Coexistence in Wireless Networks with Heterogeneous Self-interference Cancellation Capabilities

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Abstract

Recently, tremendous progress has been made in self-interference cancellation (SIC) techniques that enable a wireless device to transmit and receive data simultaneously on the same frequency channel, a.k.a. in-band full-duplex (FD) communications. Although operating in a FD mode significantly improves the throughput of a single wireless link, it doubles the number of concurrent transmissions, which limits the potential for coexistence between multiple FD-enabled links. In this paper, we consider the problem of concurrent transmissions between two FD-enabled links with different SIC capabilities; each link can operate in either FD or half-duplex (HD) mode. Following a game-theoretic framework, we aim to determine the stable behavior (FD or HD) for the two coexisting links. To achieve this objective, we first analyze a simple normal form game between the two links, which provides some insight into the coexistence problem. It turns out that the outcome of this game depends on two factors: The amount of residual self-interference (due to imperfect SIC) and the external interference from one link on the other. To capture the impact of residual self-interference, we formulate a Bayesian game between two links with heterogeneous SIC capabilities. In this game, each link (player) tries to maximize its throughput while minimizing the transmission power cost. We derive the Bayesian Nash equilibrium for this game. Furthermore, we determine the conditions on the external interference under which no outage occurs at both links. Finally, we conduct simulations and USRP hardware experiments to corroborate our analytical findings.

I. INTRODUCTION

A. Motivation

Classical wireless systems achieve bidirectional communications by separating the forward and reverse links in time, i.e., time division duplexing (TDD), or frequency, i.e., frequency division duplexing (FDD). Some wireless systems, such as 4G LTE, support both schemes (e.g., LTE-TDD and LTE-FDD). The challenge of achieving simultaneous transmission and reception on the same frequency, i.e., in-band full-duplex (FD) communications, is related to the strong self-interference that arises when a device that is receiving some information signal attempts to transmit another signal at the same time. Because of path loss, the received power of the intended signal (from the peer node) is often much weaker than the node’s self-interference. This results in saturating the ADC and prevents packet decoding. Recently, new designs for analog and digital self-interference cancellation (SIC) techniques have been proposed (see [1] for a survey), which together provide up to 110 dB SIC on a single-antenna FD transceiver [2].

From one link’s perspective, the advantage of FD communications is clear; it basically doubles the link’s throughput. However, such gain is less obvious in the case of a network with multiple interfering links. When these links operate in the same vicinity (i.e., the same collision domain), it is not always optimal for all links to operate in FD fashion [1, 3]. To illustrate, consider the three scenarios in Figure 1. In each scenario, two links are active at the same time and over the same frequency channel. As shown in Figure 1(a), transmitting in an HD fashion enables both links \(a \rightarrow b\) and \(c \rightarrow d\) to operate simultaneously over the same channel, achieving a total throughput of \(2R\) bps (for simplicity, in this example we assume that all transmissions are associated with a constant rate \(R\)). However, if link \(a \rightarrow b\) switches unilaterally from HD to FD, as shown in Figure 1(b), collisions may occur at both nodes \(a\) and \(d\), reducing the network throughput to \(R\) bps (only \(a \rightarrow b\) transmission is successful). The same argument applies if link \(c \rightarrow d\) switches to FD mode instead of \(a \rightarrow b\). If both links operate in FD mode, collisions...
Fig. 1. Implications of operating wireless networks in FD fashion. (a) (HD, HD) strategy returns total throughput of $2R$ bps, (b) (FD, HD) strategy returns total throughput of $R$ bps, while (c) (FD, FD) strategy returns zero throughput. The arcs represent the transmission ranges of different nodes.

will occur at all four nodes, reducing the network throughput to zero (see Figure 1(c)). Note that this is a simplified example of a small network with only two links. The situation worsens with a higher number of links, where the collision probability increases between concurrent transmissions from different links.

B. Related Work

The feasibility of network-wide FD communications has been discussed in [3–7]. In [3], the authors proposed an analytical framework to analyze the capacity gain of FD networks, assuming a CSMA-based MAC protocol. They found that spatial reuse and inter-link interference may cause significant reduction to the network capacity. To enable FD Wi-Fi networks, the authors in [4] proposed a MAC protocol and presented an experimental and simulation study to assess the performance of their scheme. In contrast to [3], they found that FD communications can potentially enhance the performance of Wi-Fi networks. The authors in [6] considered the inter-node interference in a three-node (base station, downlink node, and uplink node) FD OFDM network. To maximize the network sum-throughput under this scenario, they proposed different power allocation schemes while taking into account inter-node interference.

The achievable throughput of FD MIMO networks was investigated in [5, 7]. In [5] the authors considered a binary interference model of a general network topology and derived the conditions under which different technologies (MIMO, multi-user MIMO, and FD communications) achieve the best performance. In [7] the authors considered the problem of power minimization in an FD MIMO network subject to rate demands. They derived conditions under which the FD mode outperforms the HD mode.

In [8], the authors proposed several stochastic resource allocation formulations to minimize the cost of composing a virtual LTE-U network from a hybrid set of HD/FD Wi-Fi access points (APs). An AP is assumed to be either HD or FD (with perfect SIC). In general, different devices in a network may have different SIC capabilities, depending, mainly, on the employed analog and digital cancellation mechanisms. In [9] the authors proposed joint channel and base station (BS) stochastic allocation schemes for opportunistic LTE networks with heterogeneous SIC capabilities at different BSs. The resource allocation schemes in [8, 9] are centralized with high computational complexity. Instead, in this paper we develop a game-theoretic framework to analyze the performance of wireless networks with heterogeneous SIC capabilities and in the absence of any global information (i.e., a link does not know the exact SIC level of its neighboring link).

