Heterogeneous Spectrum Sharing with Rate Demands in Cognitive MIMO Networks

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Abstract—We are interested in addressing a fundamental question: what are conditions under which an ad hoc cognitive radio MIMO (CMIMO) network can support a given rate-demand profile, defined as the set of rates requested by individual links? From an information theoretic view, a rate profile can be supported if it is within the network capacity region. However, the network capacity region of interfering MIMO networks is essentially unknown. In dynamic spectrum access, the problem is even more challenging due to the dynamics of primary/legacy users (PUs), resource constraints, and the heterogeneity of opportunistic spectrum (i.e., the set of available channels varies from one to another). Considering a non-cooperative game, we address the above question in a noncooperative game framework where each CMIMO link independently optimizes its spectrum, power allocation, and MIMO precoders to meet its rate demand. We derive sufficient conditions for the existence of a NE are derived. These conditions establish an explicit relationship between the rate-demand profile and interference from PUs, CMIMO network’s interference, and CMIMO nodes’ power budget. We also show that a NE, if exists, is unique. Our results help to characterize the network capacity region of CMIMO networks.

Index Terms—Cognitive radio, MIMO, Nash equilibrium, noncooperative game, rate demand, interfering network capacity.

I. INTRODUCTION

Consider an interfering CR MIMO (CMIMO) network in which each link wishes to minimize its transmit power while meeting a given rate demand. The problem can be modeled as a noncooperative game, referred to as power minimization (PM) game. Such a PM game is different from the rate maximization (RM) game (e.g., [1] [2] [3]), in which nodes individually maximize their rates. Whereas the players’ strategic spaces in a PM game are independent, these strategic spaces exhibit complex coupling in a PM game. This is because the strategic space of a link in the PM game is defined by its available resources, e.g., power, available channels, antennas, etc., which do not depend on other players’ actions. In contrast, in a PM game with rate constraints, the strategic space of a player is not only shaped by its resources but also its achievable rate (to meet the rate demand), which is a function of other players’ actions. The interdependence of the strategic spaces makes the analysis of PM games much more challenging than RM games.

As an example, it can be proved that the RM game always admits a NE [2]. By contrast, the PM game may not have a NE (e.g., the rate demands are beyond the network capacity region). Moreover, under resource constraints (e.g., power budgets), the strategic space of a player under the PM game can be empty (e.g., when the power budget is not sufficient to support the rate demand given interference from other transmitters). In the context of a CR network, the dynamics of primary users (PUs) also affect the requested CR rate can be met or not. It is this possible emptiness of strategic spaces that prevents us from directly applying techniques used to study the RM games of MIMO systems to our setup. The projection method (onto a nonempty compact and convex space) in the context of fixed point theory [4] and the variational inequality theory [3] [5] have been instrumental in tackling the RM game. However, these techniques require nonempty strategic spaces.

Another challenge is the spectrum heterogeneity of CR communications. Due to the spatiotemporal variations of spectrum opportunities, a channel that is temporarily available for one CR user may not be available to other CR users. This leads to a CR network with heterogeneous spectrum sharing. Traditional RM and PM games [1] [2] [3] [6] [7] often assume homogeneous spectrum sharing setting in which the set of idle channels are the same at all nodes.

The goal of this paper is to investigate the conditions under which a given rate profile can be supported by an interfering CMIMO network with heterogeneous spectrum sharing. Using game theory, recession analysis, and variational inequality theory, we derive sufficient conditions that guarantee a given rate profile. Intuitively, these conditions are met if the CRs’ power budget is sufficient enough to satisfy the rate demands, the requested rates are not too high to harm PU receptions, the PUs’ interference to CRs is not too strong, and the CR interference is not too severe. The four conditions are quantified in a way that allows a node to decide its appropriate rate. We also show that if a NE exists for the underlying PM game, it must be unique. Interestingly, by removing resource constraints and set the number of antenna to 1, our sufficient conditions become necessary and reduce to those in [8] [9].

Throughout the paper, (.)∗ denotes the conjugate of a matrix, (.)H denotes its Hermitian transpose, tr(.) denotes its trace, ||.|| denotes the Euclidean (or Frobenius) norm, and (.)T denotes the transpose. eigmax(.), eigmin(,), and diag(,) indicate the maximum, minimum eigenvalue, and the diagonal element (s, s) of a matrix, respectively. Matrices and vectors are bold-faced.

