{i=1}^{M} z_{ksi} \le 1, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}.$$
(43)

The Het-SMKP<sub>2</sub> formulation is summarized below.

Problem 3 (Het-SMKP<sub>2</sub>):  

$$\min_{\substack{x_{ksm}, k \in \mathcal{K}, \\ s \in \mathcal{S}, m \in \mathcal{M}}} \left\{ \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ks} x_{ksm} + \mathbb{E}[h(\boldsymbol{x}, \tilde{\boldsymbol{w}})] \right\}$$
(44)

subject to:

where  $h(x, \tilde{w})$  is the optimal value of the second-stage problem, which is given by:

$$\underset{\substack{y_{ksm}, z_{ksm} \\ k \in \mathcal{K}, s \in \mathcal{S}, \\ m \in \mathcal{M}}}{\text{minimize}} \left\{ -\sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} \gamma_{ks} c_{ks} \left( y_{ksm} + z_{ksm} \right) \right\}$$
(45)

subject to:

(39), (40), (41), (42), (43), and  

$$y_{ksm}, z_{ksm}, \beta_{ksm} \in \{0, 1\}, \forall k \in \mathcal{K},$$
  
 $\forall s \in \mathcal{S}, \forall m \in \mathcal{M}.$  (46)

We note that Het-SMKP<sub>2</sub> has a relatively complete recourse, i.e., for every feasible first-stage decision,  $x_{ksm}$ , there exists a feasible solution to the second-stage problem under each scenario  $\omega \in \Omega$ . For example,  $y_{ksm} = z_{ksm} = 0, \forall k \in \mathcal{K}, \forall s \in S, \forall m \in \mathcal{M}$ , is always a feasible solution to the second-stage problem.

### B. Problem Reformulation and Solution Approach

Similar to Het-SMKP<sub>1</sub> and Hom-SMKP, we solve Het-SMKP<sub>2</sub> by deriving its DEP. The second-stage objective function is substituted in (44) and constraints (39)-(43) are evaluated for each scenario  $\omega \in \Omega$ . The DEP of Het-SMKP<sub>2</sub> is given below.

Het-SMKP<sub>2</sub> (DEP):  

$$\begin{cases}
\min_{\substack{x_{ksm}, y_{ksm}^{(\omega)}, z_{ksm}^{(\omega)}, w_{m}^{(\omega)} \\ k \in \mathcal{K}, s \in \mathcal{S}, m \in \mathcal{M}, \\ \omega \in \Omega}
\end{cases}
\begin{cases}
\sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ks} x_{ksm} \\
\sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{m=1}^{M} c_{ks} x_{ksm} \\
\sum_{k=1}^{K} \sum_{s=1}^{K} \sum_{m=1}^{K} \sum_{s=1}^{K} \sum_{m=1}^{K} c_{ks} x_{ksm} \\
\sum_{k=1}^{K} \sum_{s=1}^{K} \sum_{m=1}^{K} \sum_{s=1}^{K} \sum_{m=1}^{K} \sum_{s=1}^{K} \sum_{m=1}^{K} \sum_{s=1}^{K} \sum_{s=$$

subject to:

$$(9), (10), \text{ and}$$

$$y_{ksm}^{(\omega)} \leq x_{ksm}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$$

$$(48)$$

$$\sum_{i=1}^{K} \sum_{j=1}^{S} \left( x_{ijm} - y_{ijm}^{(\omega)} \right) w_i^{(\omega)} f(\eta_j) \geq R_m y_{ksm}^{(\omega)},$$

$$\forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$$

$$(49)$$

$$\sum_{i=1}^{K} \sum_{j=1}^{S} \left( x_{ijm} - y_{ijm}^{(\omega)} + z_{ijm}^{(\omega)} \right) w_i^{(\omega)} f(\eta_j) <$$

$$(K - R_m - f(\eta_s)) (1 - z_{ksm}) + R_m + f(\eta_s),$$

$$\forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$$

$$(50)$$

$$z_{ksm}^{(\omega)} \leq \sum_{i=1}^{M} y_{ksi}^{(\omega)}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}, \forall \omega \in \Omega$$

$$(51)$$

$$\sum_{i=1}^{M} z_{ksi}^{(\omega)} \le 1, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall \omega \in \Omega$$
(52)

$$x_{ksm}, y_{ksm}^{(\omega)}, z_{ksm}^{(\omega)}, u_m^{(\omega)} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall s \in \mathcal{S}, \forall m \in \mathcal{M}, \forall \omega \in \Omega.$$
(53)