C. Contributions

1) We analyze a simple (HD vs. FD) game between two bidirectional links to gain insight into the coexistence problem. In this game, links (players) know the exact SIC capabilities of each other. Simple utilities and interference models are considered. We find that the outcome of this game depends on two factors: Residual self-interference (RSI) (due to imperfect SIC) and external interference (from one link on the other).

2) To capture the heterogeneity in SIC capabilities, we then formulate a Bayesian game between the two links. In this game, the SIC capability of each link specifies its ‘type,’ which is unknown to the other link. We derive the Bayesian Nash equilibrium (NE) for this game. From our analysis, we observe that the range of possible SIC values a given player may take can be divided into three regions (types). In two types, either the HD or the FD strategy is dominant, while the Bayesian NE in the third type depends mainly on the probability distribution of the other player’s types. Second, the thresholds that specify the
Furthermore, we consider simple utility functions and interference models. The game can be defined in the strategic form as:

\[ G = (\mathcal{P}, \mathcal{S}, \mathcal{U}) \]

where:
- \( \mathcal{P} = \{P_1, P_2\} \) is the set of players. Let \( a \) and \( b \) be the two nodes associated with \( P_1 \), and let \( c \) and \( d \) be the two nodes associated with \( P_2 \).
- \( \mathcal{S} = \{S_1, S_2\} \), where \( S_i = \{FD, HD\} \) is the set of pure strategies of \( P_i \), \( i = 1, 2 \). An FD strategy of a given player indicates that both nodes of that player will transmit simultaneously on the same channel. On the other hand, under an HD strategy, only a single node (node \( a \) in case of \( P_1 \) and node \( c \) in case of \( P_2 \)) will transmit data to its peer.
- \( \mathcal{U} = \{U_1, U_2\} \), where \( U_i \) is the utility of \( P_i \), a map giving the total throughput of link \( i \) under each strategy profile.

The utilities of the two links, and hence their operational modes at the NE, depend on two factors: External interference from one link to the other and self-interference. The amount of external interference depends on the channel gains and the transmission powers, whereas self-interference is function of the SIC capabilities. Let \( R \) be the throughput of a link in one direction when the communication is successful. Table I shows two instances of the game \( G_s \). In the first instance (Table I(a)), both external and self-interference are assumed to be high. As a result, if both \( P_1 \) and \( P_2 \) operate in the FD mode, then their utilities will be zero. Although (FD, FD) is a NE, it is not an efficient one. Moreover, (FD, FD) is not a unique NE. Specifically, (FD, HD) and (HD, FD) are also NE and, in fact, they are Pareto efficient. In another instance of \( G_s \) (shown in Table I(b)), the external interference is low and all nodes have perfect SIC. In this case, (FD, FD) is an efficient and unique NE.

The above game illustrates that FD communications is not always optimal from a network’s perspective. To provide a complete characterization of the stable strategy profiles for coexisting FD-enabled links, we account for nodes’ SIC capabilities as well as the amount of induced interference from one link on the other in the following Bayesian game.

### III. Game with Heterogeneous SIC Capabilities

#### A. Game Formulation

The two-player Bayesian game can be defined as \( G = (\mathcal{P}, \mathcal{S}, \Theta, \mathcal{D}, \mathcal{U}) \), where \( \mathcal{P} \) and \( \mathcal{S} \) are as in \( G_s \). The exact expressions for the utility set \( \mathcal{U} = \{U_1, U_2\} \) will be defined later. As for \( \Theta \) and \( \mathcal{D} \), they are defined as follows:

- \( \Theta = \{\Theta_1, \Theta_2\} \), where \( \Theta_i = \{1, 2\} \) is the set of types of \( P_i \). Here, the type represents a range of values for the SIC capability of \( P_i \) that dictates a certain strategy. Let \( \chi_i \in [0, 1] \) be the SIC parameter of \( P_i \). For simplicity, we assume that both nodes of \( P_i \) have the same \( \chi_i \). Specifically, \( \chi_i \) is the ratio between the RSI signal and the original one. If \( \chi_i = 0 \), the corresponding node can totally suppress its self-interference signal (perfect SIC); otherwise, it can only suppress a fraction \( 1 - \chi_i \) of its self-interference. As we later show, the outcome of the game does not depend on the exact value of \( \chi_i \), but rather the relationships between \( \chi_i \) and two thresholds, \( \chi_i^\ast \) and \( \chi_i^{\ast\ast} \), to be derived. As a result, \( P_i \) will be of one of three types: Type 1 (\( \theta_1 = 1 \)), type 2 (\( \theta_2 = 2 \)), and type 3 (\( \theta_3 = 3 \)).

- \( \mathcal{D} = \{D_1, D_2\} \), where \( D_i \) is a probability distribution over the types of \( P_i \), \( i = 1, 2 \). \( P_i \) is of type 1 with probability \( p_{i1} \), of type 2 with probability \( p_{i2} \), and of type 3 with probability \( p_{i3} \). Note that \( \theta_1 \) and \( \theta_2 \) are independent.