II. PROBLEM STATEMENT

A. System Model

Consider a multi-channel CMIMO network that coexists with several PU networks in a rich-scattering environment (to facilitate MIMO spatial multiplexing). The network consists of N transmitter-receiver pairs (links), denoted by ΦN = {1, 2, ..., N}. Each CR node is equipped with M antennas. The set of temporarily idle channels at link i is denoted by Si. In general, Si ≠ Sj for two links i and j. The network’s opportunistic spectrum is the union of available-channel sets from all links, consisting of K orthogonal (not necessarily contiguous) channels with central frequencies f1, f2, ..., fK, denoted by ΨK = {1, 2, ..., K}. Each CR
i can simultaneously communicate over multiple frequencies (e.g., using non-contiguous OFDM).

The transmitter of a CR link can send up to \( M \) independent data streams over each channel. Let \( x^{(k)}_u \) be an \( M \times 1 \) column vector, consisting of \( M \) information symbols (from \( M \) data streams), sent on link \( u \) using the channel with central frequency \( f_k \) (hereon also referred to as channel \( f_k \) for short). The radiation pattern and power allocation for the \( M \) streams of link \( u \) on channel \( f_k \) are determined by its precoding matrix \( \mathbf{T}^{(k)}_u \). The actual transmit vector on channel \( f_k \) at the radio interface is \( \tilde{x}^{(k)}_u \). We allow for spectrum sharing among various CR links. On channel \( f_k \), the signal vector \( \tilde{y}^{(k)}_u \) at the receiver of link \( u \) is given by:

\[
y^{(k)}_u = H^{(k)}_{u,u} \tilde{x}^{(k)}_u + \sum_{j \in \Phi_N \setminus\{u\}} H^{(k)}_{u,j} \tilde{T}^{(k)}_u \tilde{x}^{(j)}_u + N^u_k
\]

where \( H^{(k)}_{u,k} \) is an \( M \times M \) channel gain matrix on channel \( f_k \) of link \( u \). Each element of \( H^{(k)}_{u,k} \) is a multiplication of a distance- and channel-dependent attenuation term, and a complex Gaussian variable (with zero mean and unit variance) that reflects multi-path fading. \( H^{(k)}_{u,j} \) denotes the cross-channel gain matrix from the transmitter of link \( j \) to the unintended receiver of link \( u, j \neq u \). The second term in (1) represents interference from transmitters of \( \mathcal{C}_j \) links \( j \neq u \) that share channel \( f_k \) with link \( u \). \( N^u_k \) is an \( M \times 1 \) complex Gaussian noise vector with covariance matrix \( I_k = \mathbb{E} \{ (I_1)^2 \} \), representing the floor noise with unit variance plus (whitened) interference \( I_1 \) from \( \mathcal{N} \) on channel \( f_k \).

We assume that interference cancellation is not used. A receiver decodes its data streams by treating interference from other transmitters as colored noise. The Shannon rate over link \( u \) on channel \( f_k \) is [10]:

\[
R^{(k)}_u = \log |I + \tilde{T}^{(k)}_u H^{(k)}_{u,u} C^{(k)}_{u} H^{(k)}_{u,u} \tilde{T}^{(k)}_u|
\]

where \( C^{(k)}_{u} \) is the noise-plus-interference covariance matrix at the receiver of link \( u \) on channel \( f_k \):

\[
C^{(k)}_{u} = (1 + I_1)I + \sum_{j \in \Phi_N \setminus\{u\}} H^{(k)}_{u,j} \tilde{T}^{(k)}_u \tilde{T}^{(k)}_u H^{(k)}_{u,j}.
\]

The total channel rate over all frequencies of link \( u \) is:

\[
R^{(k)}_u = \sum_{k \in \mathcal{S}_u} R^{(k)}_u.
\]

