Consider the scenario in the figure below. There are two available paths between a source $S$ and a destination $D$. The first path goes through routers $R_1$ and $R_2$, while the second path goes through $R_3$ and $R_4$. The fixed propagation delays are shown in the figure. For $i = 1, 2, 3, 4$, the queueing delay at router $i$ is gamma distributed with scale parameter $\lambda_i$ and shape parameter $r_i$. Let

- $\lambda_1 = \lambda_2 = 0.1$
- $\lambda_3 = \lambda_4 = 0.2$
- $r_1 = 2$
- $r_2 = 1$
- $r_3 = 2$
- $r_4 = 3$

1. Numerically compute the exact $0.9$-quantile of the end-to-end delay for $\alpha = 0.1$ for both routes (hint: Suppose that $X_1$ and $X_2$ are two independent and gamma distributed random variables with scale parameters $\lambda_1$ and $\lambda_2$, and with shape parameters $r_1$ and $r_2$, respectively. If $\lambda_1 = \lambda_2$, then $X \overset{\Delta}{=} X_1 + X_2$ is also gamma distributed with scale parameter $\lambda_1$ and shape parameter $r_1 + r_2$).

2. Repeat part (1) but using the simple additive scheme for QoS accumulation.

3. Repeat part (1) but using the asymptotic scheme for QoS accumulation.

4. Repeat part (1) but using the Chernoff-bound scheme for QoS accumulation.

5. According to your calculations for each part, which route is preferable?