Finally, in Bayesian games a pure strategy of \( P_i \) is a map \( s_i : \Theta_i \rightarrow S_i \), prescribing an action for each type of \( P_i \). Strategy spaces, player types, utility functions, and the probability distributions over the types are assumed to be common knowledge to

### TABLE I. Utilities in the Simple Game.

<table>
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<tr>
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<th>(a) High external and self-interference</th>
<th>(b) Low external interference and perfect SIC</th>
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<tbody>
<tr>
<td></td>
<td>( P_1 )</td>
<td>FD</td>
</tr>
<tr>
<td>FD</td>
<td>( (0, 0) )</td>
<td>( (2R, 0) )</td>
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<tr>
<td>HD</td>
<td>( (0, 2R) )</td>
<td>( (R, R) )</td>
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<tr>
<td></td>
<td>( P_2 )</td>
<td>FD</td>
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<tr>
<td>FD</td>
<td>( (2R, 2R) )</td>
<td>( (2R, R) )</td>
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<tr>
<td>HD</td>
<td>( (R, 2R) )</td>
<td>( (R, R) )</td>
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all players. However, a given player knows only its current strategy and type, and does not know the strategies selected by the other player or its true type.

Next, we analyze the utility $U_1$ for $P_1$ ($U_2$ can be analyzed similarly). First, consider $U_1$ under the (FD, FD) strategy profile. $P_1$ benefits from an FD transmission only if no outage/collision occurs at the receivers of $a$ and $b$, i.e., when $\gamma_{ij}^{(FF)} \geq \gamma_{ij}^*, i, j \in \{a, b\}$, where $\gamma_{ij}^{(FF)}$ is the SNR at node $j$ while receiving data from node $i$ under (FD, FD) and $\gamma_{ij}^*$ is a given reception threshold. Define $1_{ij}$ to be an indicator function for a transmission from $i$ to $j$ under (FD, FD). $1_{ij}$ = 1 if $\gamma_{ij}^{(FF)} \geq \gamma_{ij}^*$ and 0 otherwise. Denote the transmission cost from $i$ to $j$ by $C_{ij}$. We set $C_{ij} = c_{ij}P_{t,i}$, where $P_{t,i}$ is the transmission power of node $i$, $i \in \{a, b, c, d\}$, $j \in \{a, b\}$, and $c_{ij}$ is a constant. Then, the utility of $P_1$ under (FD, FD), denoted by $U_1^{(FF)}$, can be expressed as follows:

$$U_1^{(FF)} = 1_{ab}R_{ab}^{(FF)} + 1_{ba}R_{ba}^{(FF)} - C_{ab} - C_{ba}$$

where $R_{ab}^{(FF)}$ and $R_{ba}^{(FF)}$ are, respectively, the forward and reverse link throughputs for link $(a, b)$. $R_{ij}^{(FF)}$, $i, j \in \{a, b\}, i \neq j$, is given by:

$$R_{ij}^{(FF)} = \log \left(1 + \gamma_{ij}^{(FF)}\right) = \log \left(1 + \frac{P_{t,i}g_{ij}}{\chi_{ij}^2P_{t,j}g_{jj} + P_{t,c}g_{ij} + P_{t,d}g_{dj} + \sigma_j^2}\right)$$

where $g_{ij}$ is the channel gain between any two nodes $i$ and $j$, $i \neq j$, and $\sigma_j^2$ is the noise power at node $j$. The notation $g_{jj}$ indicates the self-interference channel gain at node $j$. Note the RSI term in the denominator of the SNR. According to our earlier assumption, $\chi_a = \chi_b \equiv \chi_1$ (also, $\chi_c = \chi_d \equiv \chi_2$).

Under (HD, FD) (i.e., $P_1$ selects HD while $P_2$ selects FD), $P_1$’s utility, denoted by $U_1^{(HF)}$, includes the throughput of the forward link only minus its transmission cost. The same amount of external interference is still induced on $P_1$, as $P_2$ still operates in FD mode. Formally, $U_1^{(HF)}$ can be written as:

$$U_1^{(HF)} = 1_{ab}R_{ab}^{(HF)} - C_{ab} = 1_{ab}^{(HF)}\log \left(1 + \gamma_{ab}^{(HF)}\right) - c_{ab}P_{t,a}$$

where $\gamma_{ab}^{(HF)} = P_{t,a}g_{ab}/(P_{t,c}g_{cb} + P_{t,d}g_{db} + \sigma_b^2)$ is the SNR at $b$ when receiving data from $a$ under (HD, FD).

For the other two strategy profiles, $P_1$’s utility can be written as follows:

$$U_1^{(FH)} = 1_{ab}^{(FH)}\log \left(1 + \gamma_{ab}^{(FH)}\right) - c_{ab}P_{t,a} + 1_{ba}^{(FH)}\log \left(1 + \gamma_{ba}^{(FH)}\right) - c_{ba}P_{t,b}$$

where $\gamma_{ab}^{(FH)} = P_{t,a}g_{ab}/(\chi_a^2P_{t,b}g_{bb} + P_{t,c}g_{cb} + \sigma_b^2)$ and $\gamma_{ba}^{(FH)} = P_{t,b}g_{ba}/(\chi_a^2P_{t,a}g_{aa} + P_{t,c}g_{ca} + \sigma_a^2)$.

To find the Bayesian NE, we first define technical conditions under which no outage/collision occurs at the receivers of both players. We start by analyzing the no-outage case to facilitate understanding the Bayesian game analysis. Then, we conclude this section by stating the Bayesian NE in the general case when outage may occur.