PU protection is provided in the form of database-authorized access and frequency-dependent power masks on CR transmit powers. Note that the FCC [11] recently imposed power masks even for idle channels, if such channels are adjacent to PU-occupied channels. Let \( P^{(k)}_{\text{mask}} \equiv (P^{(k)}_{\text{mask}}(f_1), P^{(k)}_{\text{mask}}(f_2), \ldots, P^{(k)}_{\text{mask}}(f_M)) \) denote the power mask vector. We require:

\[
\sum_{s \neq k} P^{(s)}_{\text{mask}}(f_k) = \text{tr}(\tilde{T}^{(k)}_u \tilde{T}^{(k)}_u) \leq P^{(k)}_{\text{mask}}(f_k)
\]

where \( P^{(s)}_{\text{mask}}(f_k) \) denotes the power allocated on channel \( f_k \) (frequency dimension) over antenna \( s \) (space dimension) for the transmitter of link \( u \). If channel \( f_k \) is not available for link \( u \), \( P^{(s)}_{\text{mask}}(f_k) = 0 \). Note that \( P^{(k)}_{\text{mask}}(f_k) \) otherwise. Note that \( P^{(k)}_{\text{mask}}(f_k) \) differs from one link to another. We impose following constraints:

\[
\begin{align}
C_1: & \quad c_u \leq R^{(k)}_u, \quad \forall u \in \Phi_N \\
C_2: & \quad \text{tr}(\tilde{T}^{(k)}_u \tilde{T}^{(k)}_u) \leq P^{(k)}_{\text{mask}}(f_k), \quad \forall k \in \Psi_K, \forall u \in \Phi_N \\
C_3: & \quad \sum_{k \in \Psi_K} \text{tr}(\tilde{T}^{(k)}_u \tilde{T}^{(k)}_u) \leq P^{(k)}_{\text{max}}, \quad \forall u \in \Phi_N.
\end{align}
\]

where \( C_1 \) ensures that all links achieve their rate demands, \( C_2 \) ensures that the frequency-dependent power masks are satisfied, and \( C_3 \) presents a maximum-power budget constraint (\( P^{(k)}_{\text{max}} \)) at node \( u \) (we assume nodes have an identical power budget).

B. Noncooperative Game Formulation

Each CR link represents a player in the PM game who aims at maximizing its utility, defined as the negative of its transmit power. The game’s strategic space \( \mathcal{Q} \) is the union of the strategic spaces of various players, subject to constraints \( C_1, C_2, C_3 \) in (5). Each player \( u \) competes against others by selecting its strategic action of \( K \) precoders, denoted by \( \tilde{T}^{(k)}_u \equiv (\tilde{T}^{(k)}_u(1), \tilde{T}^{(k)}_u(2), \ldots, \tilde{T}^{(k)}_u(K)) \). \( \tilde{T}^{(k)}_u \) is an \( M \times KM \) block matrix, comprised of \( K \) \( M \times M \) matrices. The payoff for player \( u \), given below, is a function of its action \( \tilde{T}^{(k)}_u \) as well as other players’ actions, \( \tilde{T}^{(k)}_{-u} \equiv (\tilde{T}^{(k)}_{1,-u}, \tilde{T}^{(k)}_{2,-u}, \ldots, \tilde{T}^{(k)}_{K,-u}) \):
We observe that the unit matrix $I$ is positive definite, so the objective function in (8) is non-decreasing in every element of $p_u$. In other words, at a NE of the game (if one exists), the inequality constraint $C1'$ becomes equality. Otherwise, one can still lower the power consumption to achieve a smaller value for the objective function while meeting the rate demand. This fact defines a feasible set for $p$, denoted by $Q_{\text{feasible}}(c)$, corresponding to a given requested rate profile $c = (c_1, c_2, \ldots, c_N)$ at a NE. For a given rate profile $c$, the game (8) has at least one bounded NE and only bounded NEs, if $Q_{\text{feasible}}(c)$ is nonempty and empty.

**Theorem 1:** Let $G_k'$ be defined in (11) and matrix $G_k'$ be obtained from $G_k$ by deleting rows $G_u(u, \cdot)$ and columns $G_k(\cdot, u)$ for all $\{u|k \notin S_u\}$. If $G_k'$ is a P-matrix\(^1\) \(\forall k \in \Psi_K\), then $Q_{\text{feasible}}(c)$ contains at least one bounded vector $p \in R^{NK_M}$ and only bounded vectors $p$. In other words, the game (8) admits at least one bounded NE and only bounded NEs.

**Proof:** We first claim that $Q_{\text{feasible}}(c)$ contains at least one bounded vector $p \in R^{NK_M}$ or the existence of a bounded NE to the game (8):

**Lemma 1:** Given that $G_k'$ is a P-matrix \(\forall k \in \Psi_K\), then there exists at least one bounded vector $p \in Q_{\text{feasible}}(c) \subset R^{NK_M}$.

