LDPC-Coded Modulation for Beyond 100-Gb/s Optical Transmission

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ABSTRACT

The future Internet traffic growth will require deployment of optical transmission systems with bit rates higher than rate of currently available 40-Gb/s systems, such as 100-Gb/s and above. However, at data rates beyond 100-Gb/s the signal quality is significantly degraded mainly due to impact of polarization mode dispersion (PMD), and intra-channel nonlinear effects. All electrically time-division multiplexed (ETDM) multiplexers and de-multiplexers operating at ~100-Gb/s are becoming commercially available. However, the modulators operating ~100-Gb/s are not widely available so that alternative approaches to enable 100-Gb/s transmission and beyond using commercially available components operating at 40-Gb/s are of high practical importance. In this invited paper, several joint coded-modulation and multiplexing schemes enabling beyond 100-Gb/s transmission using commercially available components operating at 40-Gb/s are presented. Using this approach, modulation, coding and multiplexing are performed in a unified fashion so that, effectively, the transmission, signal processing, detection and decoding are done at much lower symbol rates, where dealing with the nonlinear effects and PMD is more manageable, while the aggregate data rate is maintained above 100-Gb/s. The main elements of our approach include: (i) bit-interleaved LDPC-coded modulation, (ii) multilevel coding (MLC) with LDPC component codes, and (iii) LDPC-coded orthogonal frequency division multiplexing (OFDM). The modulation formats of interest in this paper are M -ary quadrature-amplitude modulation (QAM) and M -ary phase-shift keying (PSK), where $M=2, \dots, 16$, both combined with either Gray or natural mapping rule. It will be shown that coherent detection schemes significantly outperform direct detection ones and provide an additional margin that can be used either for longer transmission distances or for application in an all-optical networks.

Keywords: Bit-interleaved coded modulation, multilevel coding, orthogonal frequency division multiplexing, low-density parity-check (LDPC) codes, coherent detection, direct detection, optical communications

1. INTRODUCTION

Higher bit rates and larger numbers of closely spaced wavelengths are common ways to meet ever-increasing demands for higher capacity in optical transmission. Future optical communication systems will also require longer amplifier distance, and flexible wavelength management. The deployment of transmission systems based on high bit rates, such as 100-Gb/s and above, is advantageous for many reasons. From a transmission system point of view, smaller number of high-speed channels for a given bandwidth results in better spectral efficiency because no bandwidth is wasted on separating the channels. At the same time, a single high-speed transponder may replace many low-speed ones resulting in reduced number of optoelectronic devices and easier monitoring. From a network application point of view, reduced number of channels allows us to employ simpler optical switching devices and simpler routing algorithms. The technology that can directly benefit from beyond 100-Gb/s transmission is Ethernet [1],[2]. Ethernet was initially introduced as a communication standard for short-distance connection of hosts in local area networks (LANs) [2]. Thanks to its simplicity with respect to other protocols, low-cost and high speed it has rapidly evolved, and has already been used to establish campus-size distance connections, and beyond, in metropolitan area networks. Moreover, the Network Interface Cards (NICs) for 1-Gb/s and 10-Gb/s Ethernet are already commercially available [1]. Because so far the Ethernet has grown in 10 fold increments, the 100-Gb/s is being envisioned as the speed of the next generation of Ethernet [1]-[11].

Unfortunately, the increase of data rates of fiber optics communication systems is connected with numerous technological obstacles such as (i) increased sensitivity to fiber nonlinearities, (ii) high sensitivity to PMD, and (iii) increased demands in dispersion accuracy. Moreover, in contrast to transmission at lower data rates (10-Gb/s) that is impacted mostly by inter-channel interactions such as four-wave mixing (FWM) and cross-phase modulation (XPM), at

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speeds of 40-Gb/s and above the major nonlinear penalties are due to intra-channel interactions, such as intra-channel four-wave-mixing (IFWM) and intra-channel cross-phase modulation (IXPM) [12]-[16]. The multiplexers and demultiplexers related to all electrically time-division multiplexed (ETDM) transceivers operating at ~100-Gb/s are becoming commercially available. However, the modulators with bandwidths required for beyond 100-Gb/s are not widely available. In order to deal with limited modulator bandwidth someone may use optical duobinary modulation or optical equalization [7]. On the other hand, there is an option to use commercially available components operating at lower speed as an alternative approach based on multilevel modulations [5], [9]-[11], [17] to enable beyond 100-Gb/s optical transmission.