Outage will not occur for $P_1$ iff $\gamma_{ij}^{(mn)} \geq \gamma_{ij}^*$ and $\gamma_{ij}^{(mn)} \geq \gamma_{ij}^*$, $\forall m, n \in S_1$. However, by examining the SNR expressions for different strategy profiles, we notice that the minimum SNR is encountered under (FD, FD) due to the existence of two interference terms from nodes $c$ and $d$, in addition to the RSI. With the imposed conditions on $\gamma_{ij}^{(FF)}$ and $\gamma_{ij}^{(FH)}$, we can implicitly ensure that no outage occurs under the other strategy profiles. Formally, we define the following two conditions on $P_1$:

$$TC_{11}: P_{t,c}g_{cb} + P_{t,d}g_{db} - \frac{P_{t,a}g_{ab}}{\gamma_{ab}^*} + \chi_a^2P_{t,b}g_{bb} + \sigma_b^2 < 0$$

$$TC_{12}: P_{t,c}g_{ca} + P_{t,d}g_{da} - \frac{P_{t,b}g_{ba}}{\gamma_{ba}^*} + \chi_a^2P_{t,a}g_{aa} + \sigma_a^2 < 0.$$
with $\chi_1$ than that under (FD, FD). The reason for this is that under (FD, HD), the RSI dominates the external interference (since $P_2$ is playing HD), which is not the case for the (FD, FD). The same argument applies to player $P_2$. $\chi_1^{**}$ is defined as the value of $\chi_1$ at which $U_1^{(FF)} = U_1^{(HF)}$ and $\chi_2^{**}$ is defined as the value of $\chi_2$ at which $U_2^{(FF)} = U_2^{(FH)}$.

**Type 3 ($\chi_1 \in [\chi_1^{**}, 1]$):** As $\chi_1$ continues to increase beyond $\chi_1^{**}$, the RSI signal will be very high, forcing the utilities under the FD strategy to be lower than that of the HD strategy (irrespective of the other player’s strategy). Towards the end of this section, we discuss how $\chi_1^*$ and $\chi_2^*$ can be computed.

### B. Existence and Uniqueness of a Bayesian NE

**Theorem 1:** Game $G$ described above has a Bayesian NE $(s_1^*, s_2^*)$, which is given by:

$$s_i^* = \begin{cases} 
FD, & \text{if } \theta_i = 1 \\
FD, & \text{if } p_{i1} > \alpha_i \\
FD (HD), & \text{if } p_{i1} \leq \alpha_i \text{ and } p_{i1} + p_{i2} \geq \alpha_i \\
HD, & \text{if } p_{i1} + p_{i2} < \alpha_i \\
HD, & \text{if } \theta_i = 3 
\end{cases}$$  \hfill (8)

for $i \in \{1, 2\}$, $\hat{i} = (i \mod 2) + 1$ is the index of node $i$’s peer node (i.e., if $i = 1$, $\hat{i} = 2$, and vice versa). $\alpha_1, \alpha_2 \in [0, 1]$ are defined as:

$$\alpha_1 \overset{\text{def}}{=} \frac{U_1^{(HH)} - U_1^{(HF)}}{U_1^{(HH)} - U_1^{(HF)} + U_1^{(FF)} - U_1^{(HF)}}$$  \hfill (9)

$$\alpha_2 \overset{\text{def}}{=} \frac{U_2^{(HH)} - U_2^{(HF)}}{U_2^{(HH)} - U_2^{(HF)} + U_2^{(FF)} - U_2^{(HF)}}.$$  \hfill (10)

Note that $\alpha_i$ is a function of $\chi_i, i \in \{1, 2\}$.

**Proof:** If player $P_1$ is of type 1, then $U_1^{(FF)} > U_1^{(HF)}$ and $U_1^{(HH)} > U_1^{(HF)}$, i.e., the FD strategy strictly dominates the HD strategy. On the other hand, if $P_1$ is of type 3, then $U_1^{(FF)} < U_1^{(HF)}$ and $U_1^{(HH)} < U_1^{(HH)}$, i.e., the HD strategy strictly dominates the FD strategy. Applying the same argument to player $P_2$, we can determine the Bayesian NE if the players are of type 1 or 3, as shown in (8).

The more challenging case occurs when at least one of the players is of type 2. If $P_1$ is of type 2, then $U_1^{(FF)} > U_1^{(HF)}$ and $U_1^{(HH)} < U_1^{(HH)}$. Therefore, $P_1$’s decision depends on $P_2$’s action. As mentioned earlier, $P_2$’s action will be FD if $\theta_2 = 1$ and HD if $\theta_2 = 3$, however, its action when $\theta_2 = 2$ is not determined yet. First, consider the case when $P_2$’s action is FD and $\theta_2 = 2$.