**Proof:** See Appendix I of [13].

The remaining task is to show that the game (8) admits only bounded NEs if $Q_{\text{feasible}}(c)$ is bounded. To that end, we rely on the concept of asymptotic cone of a nonempty set in recession analysis [14]. For a nonempty set $Q \subseteq R^N$, its asymptotic cone, denoted by $Q_{\text{asympt}}$, consists of vectors $d \in R^N$ referred to as limit directions. Each limit direction vector $d$ is defined through the existence of a sequence of vectors $p_n \in Q$ and a sequence of scalars $\nu_n$ tending to $+\infty$ such that [14]:

$$\lim_{n \to \infty} \frac{p_n}{\nu_n} = d. \quad (14)$$

The set $Q$ is bounded if its asymptotic cone $Q_{\text{asympt}}$ contains only the zero vector $0$ [14]. Applying this to the set $Q_{\text{feasible}}(c)$, the game (8) admits only bounded NEs if its asymptotic cone $Q_{\text{asympt}}(c)$ contains only the zero vector. The asymptotic cone $Q_{\text{asympt}}(c)$ is formally defined in (12).

Given that $Q_{\text{feasible}}(c)$ has at least one bounded $p$ (Lemma 1), it is clear that the vector zero $0$ belongs to its asymptotic cone $Q_{\text{asympt}}(c)$ (by the definition of limit directions). We now construct a set $Q(c)$ of which $Q_{\text{asympt}}(c)$ is a subset and prove that $Q(c) = \{0\}$ if $G_k'$ is a P-matrix $\forall k \in \Psi_K$.

**Lemma 2:** If $d \in Q_{\text{asympt}}(c)$ then $d$ belongs to $Q(c)$, defined in (13).

**Proof:** See Appendix II in [13].

Assuming that there exists at least one $d \neq 0$ and that $d \in Q(c)$, then \(\forall u \in \Phi_N\) and $S_u \supset k$:

$$\log \left(1 + \frac{\text{tr}(T_u(k)^H T_u(k))H_u(k)H_u(k)^H}{\sum_{(j,k) \in S_u} \text{tr}(T_j(k)^H T_j(k))} \right) \leq c_u \quad (15a)$$

$$\text{tr}(T_u(k)^H T_u(k))H_u(k)H_u(k)^H \cdot \frac{1}{\sum_{(j,k) \in S_u} \text{tr}(T_j(k)^H T_j(k))}$$

$$\left[2^{c_u} - 1\right] \sum_{(j,k) \in S_u} \frac{\text{tr}(T_j(k)^H T_j(k))H_u(k)}{M} \leq 0 \quad (15b)$$

$$G_k' \times [\text{tr}(T_u(k)^H T_u(k)), \ldots, \text{tr}(T_j(k)^H T_j(k))]^T \leq 0. \quad (15c)$$

\(^1\)A matrix is a P-matrix if all of its principal minors are positive [12].

As $G_k'$ is a P-matrix for all $k \in \Psi_K$ and $[\text{tr}(T_u(k)^H T_u(k)), \ldots, \text{tr}(T_j(k)^H T_j(k))]^T$ is a nonnegative vector, (15c) implies $\text{tr}(T_u(k)^H T_u(k)) = 0$ \(\forall u \in \Phi_N, \forall k \in \Psi_K\) [12] or $d = 0$. This contradicts the above assumption. Hence, $Q(c)$ and its subset $Q_{\text{asympt}}(c)$ equal to $\{0\}$. Theorem 1 is proved.

We now give some intuitions behind Theorem 1. As the diagonal elements of $G_k'$ are positive (under rich-scattering environment), then a sufficient condition for $G_k'$ to be a P-matrix is $|G_k'(u, u)| \geq \sum_{j \neq u} |G_k'(u, j)|$ (i.e., row diagonally dominant) [12]. The following inequality guarantees that game (8) has at least one bounded NE and only bounded NEs:

$$\frac{\text{Mdet}(H_u(k)H_u(k)^H)}{\sum_{(j,k) \in S_u} \text{tr}(H_u(k)H_u(k)^H)} \geq (2^{c_u} - 1) \quad \forall k, \forall u. \quad (16)$$

The nominator of the RHS in (16) represents the strength of the channel gain of link $u$ on channel $f_k$, while its denominator describes the strength of cross-interfering channel gains from other links $j, j \neq u$, on the receiver of link $u$. First, for the game (8) to have at least one NE (at which the required powers of all links are bounded), the multi-user interference in each channel $f_k$ should not be too strong. Second, the acceptable multi-user interference is explicitly quantified in (16), and it is a function of the rate demand $c_u$ of each link $u$. For higher rate demands, inequality (16) becomes stringent, meaning that lower multi-user interference is necessary. Hence, inequality (16) can be used as a criterion to reject or admit a newly requested transmission rate. When links set their target rate too high that inequality (16) does not hold, a bounded NE may not exist. In this case, nodes keep increasing their transmit powers to meet their rate demands. Network interference becomes more severe and no link reaches its requested rate (interference-limited communications).

