



# Rendezvous Under Smart Jamming

Mohammad J. Abdel-Rahman and Marwan Krunz

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## Background

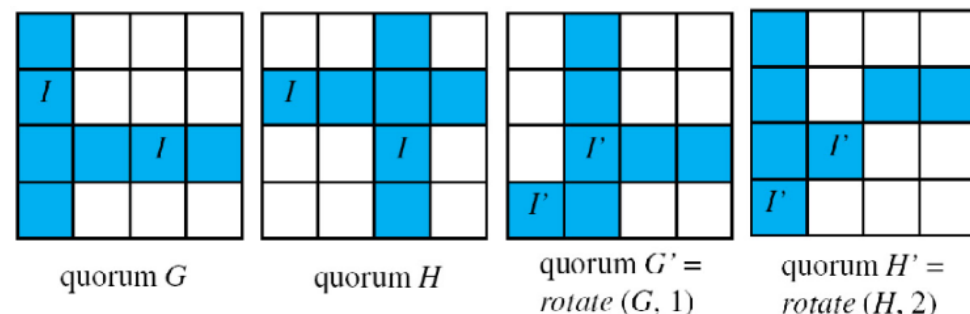
**Quorum System:** A collection of nonempty sets (called quorums) that pairwise overlap by one or more elements.  
Example:  $Q = \{ \{3, 4\}, \{2, 3\}, \{2, 4\} \}$  is a quorum system on  $\{2, 3, 4\}$ .

**Grid Quorum System (GQS):** The elements of the set are arranged into a square array. Each quorum consists of one column and one row.

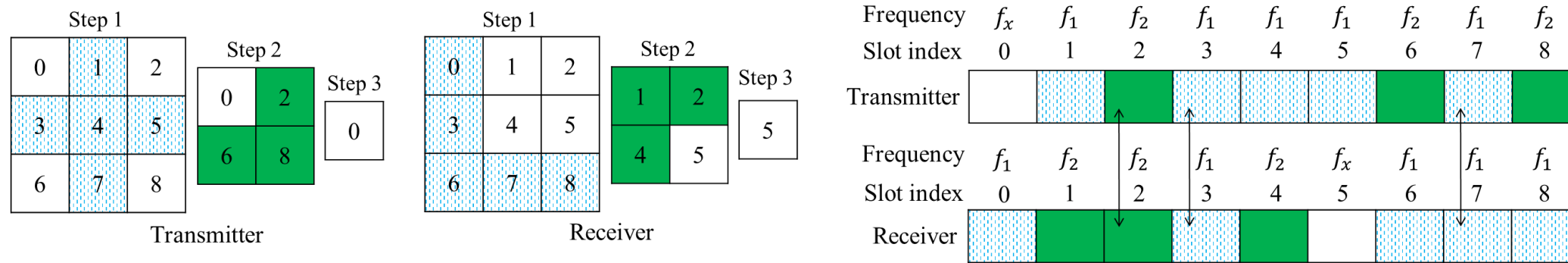
Example:  $Q = \{ \{1,2,3,4,7\}, \{1,2,3,5,8\}, \{1,2,3,6,9\}, \{1,4,5,6,7\}, \{2,4,5,6,8\}, \{3,4,5,6,9\}, \{1,4,7,8,9\}, \{2,5,7,8,9\}, \{3,6,7,8,9\} \}$  is a GQS on  $\{1, \dots, 9\}$ .