The basic idea behind our approach is to combine modulation, coding and multiplexing so that the transmission takes place at lower symbol rate, such as 40 Giga symbol/s, where dealing with the nonlinear effects and PMD is more manageable, while the aggregate data rate is maintained above 100-Gb/s.

This invited paper, based on our several recent publications [9]-[11], [17], [26], [27], [29], is organized as follows. In Section 2 we describe our first approach to achieve beyond 100-Gb/s optical transmission. It is based on bit-interleaved low-density parity-check (LDPC)-coded modulation. In Section 3 the second approach to achieve beyond 100-Gb/s optical transmission, based on multilevel coding with LDPC component codes, is described. The use of LDPC-coded OFDM to achieve ≥ 100 -Gb/s optical transmission is described in Section 4. LDPC codes used in different coded modulation schemes described here are briefly described in Section 5. Some important conclusions are given in Section 6.

2. BEYOND 100-Gb/s OPTICAL TRANSMISSION USING BIT-INTERLEAVED LDPC-CODED MODULATION

In this Section we describe our first technique to achieve beyond 100-Gb/s optical transmission. This approach is based on bit-interleaved coded modulation [18], and low-density parity-check (LDPC) codes [19]-[21]. It was recently introduced by author in [10], [11].

The transmitter configuration is shown in Fig. 1(a). The source bit streams coming from m information sources carrying 40-Gb/s traffic are encoded by using identical (n, k) LDPC codes of code rate $r=k/n$, where k denotes the number of information bits, and n denotes the codeword length. The LDPC encoder outputs are row-wise written into the $m \times n$ block-interleaver. The mapper accepts m bits, $c=(c_1, c_2, \dots, c_m)$, from $(m \times n)$ interleaver at time instance i column-wise and determines the corresponding M -ary ($M=2^m$) constellation point $s_i=(I_i, Q_i)=|s_i|\exp(j\phi_i)$ using appropriate mapping rule. In coherent detection case the data phasor $\phi_i \in \{0, 2\pi/M, \dots, 2\pi(M-1)/M\}$ is sent at each i th transmission interval. On the other side, in direct detection case, the differential encoding is required so that the data phasor $\phi_i = \phi_{i-1} + \Delta\phi_i$, where $\Delta\phi_i \in \{0, 2\pi/M, \dots, 2\pi(M-1)/M\}$, is sent instead at each i th transmission interval.

The receiver input electrical field at i th transmission interval, in direct detection case, is denoted by $E_i = |E_i|\exp(j\phi_i)$. The optical M -ary differential phase-shift keying (DPSK) receiver, shown Fig. 1(b), is implemented using two Mach-Zehnder delay interferometers (MZDIs). The outputs of I- and Q-branches (upper- and lower-balanced detectors in Fig. 1(b)) are proportional to $\text{Re}\{E_i E_{i-1}^*\}$ and $\text{Im}\{E_i E_{i-1}^*\}$, respectively. The corresponding coherent detector receiver architecture is shown in Fig. 1(c). $S_i = |S|e^{j\phi_{S,i}}$ ($\phi_{S,i} = \omega_s t + \phi_i + \phi_{S,PN}$) denotes the coherent receiver input electrical field at time instance i , while $L = |L|e^{j\phi_L}$ ($\phi_L = \omega_L t + \phi_{L,PN}$) denotes the local laser electrical field. For homodyne detection the frequency of the local laser (ω_L) is the same as that of the incoming optical signal (ω_s). In that case, the balanced outputs of I- and Q-channel branches (upper- and lower-branches of Fig. 1 (c)) can be written as

$$\begin{aligned} v_I(t) &= R_{PIN} |S_k| |L| \cos(\phi_i + \phi_{S,PN} - \phi_{L,PN}) \quad (i-1)T_s \leq t < iT_s \\ v_Q(t) &= R_{PIN} |S_k| |L| \sin(\phi_i + \phi_{S,PN} - \phi_{L,PN}) \quad (i-1)T_s \leq t < iT_s \end{aligned} \quad (1)$$

where R_{PIN} denotes the photodiode responsivity. $\phi_{S,PN}$ and $\phi_{L,PN}$ represent the laser phase noise of transmitting and receiving (local) laser, respectively. These two noise sources are commonly modeled as Wiener-Lévy process [22],

which is a zero-mean Gaussian process with variance $2\pi(\Delta\nu_S + \Delta\nu_L)|t|$, where $\Delta\nu_S$ and $\Delta\nu_L$ denote the laser linewidths of transmitting and receiving laser, respectively.