### V. PERFORMANCE EVALUATION

In this section, we evaluate the proposed allocation schemes. All schemes are implemented in CPLEX. The numerical values of various parameters are listed in Table I.

### A. Static Joint Channel and BS Allocation

1) Heterogeneous (Per-cell) Allocation: In Figure 3, we compare between Het-SMKP<sub>1</sub> and Het-SMKP<sub>1</sub><sup>\*</sup> based on the total cost of the BSs used in composing

TABLE I: Numerical values of various parameters.

Parameter	Het-SMKP <sub>1</sub> , Het-SMKP <sup>*</sup> <sub>1</sub> , and Het-SMKP <sub>2</sub>	Hom-SMKP
K	8	5
S	3	3
N	1	2
$\eta_j, j \in S$	[0, 0.5, 1]	[0, 0.5, 1]
$f(\eta_j), j \in \mathcal{S}$	[1, 0.75, 0.5]	[1, 0.75, 0.5]
$c_j, j \in S$	[1, 0.3, 0.2]	[1, 0.3, 0.2]
$w_i, i \in \mathcal{K}$	$\left[0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 ight]$	[0.2, 0.4, 0.6, 0.8, 0.9]
$c_{ij}, i \in \mathcal{K}, j \in \mathcal{S}$	$c_j \times w_i$ for Het-SMKP <sub>1</sub> and Het-SMKP <sub>2</sub> , and $c_j$ for Het-SMKP <sub>1</sub> <sup>*</sup>	$c_j \times w_i$
$\gamma_{ij}, i \in \mathcal{K}, j \in \mathcal{S}$	0.8 for Het-SMKP <sub>2</sub>	-



Fig. 3: Total cost of the LTE-A network in Het-SMKP<sub>1</sub> and Het-SMKP<sub>1</sub><sup>\*</sup> as a function of  $R_m$  for different values of  $\alpha$  (M = 1).

an LTE-A network for satisfying the same probabilistic user demand. Figure 3 depicts the values of the objective function in equation (5) for both Het-SMKP<sub>1</sub> and Het-SMKP<sub>1</sub><sup>\*</sup> as a function of  $R_m$  when M = 1. As shown in the figure, Het-SMKP<sub>1</sub><sup>\*</sup> incurs less BSs cost compared to Het-SMKP<sub>1</sub>; because in Het-SMKP<sub>1</sub><sup>\*</sup> the cost of using a particular BS does not increase with the number of channels assigned to this BS.

The reduction in the total cost of allocated BSs in Het-SMKP<sub>1</sub><sup>\*</sup> comes at the expense of increasing the number of allocated channels, as can be observed by comparing Figures 4 and 5. Figures 4 and 5 illustrate the number of channels assigned to each BS according to Het-SMKP<sub>1</sub> and Het-SMKP<sub>1</sub><sup>\*</sup>, respectively.

In the following, we only consider Het-SMKP<sub>1</sub>. In Figure 6, we study the effect of increasing M on the objective function value of Het-SMKP<sub>1</sub> (i.e., equation (1)). As shown in the figure, increasing M (while fixing the total rate demand) reduces the cost of the composed LTE-A network. Furthermore, when the total requested rate exceeds a certain threshold, the allocation problem becomes infeasible when M = 1. This is because when M = 1 the total rate demand is required to be available with probability  $> \alpha$ , whereas when M > 1, only rate  $R_m$  is required to be available for each user m with probability  $> \alpha$  (i.e., when M > 1, it is not required that the total requested rate by all users is simultaneously available for  $\alpha$  fraction of the scenarios).