1	2	3
4	5	6
7	8	9

Intersection property  
Rotation closure property



## Nested Grid-quorum-based Frequency Hopping Algorithm (NGQFH)



## Problem Statement

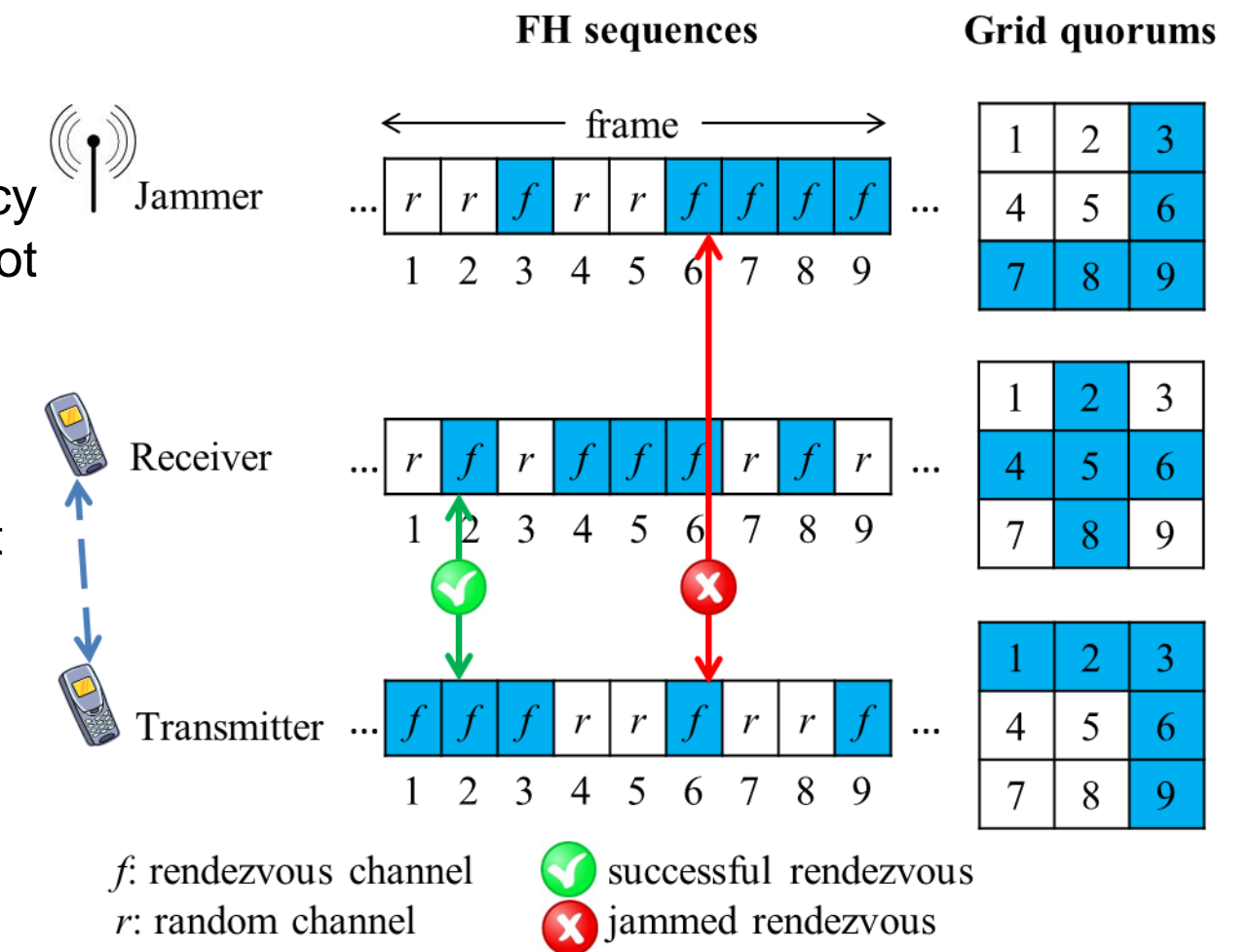
Two nodes aim at rendezvousing in the presence of an adversary.

### System Model:

- Single link (unicast).
- Nodes operate in frequency hopping mode, with slot duration  $T$ .
- NGQFH algorithm is used.

### Adversarial Model:

- Time-slotted jammer, with slot duration  $T$ .
- Jamming is carried out by a compromised node.
- Jammer is aware of the used NGQFH algorithm.



What does the jammer exactly know?