In this case, $P_1$’s FD strategy dominates the HD strategy iff:

$$p_{21}U_1^{(FF)} + p_{22}U_1^{(FF)} + p_{23}U_1^{(HH)} > p_{21}U_1^{(HF)} + p_{22}U_1^{(HF)} + p_{23}U_1^{(HH)}.$$  

Second, if $P_2$’s action is HD when $\theta_2 = 2$, then $P_1$’s FD strategy dominates the HD strategy iff:

$$p_{21}U_1^{(FF)} + p_{22}U_1^{(HF)} + p_{23}U_1^{(HH)} > p_{21}U_1^{(HF)} + p_{22}U_1^{(HF)} + p_{23}U_1^{(HH)}.$$  

Solving the above two equations along with $p_{21} + p_{22} + p_{23} = 1$, we obtain the value of $\alpha_1$ and derive the conditions on $P_2$’s probability distribution as in (8). A similar approach is used to determine $\alpha_2$ for $P_2$.  

\[ \blacksquare \]
Corollary 1: If each of the two players $P_1$ and $P_2$ satisfies one of the following two conditions:

\begin{align}
    p_{i1} & > \alpha_i \quad (11) \nonumber \\
    p_{i1} + p_{i2} & < \alpha_i \quad (12) 
\end{align}

where $i \in \{1,2\}$ and $\hat{i} = (i \mod 2) + 1$, then, game $G$ has a unique Bayesian NE. That is given by:

- $(F,F,H), (F,F,H)$ if $p_{21} > \alpha_1$ & $p_{11} > \alpha_2$ \quad (13)
- $(F,F,H), (F,H,H)$ if $p_{21} > \alpha_1$ & $p_{11} + p_{12} < \alpha_2$ \quad (14)
- $(F,H,H), (F,F,H)$ if $p_{21} + p_{22} < \alpha_1$ & $p_{11} + p_{12} < \alpha_2$ \quad (15)
- $(F,H,H), (F,H,H)$ if $p_{21} + p_{22} < \alpha_1$ & $p_{11} + p_{12} < \alpha_2$ \quad (16)

where the notation $((F,F,H), (F,H,H))$, for instance, means that $P_2$ operates in the FD mode when $\theta_1 = 1$ or 2 and in the HD mode when $\theta_1 = 3$, while $P_2$ operates in the FD mode when $\theta_2 = 1$ and in the HD mode when $\theta_2 = 2$ or 3.

Proof: The two-player Bayesian game $G$ (with each player having one of three possible types) can be equivalently formulated as a $2^3$ by $2^3$ strategic game. From Theorem 1 and its proof, it can be shown that under each of the conditions stated in (13)-(16), the corresponding strategy profile consists of a strictly dominating strategy for each player in the 8-by-8 game. Hence, the Bayesian NE is unique.

Corollary 2: If exactly one of the players neither satisfies (11) nor (12), then the game $G$ has two Bayesian NEs (from the strategy profiles given by (13)-(16)).

Proof: Follows from Theorem 1 and Corollary 1.

Corollary 3: If both players neither satisfy (11) nor (12), then the game has the four Bayesian NEs, given by (13)-(16).

Proof: Follows from Theorem 1 and Corollary 1.

C. Accounting for Link Outages

We now relax the technical conditions ((6) and (7) for $P_1$ and the two similar conditions for $P_2$), and assume that any of the forward/reverse links may experience outage. In this case, the relation between $\chi_i^*$ and $\chi_i^{**}$ is not fixed (i.e., $\chi_i^*$ could be larger or smaller than $\chi_i^{**}$). The reason is that in the case of outage, if $P_2$ operates in the FD mode, then $U_{1(FF)}$ may decrease faster with $\chi_1$ compared to $U_{1(HF)}$, and hence the condition $U_{1(FF)} = U_{1(HF)}$ may be satisfied before the condition $U_{1(FH)} = U_{1(HF)}$. Therefore, $\chi_1^{**} < \chi_1^*$ (recall the definition of $\chi_1^*$ and $\chi_1^{**}$ in Section III-A). The same situation arises in the case of $P_2$ when outage is likely to happen.

Theorem 2: Theorem 1 and Corollaries 1-3 hold true for game $G$ under the possibility of outage. However, $\theta_i, i \in \{1,2\}$, in this case, is given by:

\begin{align}
    \theta_i = \begin{cases} 
    1, & \text{if } \chi_i \leq \min(\chi_i^*, \chi_i^{**}) \\
    2, & \text{if } \min(\chi_i^*, \chi_i^{**}) < \chi_i < \max(\chi_i^*, \chi_i^{**}) \\
    3, & \text{if } \chi_i \geq \max(\chi_i^*, \chi_i^{**}). 
    \end{cases} 
\end{align}

Proof: From the definitions of $\chi_i^*$ and $\chi_i^{**}$ in Section III-A, it can be shown that, irrespective whether outage occurs or not, the dominating strategy of player $P_i$ when $\chi_i \leq \min(\chi_i^*, \chi_i^{**})$ is FD, and when $\chi_i \geq \max(\chi_i^*, \chi_i^{**})$ is HD. When $\min(\chi_i^*, \chi_i^{**}) < \chi_i < \max(\chi_i^*, \chi_i^{**})$, following the same approach in the proof of Theorem 1 (by deriving the conditions under which $P_i$’s expected utility under the FD strategy is higher than that under the HD strategy while fixing the other player strategy, and solving the two resulting inequalities along with $p_{21} + p_{22} + p_{23} = 1$, it can be shown that the dominating strategy is given by (8) when $\theta_i = 2$. Hence, Theorem 2 holds.

Next, we derive closed-form expressions for $\chi_1^*$ and $\chi_1^{**}$. In this derivation, we focus on player $P_1$; a similar approach could be used to derive the thresholds of player $P_2$. The challenge in this derivation is the presence of the indicator functions along with the logarithmic terms.