Fig. 1. Bit-interleaved LDPC-coded modulation scheme: (a) transmitter architecture, (b) direct detection receiver architecture, and (c) coherent detection receiver architecture. $T_s = 1/R_s$, R_s is the symbol rate. S_i is the received electrical field in i th slot, and L is the local laser electrical field.

In either direct detection or coherent detection case, the outputs at upper- and lower-branches are sampled at symbol rate, the symbol reliabilities are calculated in a posteriori probability (APP) demapper block (see Fig. 1(b,c)), the bit reliability are calculated from symbol reliabilities as shown below, and forwarded to the LDPC decoder. The LDPC decoder is implemented using the sum-product algorithm description due to Xiao-Yu *et al.* [23].

Let us now provide more details about APP demapper and bit LLRs calculation blocks shown in Fig. 1(b,c). The symbol log-likelihood ratios (LLRs) are calculated in APP demapper block as follows

$$\lambda(s) = \log \frac{P(s = s_0 | \mathbf{r})}{P(s \neq s_0 | \mathbf{r})}. \quad (2)$$

In (2) $s = (I_i, Q_i)$ denotes the transmitted signal constellation point at time instance i , while $\mathbf{r} = (r_I, r_Q)$, $r_I = v_I(t = iT_s)$, and $r_Q = v_Q(t = iT_s)$ denote the samples of I- and Q-detection branches from Fig. 1(b,c). s_0 is arbitrary symbol from the constellation, used as a referent symbol. The posterior probability $P(s|\mathbf{r})$ is determined by using Bayes' rule

$$P(s|\mathbf{r}) = \frac{P(\mathbf{r}|s)P(s)}{\sum_{s'} P(\mathbf{r}|s')P(s')}. \quad (3)$$

The probability $P(\mathbf{r}|s)$ from Eqn. (3) is estimated by evaluation of histograms, employing sufficiently long training sequence. With $P(s)$ we denoted the *a priori* probability of symbol s , which is for equally probable transmission $1/M$,

with M being the number of points in a signal constellation diagram. The referent symbol is introduced to cancel denominator from Eqn. (3). The bit LLRs c_j ($j=1,2,\dots,m$) (the bit LLRs calculation block in Fig. 1(b,c)) are determined from symbol LLRs of Eqn. (2) as

$$L(\hat{c}_j) = \log \frac{\sum_{s:c_j=0} \exp[\lambda(s)]}{\sum_{s:c_j=1} \exp[\lambda(s)]}. \quad (4)$$

In order to improve the BER performance we allow the iteration of extrinsic information between APP demapper and LDPC decoder. The APP demapper extrinsic LLRs is defined as the difference of demapper bit LLRs and LDPC decoder LLRs from previous step, and can be found by

$$L_{M,e}(\hat{c}_j) = L(\hat{c}_j) - L_{D,e}(c_j). \quad (5)$$

In (5) $L_{D,e}(c)$ denotes LDPC decoder extrinsic LLRs from previous iteration, which is initially set to zero value (because we assume equally probable transmission and $P(s)$ and $P(s')$ in (3) cancels each other). The LDPC decoder is, as mentioned above, implemented by using the sum-product algorithm. The LDPC decoders extrinsic LLRs, $L_{D,e}$, are defined as the difference between LDPC decoder output and the input LLRs. Those LLRs are forwarded to the APP demapper as *a priori* bit LLRs (denoted as $L_{M,a}$), so that the symbol *a priori* LLRs for APP demapper are calculated as

$$\lambda_a(s) = \log P(s) = \sum_{j=0}^{m-1} (1 - c_j) L_{D,e}(c_j). \quad (6)$$

Fig. 2 EXIT chart for different mappings, and LDPC(4320,3240) code for: (a) 8-DPSK at optical SNR 11.04 dB, and (b) 16-DPSK at optical SNR 14.04 dB.