Frame length	Used grid quorum	Used rendezvous channel(s)
✓	✗	✓ ✗

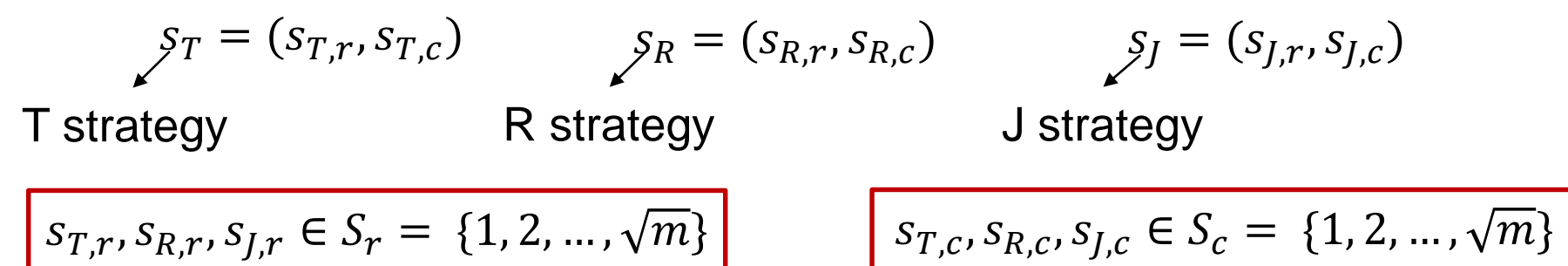
Synchronous and asynchronous cases are considered.

## Synchronous Rendezvous Over a Known Channel

### Part I. Three-player Game

**Players:** Transmitter (T), Receiver (R), and Jammer (J)

**Strategy:** Which quorum to select



**Utility:**  $u_T(S_T, S_R, S_J)$  = number of unjammed rendezvous slots per frame - number of jammed rendezvous slots per frame.

	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Rx	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Tx	(1, 1)	(1, 2)	(2, 1)	(2, 2)

**Theorem 1:** The three-player game does not have a pure-strategy NE.

### Part II. Two-player Game

**Theorem 2:** For any  $s_T = (s_{T,r}, s_{T,c})$ , the  $\sqrt{m} \times \sqrt{m}$  R/J game has at least  $(\sqrt{m} - 1)^2$  NEs, all of them result in  $u_T = -2$ . These NEs are given by:

$$S_{J,r} = s_{T,r}, S_{J,c} = s_{T,c}$$

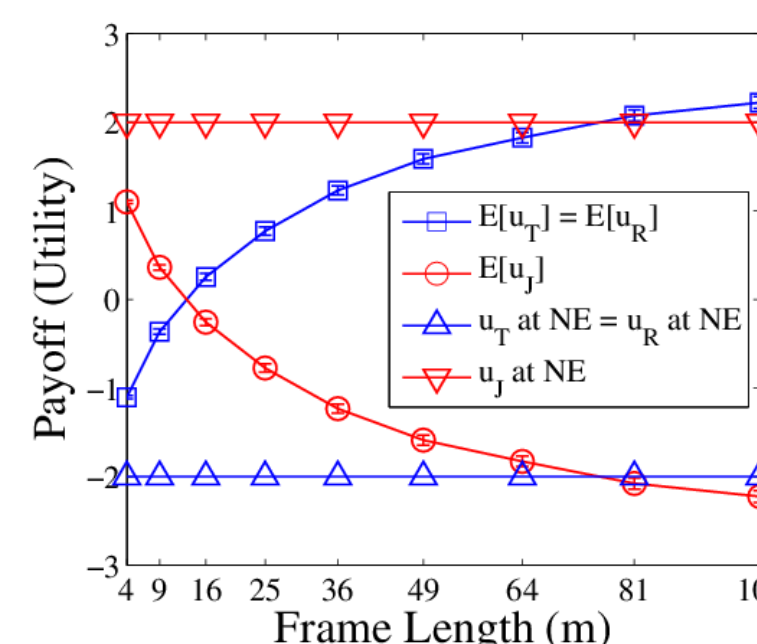
$$S_{R,r} \neq s_{T,r}, S_{R,c} \neq s_{T,c}$$

**Proposition:** The  $(\sqrt{m} - 1)^2$  NEs in Theorem 2 are the only NEs for the  $\sqrt{m} \times \sqrt{m}$  game when  $m \geq 9$ . When  $m = 4$ , the game has additional NEs, given by:

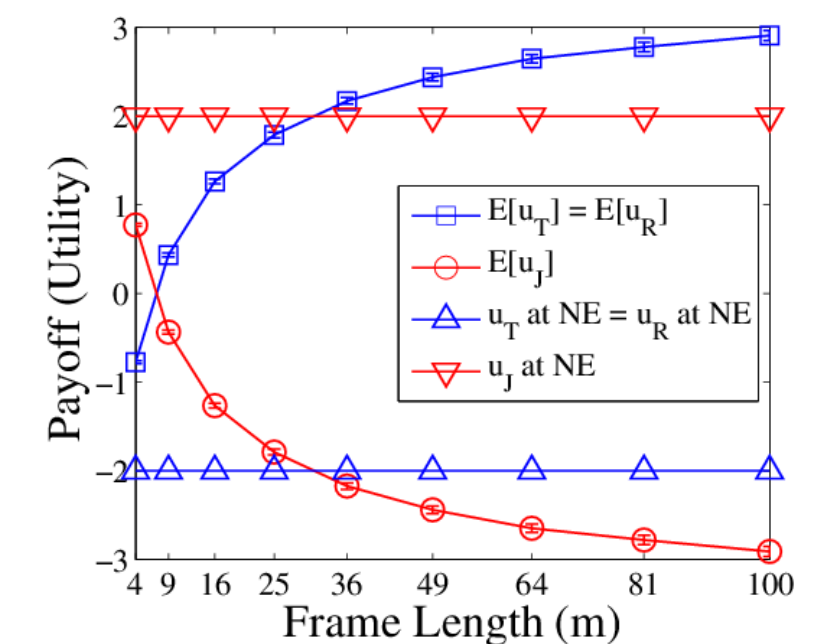
$$S_{T,r} = S_{R,r} = S_{J,r} \text{ and } S_{J,c} = S_{T,c} \neq S_{R,c}$$

$$S_{J,r} = S_{T,r} \neq S_{R,r} \text{ and } S_{T,c} = S_{R,c} = S_{J,c}$$

	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Rx	(1, 1)	(1, 2)	(2, 1)	(2, 2)
Tx	(1, 1)	(1, 2)	(2, 1)	(2, 2)



R and J have different beliefs about  $s_T$



R and J have a common belief about  $s_T$

## Synchronous Rendezvous Over an Unknown Channel (Bayesian Game)

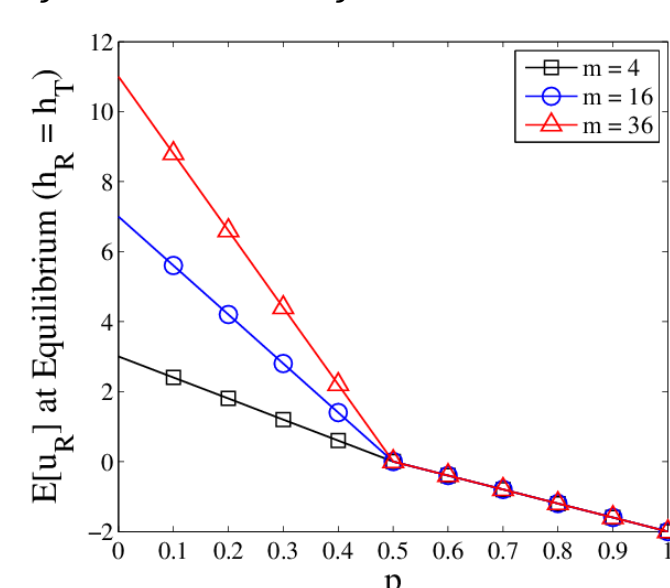
Let  $h_T, h_R$ , and  $h_J$  denote the channels selected by T, R, and J, respectively. Then, R has two types:  $h_R = h_T$  and  $h_R \neq h_T$  and J has two types:  $h_J = h_T$  and  $h_J \neq h_T$ .

**Theorem 3:** Let  $p = \Pr\{h_J = h_T | h_R = h_T\}$ , then the Bayesian NE of the above game is:

$$S_R = \begin{cases} = s_T, & \text{if } p < 0.5 \\ \neq s_T, & \text{if } p > 0.5, \\ \text{Does not matter,} & \text{if } p = 0.5 \end{cases} \text{ if } h_R = h_T$$