We start by deriving $\chi_1^{**}$, which occurs when $U_{1(FF)} = U_{1(HF)}$. Substituting in the previous equation using (1) and (3), we get the following:

\begin{align}
    1_{ab} R_{ab}^{(FF)} + 1_{ba} R_{ba}^{(FF)} - C_{ab} - C_{ba} = 1_{ab} R_{ab}^{(HF)} - C_{ab}. 
\end{align}
where

\[ \hat{\beta}_1 = P_{t,a} P_{t,b} g_{ab} \] (1 - \( e^w \)), \ \hat{\beta}_2 = (P_{t,b} g_{bb} + P_{t,a} g_{aa} u) (1 - \( e^w \)) + P_{t,b} g_{ab} g_{bb} + P_{t,a} g_{aa} g_{ba}, \quad \text{and} \ \hat{\beta}_3 = \hat{w} + P_{t,b} g_{bb} \hat{v} + P_{t,a} g_{aa} \hat{u} + P_{t,b} g_{ab} \hat{u} + P_{t,a} P_{t,b} g_{ab} g_{ba}.

The above approach can also be used to derive \( \chi_1^* \), resulting in:

\[
\chi_1^* = \begin{cases} 
0, & \text{if } \hat{I}_{ab}^{(HF)} = \hat{I}_{ba}^{(HF)} = 0 \\
\sqrt{\frac{1}{P_{t,a} g_{aa}} \left( \frac{P_{t,b} g_{bb}}{e^w - 1} - \hat{\nu} \right)}, & \text{if } \hat{I}_{ab}^{(HF)} = 0, \hat{I}_{ba}^{(HF)} = 1 \\
\sqrt{\frac{1}{P_{t,b} g_{bb}} \left( \frac{P_{t,a} g_{ab}}{e^w - 1} - \hat{\nu} \right)}, & \text{if } \hat{I}_{ab}^{(HF)} = 1, \hat{I}_{ba}^{(HF)} = 0 \\
-\hat{\beta}_2 - \sqrt{\frac{\hat{\beta}_2^2 - 4 \hat{\beta}_1 \hat{\beta}_3}{2 \hat{\beta}_1}}, & \text{if } \hat{I}_{ab}^{(HF)} = \hat{I}_{ba}^{(HF)} = 1
\end{cases}
\]
In this section, we conduct numerical evaluations, LabVIEW simulations, and hardware experiments to assess the performance of our proposed game-theoretic analysis.

A. Numerical Results

For the numerical results, unless stated otherwise, we use the following parameter values: The channel gain between nodes $i$ and $j$, $g_{ij}$, is set to 1, $\forall i, j \in \{a, b, c, d\}$, $\sigma_i^2 = 1$ Watt, and the normalized transmission cost $c_{ij}$ is set to 0.001.

Figure 2 depicts the expected utility for $P_1$ vs. $\chi_1$ at different values of interference channel gains (ICGs) (i.e., channel gains between the transmitters of $P_1$ and receivers of $P_2$, and vice versa). As shown in the figure, increasing the ICGs reduces the expected utility of $P_1$. If $P_1$ is of type 1, its expected utility decreases with $\chi_1$, as $P_1$ operates in the FD mode, whereas when it is of type 3, its expected utility is constant because it operates in the HD mode. When $P_1$ is of type 2, $P_1$ starts operating in the FD mode as $\chi_1$ increases (since $p_{21} > \alpha_1$), and eventually it switches to the HD mode when $\alpha_1$ exceeds $p_{21}$. As shown in the figure, the expected utility when $\theta_1 = 2$ first decreases with $\chi_1$ and then switches to a fixed value. Note that the SIC thresholds are different for different ICGs (we only show these thresholds when $\text{ICG} = 0.01$). In Figure 3, we show $P_1$’s expected utility for different values of $p_{21}$ (apriori probability that $P_2$ is of type 1). As $p_{21}$ increases, $P_1$’s expected utility decreases. The reason is that if $p_{21}$ is sufficiently small (e.g., $p_{21} = 0.1$), the expected interference level is relatively low and hence higher expected utility can be achieved.

Figure 4 shows the SIC thresholds of players $P_1$ and $P_2$ vs. $P_{t,c}$. Under conditions (6) and (7) for $P_1$ and the two similar conditions for $P_2$, $\chi_1^* < \chi_2^*, \forall i$. Since $P_{t,c}$ is the dominating term in $\gamma_{al}^{(HH)}$, increasing $P_{t,c}$ causes $U_1^{(HH)}$ to decrease faster than $U_1^{(FH)}$. Hence, $\chi_1^*$ increases with $P_{t,c}$. A similar argument can be made regarding $\chi_2^*$. On the other hand, increasing $P_{t,c}$ causes $U_2^{(FH)}$ to decrease faster than $U_2^{(HH)}$. Hence, $\chi_2^*$ decreases with $P_{t,c}$. In Figure 5, we plot the players’ expected utilities vs. $p_{21}$ for different values of $p_{11}$. As $p_{21}$ increases, $P_1$’s expected utility decreases. Similarly, $P_2$’s expected utility decreases with $p_{11}$, whereas it is constant with $p_{21}$, because $P_2$ already knows its type.

IV. PERFORMANCE EVALUATION

Fig. 2. $P_1$’s expected utility at Bayesian NE vs. $\chi_1$ ($\chi_2 = 0$, uniform distribution over the types of both players, and $P_{t,i} = 100$ Watts $\forall i$).

Fig. 3. $P_1$’s expected utility at Bayesian NE vs. $\chi_1$ ($\chi_2 = 0$, uniform distribution over $P_1$’s types, $p_{22} = p_{23} = (1 - p_{21})/2$, ICG= 0.05, and $P_{t,i} = 100$ Watts $\forall i$).