To better interpret inequality (16), recall that each element of channel gain matrices in (16) is the product of a complex Gaussian variable with zero mean and unit variance (in the $H_u(k)$ matrix) and the distance-dependence attenuation factor: $h_{u, u} = d_{u, u}^{-n}$, where $n$ is the free-space attenuation factor and $d_{u, u}$ is the transmission distance of link $u$. Inequality (16) can be rewritten as:

$$\frac{\text{Mdet}(H_u(k)H_u(k)^H)}{\sum_{(j,k) \in S_u} d_{u, j}^{-n} \text{tr}(H_u(k)H_u(k)^H)} \geq (2^{c_u} - 1) \quad \forall k, \forall u. \quad (17)$$

(17) holds if the distance between the transmitter and the receiver is small enough compared with distances between the receiver and its interferers, the channel gain matrix of link $u$ is full-rank (this is often the case in a rich-scattering environment) and its requested rate is not too high. Given the existence of bounded NEs to the game in (8), we now incorporate the power mask and power budget constraints in the following theorem.

**Theorem 2:** The game (7) admits at least one bounded NE and only bounded NEs if $G_k'$ is a P-matrix and the vector-inequality...
Theorem 1 ensure that network interference is mild enough to
for a NE to exist. Hence, besides the PU protection requirement,
LHS of (18)) when PUs become more active, it is less likely
should reduce their transmission power on this channel to avoid in-
interfering PUs. Moreover, as the inequality becomes tighter (smaller

Proof: See proof of Theorem 2 in [13].

From (18), if PUs are more active on a given channel (higher
I_{pu}(k)), the inequality becomes stricter. This means that CRs
should reduce their transmission power on this channel to avoid inter-
tering PUs. Moreover, as the inequality becomes tighter (smaller LHS of (18)) when PUs become more active, it is less likely for a NE to exist. Hence, besides the PU protection requirement, inequality (18) also shows the interference effect from PUs to CRs.

So far, we have derived conditions that capture the factors that
affect the existence of a NE of the game (7). The conditions in Theorem 1 ensure that network interference is mild enough to support the requested rates. The conditions in the first inequality in Theorem 2 enforce that the requested rates are not too high to harm PUs reception given PUs’ activities (indirectly captured by PUS’s interference). The last inequality in Theorem 2 guarantees that rate demands are affordable given nodes’ power budgets.

When the spectrum opportunities are homogeneous (i.e., $S_u = \Psi_K, \forall u$), one can verify that by removing the resource and PUS protection constraints and setting the number of antenna to be one, the conditions in Theorem 1 reduce to the conditions derived for the NE existence in single-antenna (legacy) networks (in Theorem 5 of [8]). The authors of [8] proved that their sufficient conditions become necessary when $K = 1$ and $M = 1$, i.e., a single-channel SISO network (Proposition 11 of [8]). They also showed that for the case $K = 1$ and $M = 1$, their sufficient conditions are identical to those in [9]. Hence, though we cannot show that the sufficient

2Since $G_k'$ is a P-matrix, it is invertible.
a rate demand of 3 bps/Hz. Channel gain matrices among the 4 nodes are in Section IV.B of [13] (where \( H(:, :, i, j) \) is the channel gain matrix from node \( i \) to node \( j \)).

**Conditions in Theorem 1:**

\[
G_1' \overset{\text{def}}{=} \left[ \frac{\sqrt{(1 + I_{pu}(1))}H(:, 2, 1)H(:, 2, 1)}{(2^3 - 1)(1 + I_{pu}(1))} - \frac{1}{2} \frac{(2^3 - 1)\frac{H(:, 2, 3)H(:, 2, 3)}{P_{mask}(f_1)}}{|H(:, 4, 3)H(:, 4, 3)|^{1/2}} \right]
\]

The above \( G_1' \) is a P-matrix as it meets the sufficient conditions in (16). Hence, Theorem 1 holds.