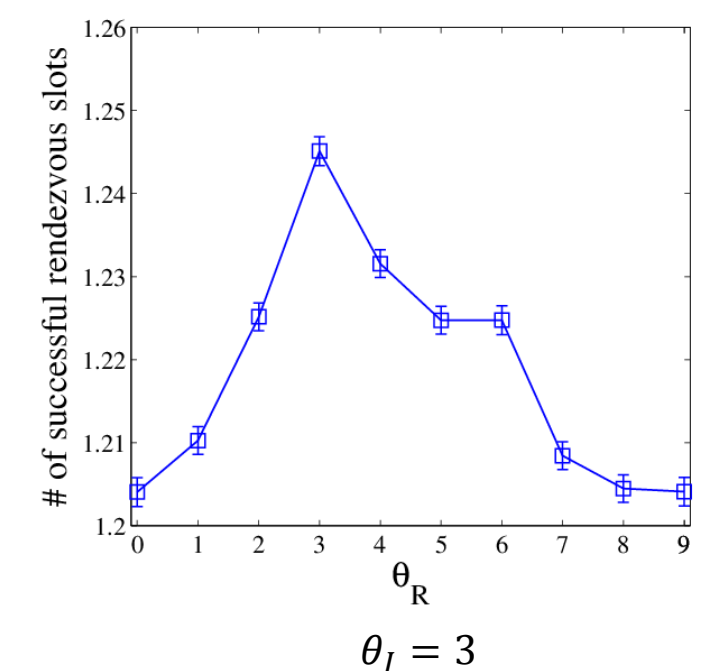
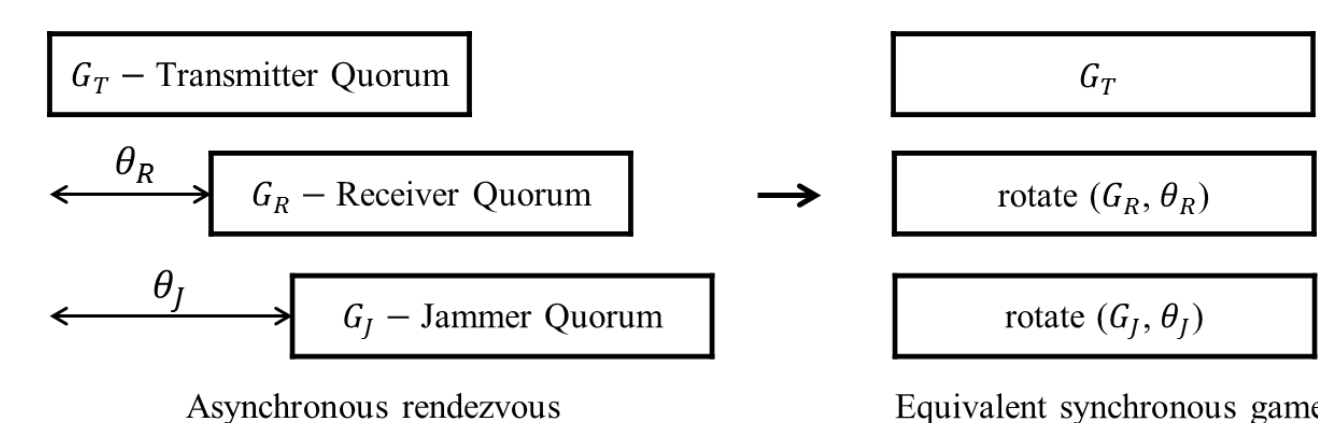
$$S_R = \begin{cases} \text{Does not matter,} & \text{if } h_R \neq h_T \end{cases}$$

$$S_J = \begin{cases} = s_T, & \text{if } h_J = h_T \\ \text{Does not matter,} & \text{if } h_J \neq h_T. \end{cases}$$



## Asynchronous Rendezvous

The strategy of the player consists of a column and a sequence of  $\sqrt{m}$  consecutive elements that do not necessarily form a row.



## Main Conclusions

### Synchronous Case

- R benefits from being, along with J, unaware of  $s_T$ . Furthermore, the benefits of R increase with the frame length.
- It is beneficial for R if J has the same belief about  $s_T$  as it has.

### Asynchronous Case

The number of successful rendezvous slots is maximized when  $\theta_R = \theta_J$ .

## Ongoing/Future Work

- Examine the more general case when the nesting degree is greater than one.
- Design different sequential/parallel update mechanisms, including best-response update.
- Study the convergence behavior of various updating mechanisms.
- Consider other utility functions for the game formulation.
- Consider the multicast rendezvous problem under smart jamming.