Fig. 4. SIC thresholds for $P_1$ and $P_2$ vs. $P_{t,c}$ (ICG= 0.05 and $P_{t,i} = 100$ Watts $\forall i$).

Fig. 5. Expected utilities of $P_1$ and $P_2$ at Bayesian NE vs. $p_{21}$ ($\chi_1 = \chi_2 = 0$, ICGs= 0.05, and $P_{t,i} = 100$ Watts $\forall i$).

Fig. 6. Player $P_1$’s throughput vs. its transmission powers, $P_{t,a}$ and $P_{t,b}$ ($\chi_1 = 0$, $P_2$ operates in the HD mode, and $P_{t,c} = 100$ Watts).

Fig. 7. Player $P_1$’s throughput vs. its transmission powers, $P_{t,a}$ and $P_{t,b}$ ($\chi_1 = 0.4$, $P_2$ operates in the HD mode, and $P_{t,c} = 100$ Watts).
them to the pulse shaping block for transmission. The simulation parameters are the mean of the Gaussian noise (varies from 500 bits, modulated using QPSK and a code rate of 1/2). We then add control information (e.g., training sequence) and pass them to the pulse shaping block for transmission. The simulation parameters are the mean of the Gaussian noise (varies from $-20$ dBm to $-50$ dBm), the ICGs = {0.01, 0.04, 0.09}, and SIC capabilities (0 to 0.5). We focus on the more practical case, where $\chi_1$ value is low. Both, the players’ channel coefficient and the self-interference channel coefficient is fixed at $0.707 + 0.707i$ (gain = 1). For each simulation run, we take the average of 1000 iterations (i.e., 1000 back-to-back packets). We fix the channel phase to $\pi/4$ and set the BER threshold needed for correct reception to BER$_{th}$ = $10^{-4}$.

Figure 8 shows the forward-link BER of $P_1$ vs. $\chi_1$ at different noise levels when $P_2$ is not operating (switched off). Depending on the noise level and the SIC capability, $P_1$ decides to operate in the HD or FD modes. This figure shows the effect of RSI (due to incomplete SIC) on the BER of $P_1$. As expected, the BER of $P_1$ increases with $\chi_1$. Specifically, $P_1$ operates in the FD mode for all values of $\chi_1$ at which the average BER is lower than BER$_{th}$. Once the average BER exceeds BER$_{th}$ (i.e., the receiver is not able to decode the packet), the optimal communication mode for $P_1$ becomes HD. As shown in Figure 8, the threshold value of $\chi_1$ for switching from FD to HD decreases with the noise level; when the noise equals $-50$ dBm, this threshold equals 1 (i.e., $P_1$ always operates in the FD mode), whereas when the noise equals $-20$ dBm, this threshold equals 0.2 (i.e., $P_1$ always operates in the FD mode if $\chi_1 < 0.2$ and operates in the HD mode otherwise).

In the next two figures, we consider the case where an outage may occur at the receiver of any player. Even with complete SIC, when both players operate in the FD mode, outage may occur at all receivers, resulting in negative utilities for both players (because of the transmission costs). However, when $P_2$ operates in the HD mode, $P_1$ starts getting a positive utility, as shown in Figure 6. Note that $P_1$ doubles its throughput in the (FD, HD) strategy profile compared to (HD, HD) when the maximum transmission power is used. However, as $P_1$’s transmission power decreases, outage occurs. Figure 7 shows the case when $\chi_1 = 0.4$. In this case, $P_1$’s utility under (HD, HD) with $P_{2,ic}$ = 10 Watts is higher than that under (FD, HD). This is due to the increase in the level of RSI.

**B. LabVIEW Simulations**

For simulations, LabVIEW 2014 software, developed by National Instruments, is used to acquire, analyze, and visualize data. The simulation setup is as follows. We consider a single-channel FD network consisting of two links with different SIC capabilities. Each link can operate in the HD or FD modes. For transmission, we generate back-to-back packets, each consisting of 500 bits, modulated using QPSK and a code rate of 1/2. We then add control information (e.g., training sequence) and pass them to the pulse shaping block for transmission. The simulation parameters are the mean of the Gaussian noise (varies from $-20$ dBm to $-50$ dBm), the ICGs = {0.01, 0.04, 0.09}, and SIC capabilities (0 to 0.5). We focus on the more practical case, where $\chi_1$ value is low. Both, the players’ channel coefficient and the self-interference channel coefficient is fixed at $0.707 + 0.707i$ (gain = 1). For each simulation run, we take the average of 1000 iterations (i.e., 1000 back-to-back packets). We fix the channel phase to $\pi/4$ and set the BER threshold needed for correct reception to BER$_{th}$ = $10^{-4}$.
Figures 11 and 12 show the forward-link BER of $P_2$, when $P_2$ operates in the FD and HD modes, respectively. In these two figures, channel coefficients are $0.0707 + 0.0707i$ (ICGs = 0.01). As expected, the threshold values of $\chi_1$ under the (FD, FD) strategy profile are lower than those under (FD, HD) for the same noise levels. This means that the range of SIC capabilities for $P_1$ to operate in the FD mode is narrower when $P_2$ operates in the FD mode compared to the case when $P_2$ operates in the HD mode. The reason for this is that when $P_2$ operates in the FD mode, it induces higher interference on $P_1$ than when it operates in the HD mode, which causes outage to occur sooner at $P_1$ (i.e., at smaller values of $\chi_1$). Comparing Figure 8 to Figures 9 and 10, the range of $\chi_1$ under which $P_1$ operates in the FD mode is higher when link $P_2$ is switched off compared to the case when it is on. We repeat the above setup for different values of ICGs. Specifically, Figures 11 and 12 show the forward-link BER of $P_1$ vs. $\chi_1$ at different noise levels, when $P_2$ operates in the FD and HD modes, respectively. In these two figures, the ICGs equal $0.1414 + 0.1414i$ (gain = 0.2). On the other hand, Figure 13 shows the HD case only when ICGs equal 0.2121 + 0.2121i (gain = 0.3). Similar conclusions can be made regarding the regions of the HD mode as the ICGs increase.