Now, we check conditions in Theorem 2.

**Conditions in Theorem 2 to protect PUs:**

\[
G_{1-1} = \begin{bmatrix} 13.7588 & 4.4330 \\ 4.6335 & 14.8624 \end{bmatrix}
\]

The inequality (18) to protect PUs is:

\[
G_{1-1}^{-1} \times \begin{bmatrix} 2^3 - 1 \\ 2^3 - 1 \end{bmatrix} = \begin{bmatrix} 127.34 \\ 136.48 \end{bmatrix} \leq \begin{bmatrix} \frac{P_{mask}(f_1)}{1 + I_{pu}(1)} \\ \frac{P_{mask}(f_1)}{1 + I_{pu}(1)} \end{bmatrix}
\]

The inequality (22) holds if the power mask is 136.48 times greater than \((1 + I_{pu}(1))\) (note that \(1 + I_{pu}(1)\) is the total floor noise (normalized to 1) and the PUs' interference on channel 1, \(I_{pu}(1)\)). This is the case if PUs' interference is not too strong. If cognitive radios obtain temporarily idle ("white") channels from spectrum databases, then there is no active PUs (i.e., \(I_{pu}(1) = 0\)). In this case, inequality (22) holds easily.

**Conditions in Theorem 2 regarding the total power budget constraints:**

The LHS of (19) for the two links reduce to scalars in the considered example (as \(K = 1\) is):

\[
(1 + I_{pu}(1)) \times [13.7588 \quad 4.4330] \times [2^3 - 1 \quad 2^3 - 1]^T
\]

\[
= 127.34(1 + I_{pu}(1))
\]

\[
(1 + I_{pu}(1)) \times [4.6335 \quad 14.8624] \times [2^3 - 1 \quad 2^3 - 1]^T
\]

\[
= 136.48(1 + I_{pu}(1))
\]

The second inequality (regarding the total power budget constraint) can also be met if the power budgets of link 1 and link 2 are greater than 127.34(1 + \(I_{pu}(1)\)) and 136.48(1 + \(I_{pu}(1)\)). Similar to the inequality (22), these conditions can also be met easily in practice. Note that as our example has only one channel (for simplicity), then the conditions regarding the power budget constraints are similar to that for the power mask constraints.

We now simulate a CMIMO network of \(N\) links (i.e., \(2N\) nodes) which are randomly placed in a square area of length 100 meters. Each node has 4 antennas. The simulation results are averaged over 40 runs. There are 10 channels with bandwidth of 16 MHz. Due to spectrum heterogeneity, we assume that channels \(i + 1, i + 2\) are not available for link \(i\), if \(i \leq 8\). Otherwise, channels \(i - 7, i - 8\) are not available for link \(i\). We set \(P_{max} = 1000\) mW and the power mask \(P_{mask} = 0.5P_{max}\) for all channels. The channel fading is flat with free-space attenuation factor of 2. The spreading angles of the signal at the receive antennas vary from \(-\pi/5\) to \(\pi/5\). The close-in distance is 1 m. The thermal floor noise is \(-174\) dBm/Hz. The PUs interference on all channels is \(-100\) dBm/Hz. We also assume that links have identical rate demands.

For a given simulation run, there is a probability that the conditions in Theorem 2 hold and the game converges to a unique NE. Fig. 1(a) depicts the probability (percentage of runs) that the game converges to a NE (a NE exists) versus the rate profile when 10 links are active and 10 channels are used. As the rate demand increases, the probability that a NE exists decreases. This is because the conditions in Theorem 2 become more stringent. Fig. 1(b) depicts the probability that a NE exists versus \(N\) when the rate demand is 1 bps/Hz. As \(N\) increases, the network/multi-user interference becomes more severe, it is unlikely that the conditions in Theorem 2 are met. Thus, the probability of a NE existence decreases.