In Figure 7, we plot the admission rate (defined as the



Fig. 4: Number of channels assigned to each BS in Het-SMKP<sub>1</sub> as a function of  $R_m$  (M = 1).



Fig. 5: Number of channels assigned to each BS in Het-SMKP<sub>1</sub><sup>\*</sup> as a function of  $R_m$  (M = 1).

percentage of satisfied users) of Het-SMKP<sub>1</sub> vs.  $\alpha$  for different values of  $R_m$  when M = 8. As expected, the admission rate decreases with both  $\alpha$  and  $R_m$ .

### 2) Homogeneous (Multi-cell) Allocation:

In Figure 8, we study the effect of increasing M on the objective function value of Hom-SMKP (i.e., equation (20)). Similar to Figure 6, increasing M (for the same total rate demand) reduces the cost of composing the LTE-A network.



Fig. 6: Total cost of the LTE-A network vs.  $R_m$  for different values of  $\alpha$ .



Fig. 7: Admission rate of Het-SMKP<sub>1</sub> vs.  $\alpha$  for different values of  $R_m$  (M = 8).

Furthermore, when the total requested rate exceeds a certain threshold, the allocation problem becomes infeasible when M = 1.

The admission rate of Hom-SMKP is compared with that of Het-SMKP<sub>1</sub> in Figure 9. As explained in the example in Figure 2, the admission rate of Hom-SMKP is expected to be lower than that of Het-SMKP<sub>1</sub>.

### B. Adaptive Joint Channel and BS Allocation

In this section, we study the performance gain achieved by  $Het-SMKP_2$  compared to  $Het-SMKP_1$ .

In Figure 10, we increase the value of  $\alpha$  for one user while fixing the demands of the others, and plot the probability of demand dissatisfaction of both Het-SMKP<sub>1</sub> and Het-SMKP<sub>2</sub>. As shown in the figure, Het-SMKP<sub>1</sub> satisfies the chance constraint. Furthermore, the reduction in the demand dissatisfaction achieved by Het-SMKP<sub>2</sub> increases when  $\alpha_3$ decreases; because the reduction in  $\alpha_3$  results in more resource



Fig. 8: Cost of the two-cell LTE-A network in Hom-SMKP vs.  $R_m$  for different values of  $\alpha$ .



Fig. 9: Admission rates of Het-SMKP<sub>1</sub> and Hom-SMKP vs.  $\alpha$  (M = 5).

swapping (from an over-satisfied user to an under-satisfied user) in the second stage of Het-SMKP<sub>2</sub>.

Moreover, we show in Figure 11 the average demand shortage of both Het-SMKP<sub>1</sub> and Het-SMKP<sub>2</sub>. The demand shortage is averaged over all scenarios under which the demand of user 3 was not satisfied. Again, the reduction in the average demand shortage achieved by Het-SMKP<sub>2</sub> increases when  $\alpha_3$  decreases.

### VI. CONCLUSIONS AND FUTURE RESEARCH

We studied the problem of joint channel and BS allocation in OSA networks with heterogeneous self-interference cancellation (SIC) capabilities. We developed static (proactive) as well as adaptive (proactive and reactive) stochastic allocation models. In the static models, the allocation is done once such that the user demands are probabilistically met. In contrast, the adaptive model allows for correcting the initial allocation once uncertainties are partially resolved. We numerically evaluated our proposed static and adaptive allocation schemes under various system parameters. Our results corroborate the ability



Fig. 10: Probability of demand dissatisfaction of Het-SMKP<sub>1</sub> and Het-SMKP<sub>2</sub> ( $M = 3, [R_1, R_2, R_3] = [3, 2, 1]$  Mbps,  $\alpha_1 = \alpha_2 = 0.5$ ).