Figure 14 shows the received signal constellation at $P_1$ for different noise levels and SIC capabilities, when $P_2$ is switched off. Specifically, Figures 14(a), 14(b), and 14(c) show the received constellation when the noise level is $-50$ dBm and $\chi_1 = 0, 0.1$, and 0.2, respectively. As $\chi_1$ increases, the variance of the received constellation points increase due to the RSI signal, and as a result the BER increases. Figures 14(d), 14(e), and 14(f) show similar constellation graphs, but at a higher noise level ($-20$ dBm). Figure 15 shows another set of sub-figures for the received constellation when $P_2$ operates in the FD mode. Both channel coefficients are $0.0707 + 0.0707i$. The external interference from the bi-directional link of $P_2$ increases the variance of the received constellation points. These interferences, along with the RSI, cause the BER of $P_1$ to increase. As a result, in some cases (depending on the noise level), $P_1$ will switch to the HD mode.
Fig. 16. Experimental setup. $P_1$ consists of nodes $a$ and $b$, while $P_2$ consists of nodes $c$ and $d$.

As future research, we plan to extend the proposed Bayesian game into more than two links, where each link does not know the exact SIC capabilities of the neighboring links. We will also investigate a power control scheme to be used as part of the game, in addition to FD/HD mode selection.

C. Hardware Experiments

The experimental setup is shown in Figure 16. We consider two coexisting links represented by four NI USRP 2922 radios. These USRPs are connected via Ethernet cables to two host PCs, running LabVIEW 2014. For transmission, we generate back-to-back packets, with packet length of 500 bits, which are modulated as 250 symbols using QPSK. We then add control information (e.g., training sequence) and pass them to the pulse shaping block. These complex I and Q symbols are then transmitted to the USRP device via the Ethernet cable. At the USRP device, the transmitter chain starts with the digital up-conversion (DUC) block, which passes the signal to the DAC. After that, an I-Q mixing occurs to directly up-convert the signal and produce an RF signal that is amplified and transmitted over the 2.4 GHz band (transmission power is around 0 dBm).

To enable FD communications, each node/USRP is equipped with two antennas, one for transmission and the other for reception. We use antenna separation [10] to separate the transmit and receive signals in the space domain. To further reduce the level of self-interference, we also exploit the antenna gain pattern by using directional antennas. In total, we achieve around 70 dB reduction in self-interference.

We conduct two types of experiments. First, we test the high self- and external interference (HSEI) case, where we place the four USRPs close to each other inside the lab. Furthermore, we displace the antennas from their optimal position so that the amount of RSI is relatively high. We then operate our network in two modes: (FD, FD) and (HD, HD), for a duration of 500 back-to-back packets. Figure 17 shows the CDF for the achieved throughput of $P_1$ and $P_2$ under the two strategy profiles. The throughput in the case of (HD, HD) is, on average, higher than that in the case of (FD, FD). Second, we repeat the same experiment under the low self- and external interference (LSEI) case, where we place two USRPs ($c$ and $d$) outside the lab and the other two inside the lab. We also adjust the positions of the antennas to reduce the RSI signal. Under this scenario, as shown in Figure 18, the (FD, FD) strategy profile results in a much higher throughput compared to (HD, HD).

V. Conclusions

In this paper, we developed a novel Bayesian game-theoretic framework to study the coexistence problem between two FD-capable wireless links, where nodes have heterogeneous SIC capabilities. Although the throughput of a single link enhances significantly when operating in the FD mode, the additional caused interference (compared to the HD case) may limit its coexistence with a neighboring link. Our analysis revealed that the SIC capability of each link (which is the type of each player) has a double-threshold structure, i.e., the range of the SIC values can be divided into three regions. When the SIC capability is very good, operating in the FD mode strictly dominates the HD mode, whereas when the SIC capability is very poor, operating in the HD mode strictly dominates the FD mode. When the SIC capability is in the middle region, we derived the conditions on the probability distribution of the types of the other link under which HD (FD) strictly dominates FD (HD).

Our conducted experiments corroborated that FD is not always the optimal operation mode when considering an FD-enabled wireless network. The optimal mode of a link depends on (i) the external interference it encounters from neighboring links (which is a function of their transmission powers and channel gains) and (ii) its residual self-interference (which is a function of its SIC capability, its transmission power, and its channel gain). Given that the external interference will not be known a priori, a link relies on the probability distribution over the types of the other link in deriving its strategy. Our simulations demonstrated the impact of the residual self-interference and external interference on the received constellation diagram, and hence the BER. Specifically, as both interference types increase, the variance in the received constellation points increases.

As future research, we plan to extend the proposed Bayesian game into more than two links, where each link does not know the exact SIC capabilities of the neighboring links. We will also investigate a power control scheme to be used as part of the game, in addition to FD/HD mode selection.
REFERENCES