![Fig. 1.](image)

**Fig. 1.** (a) Probability of NE existence vs. rate demands, (b) Probability of NE existence vs. number of links.

**Fig. 2.** Total network power consumption vs. iterations.

**V. CONCLUSIONS**

We derived sufficient conditions under which a cognitive MIMO network with heterogeneous spectrum sharing can support a given set of rate demands. By formulating the problem as a noncooperative game, using variational inequalities theory, and recession analysis, we derived sufficient conditions for the existence and uniqueness of the NE of the game. These conditions capture the interference from PUs, network interference of CMIMO, power
In the following, we state the sufficient conditions [5] for the existence and uniqueness of a solution to the above VI problem when the set $K$ has a Cartesian structure, i.e., $K = K_1 \times K_2 \times \ldots \times K_N$ (where $K_u \subset \mathbb{R}^{n_u}$ and $\sum_{u=1}^{N} n_u = n$).

**Theorem 6:** Given that the set $K$ has a Cartesian structure, the above VI($K, \mathbb{R}^n$) problem admits a unique solution $x^{opt}$ if $K_u$ is closed and convex and F is continuous uniformly-P function, i.e., there exists a positive constant $\alpha$ such that:

$$\max_{1 \leq u \leq N} (x_u - x_u')^T (F(x_u) - F(x_u')) \geq \alpha \|x_u - x_u'\|^2,$$

$$\forall x_u, x_u' \in K_u.$$

As the set of precoding matrices of each player in the game (7) are complex matrices. To reformulate the game (7) as a VI problem, we use the isomorphism in equation (24) to map the complex matrix domain to the Euclidean domain, where $vec()$ is a matrix operator that stacks columns (from left to right) of an $m \times n$ matrix to form an $mn \times 1$ vector.

The gradient of a matrix function $(\cdot)$ w.r.t $\hat{T}_u$ is given in (25). We are now ready to map the game (7) to a VI problem. If all conditions in Theorem 2 are met, the strategy set of each player $u$, denoted by $Q_u \subset \mathbb{C}^{M \times M}$, is nonempty. Additionally, it is also easy to verify that $Q_u$ is convex and bounded. Hence, problem (7) is a convex problem. The following inequality features the necessary (and then also the sufficient) condition for a strategy $\hat{T}_u^{opt}$ to be the best response:

$$\hat{T}_u - \hat{T}_u^{opt} \cdot \nabla U_u(\hat{T}_u, \hat{T}_u - u) \leq 0 \ \forall \hat{T}_u \in Q_u$$

where $A \cdot B \triangleq vec(A)^T vec(B)$.

Let define $Q \triangleq Q_1 \times \ldots \times Q_N$ and $F \triangleq F_1 \times \ldots \times F_N$ with $F_u \triangleq -\nabla U_u(\hat{T}_u, \hat{T}_u - u)$. By comparing (28) with the above definition of a VI problem, the strategy set $\hat{T}_u^{opt} \triangleq [\hat{T}_u^{opt} \times \ldots \times \hat{T}_N^{opt}]$ is a NE of the game (7) if and only if $\hat{T}_u^{opt}$ is a solution of the VI($Q,F$) problem. Therefore we can rely on VI theory to analyze the game (7).

Let $\hat{T} \triangleq [\hat{T}_1 \times \ldots \times \hat{T}_N]$ and $\hat{T}' \triangleq [\hat{T}_1' \times \ldots \times \hat{T}_N']$ be two different strategy set of the strategic space $Q$ of the game (7), then:

$$F(\hat{T}_u') = -\nabla U_u(\hat{T}_u, \hat{T}_u - u) = \hat{T}_u' \quad F(\hat{T}_u) = -\nabla U_u(\hat{T}_u, \hat{T}_u - u) = \hat{T}_u.$$

Consequently, we have:

$$vec(\hat{T}_u - \hat{T}_u^{opt})^T vec(F(\hat{T}_u) - F(\hat{T}_u')) = 1||vec((\hat{T}_u - \hat{T}_u^{opt}))||^2.$$  

The above inequality exactly meets the condition (27) so that the mapping $F$ is a continuous uniformly-P function. Moreover, $Q$ has a Cartesian structure. Hence, the VI($Q,F$) problem has a unique NE, so does the game (7).

**APPENDIX A**

**PROOF OF THEOREM 4**

We start by introducing a VI problem.

**Definition of a VI problem:** [5] Given a subset $K$ of the Euclidean $n$-dimensional space $\mathbb{R}^n$ and a mapping $F: K \rightarrow \mathbb{R}^n$, the VI problem $VI(K, \mathbb{R}^n)$ is to find a vector $x^{opt} \in K$ so that:

$$(x - x^{opt})^T F(x^{opt}) \geq 0, \ \forall x \in K.$$  \hspace{1cm} (26)