Fig. 11: Average demand shortage of Het-SMKP<sub>1</sub> and Het-SMKP<sub>2</sub>  $(M = 3, [R_1, R_2, R_3] = [3, 2, 1]$  Mbps,  $\alpha_1 = \alpha_2 = 0.5$ ).

of the proposed schemes to guarantee the probabilistic user demands are met optimally (i.e., while incurring the minimum cost). The adaptive scheme also shows significant performance gains due to its corrective/reactive capability.

The proposed formulations in this paper represent stochastic versions of the multiple knapsack problem, which are at least NP-hard (as the deterministic knapsack problem itself is NP-hard) [13]. Furthermore, our adaptive stochastic resource allocation scheme is limited to only two stages (present and future). In future research, we plan to:

• Develop approximate stochastic resource allocation schemes that are simple but close to optimal. One approach to do this is to significantly reduce the number of considered scenarios in the stochastic allocation problem while limiting the performance degradation. We aim to adopt some of the existing scenario reduction techniques, such as [15]–[17], as well as design new techniques that are specific to our proposed resource allocation problems.

• Extend the proposed two-stage allocation formulation to multiple stages to account for several decision making stages in the future, at which the resource allocation can be adjusted.

### ACKNOWLEDGMENT

This material is based upon work supported by the National Science Foundation under Grant No. 1443978.

#### REFERENCES

- J. G. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. C. Reed, "Femtocells: Past, present, and future," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 497–508, 2012.
- [2] Qualcomm small cells project. http://tinyurl.com/l5lwgcb.
- [3] Alcatel-Lucent small cells project. http://www.alcatellucent.com/solutions/small-cells.
- [4] S. Hong, J. Brand, J. Choi, M. Jain, J. Mehlman, S. Katti, and P. Levis, "Applications of self-interference cancellation in 5G and beyond," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 114–121, February 2014.
- [5] A. Sabharwal, P. Schniter, D. Guo, D. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: Challenges and opportunities," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 9, pp. 1637–1652, September 2014.
- [6] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving single channel, full duplex wireless communication," in *Proceedings of* the ACM Mobicom Conference, September 2010, pp. 1–12.
- [7] M. Jain, J. I. Choi, T. Kim, D. Bharadia, S. Seth, K. Srinivasan, P. Levis, S. Katti, and P. Sinha, "Practical, real-time, full duplex wireless," in *Proceedings of the ACM Mobicom Conference*, September 2011, pp. 301–312.
- [8] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in Proceedings of the ACM SIGCOMM Conference, August 2013, pp. 375– 386.
- [9] M. J. Abdel-Rahman, F. Lan, and M. Krunz, "Spectrum-efficient stochastic channel assignment for opportunistic networks," in *Proceed*ings of the IEEE GLOBECOM Conference, December 2013, pp. 1272– 1277.
- [10] M. J. Abdel-Rahman and M. Krunz, "Stochastic guard-band-aware channel assignment with bonding and aggregation for DSA networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 7, pp. 3888–3898, July 2015.
- [11] Qualcomm LTE-U project.
- https://www.qualcomm.com/products/lte/unlicensed.
- [12] Ericsson LTE-U project. http://www.ericsson.com/research-blog/lte/lteevolution-continues/.
- [13] H. Kellerer, U. Pferschy, and D. Pisinger, *Knapsack Problems*. Springer-Verlag, 2004.
- [14] P. Kall and S. W. Wallace, *Stochastic Programming*. John Wiley and Sons, 1994.
- [15] H. Heitsch and W. Römisch, "Scenario reduction algorithms in stochastic programming," *Computational Optimization and Applications*, vol. 24, pp. 187–206, 2003.
- [16] H. Heitsch and W. Römisch, "A note on scenario reduction for twostage stochastic programs," *Operations Research Letters*, vol. 35, pp. 731–738, 2007.
- [17] H. Heitsch and W. Römisch, "Scenario tree reduction for multistage stochastic programs," *Computational Management Science*, vol. 6, pp. 117–133, 2009.