By substituting Eqn. (6) into Eqn. (3), and then Eqn (2), we are able to calculate the symbol LLRs for the subsequent iteration. The iteration between the APP demapper and LDPC decoder, denoted here as *outer iteration* to differentiate it from *inner* sum-product algorithm iterations, is performed until the maximum number of iterations is reached, or the valid code-words are obtained. To study the extrinsic information transfer convergence behavior, the extrinsic information transfer (EXIT) chart analysis is applied as described by ten Brink in [34]. To keep the complexity of the LDPC decoder reasonably low for high-speed implementation, the structured LDPC codes are employed as described in Section 5. By observing the mutual information (MI) between codeword bit c and corresponding demapper extrinsic LLR ($L_{M,e}$) $I_{L_{M,e}}$ as a function of the MI between codeword bit c and corresponding demapper *a priori* LLR $I_{L_{M,a}}$ and optical signal-to-noise ratio, $OSNR$, in dB, the demapper EXIT characteristic (denoted by T_M) is given by $I_{L_{M,e}} = T_M(I_{L_{M,a}}, OSNR)$. The EXIT characteristic of LDPC decoder (denoted by T_D) can be defined in a similar fashion by $I_{L_{D,e}} = T_D(I_{L_{D,a}})$. The “turbo” demapping based receiver operates by passing extrinsic LLRs between demapper and LDPC decoder, as illustrated in Fig. 2. Different mappings have the EXIT curves with different slopes. The existence of “tunnel” between the demapping and decoder curves guarantees that iteration between demapper and decoder will result in successful decoding. Because the EXIT chart for Gray-demapper is a horizontal line we expect that iteration between

the demapper and the LDPC decoder helps very little in BER performance improvement. However, as shown in [10], the iteration between APP demapper and LDPC decoder in the case of anti-Gray mapping provides more than 1 dB improvement at $BER=10^{-7}$.

Fig. 3 (a) BER performance of BI-LDPC-CM scheme, and (b) performance comparison for different modulation schemes (Gray mapping rule is applied).

The results of simulations of BI-LDPC-CM for 30 inner iterations in the sum-product algorithm and 10 outer iterations for an additive Gaussian noise (AWGN) channel model are shown in Fig. 3(a). The information symbol rate is set to 40 Giga symbols/s, while 8-PSK is employed, so that the aggregate bit rate becomes 120-Gb/s. Two different mappers are considered: Gray, and natural mapping. The coding gain for 8-PSK at BER of 10^{-9} is about 9.5dB. Much larger coding gain is expected at BERs below 10^{-12} . The coherent detection scheme offers an improvement of about 2.5 dB as compared to the corresponding direct detection scheme. The BER performance of coherent BICM with LDPC(4320,3242) component code, for different modulations, is shown in Fig. 3(b). We can see that 16-QAM (with an aggregate rate of 160-Gb/s) outperforms 16-PSK by more than 3 dB. It is also interesting that 16-QAM slightly outperforms 8-PSK scheme of lower aggregate data rate (120-Gb/s). The 8-PSK scheme of aggregate rate of 120-Gb/s outperforms BPSK scheme of data rate 120-Gb/s. Moreover, since the transmission symbol rate for 8-PSK is 53.4 Giga symbols/s, the impact of PMD and intrachannel nonlinearities is much less important than that at 120-G/s. Consequently, for 100-Gb/s Ethernet transmission, it is better to multiplex two 50-Gb/s channels than four 25-Gb/s channels.

The results of Monte Carlo simulations for the dispersion map shown in Fig. 4 are shown in Fig. 5. The simulations were carried out with an average transmitted power per symbol of 0 dBm and the central wavelength is set to 1552.524 nm, while 8-DPSK/8-PSK with RZ pulses of duty cycle 33% are considered. The propagation of a signal is modeled by the nonlinear Schrödinger equation (see Eqn (11)). The effects of self-phase modulation, nonlinear phase-noise, intrachannel cross-phase modulation, intrachannel four-wave mixing, stimulated Raman scattering, chromatic dispersion, laser phase noise, ASE noise and intersymbol interference are all taken into account. The fiber parameters are given in Table 1. While, by using BI-LDPC-CM and direct detection in a point-to-point transmission scenario it was

possible to achieve the transmission distance of 2760km at 120-Gb/s aggregate rate with LDPC codes having BER threshold of 10^{-2} , the coherent detection scheme is able to extend the transmission distance by about 600 km.

Fig. 4. Dispersion map under study is composed of N spans of length $L=120$ km, consisting of $2L/3$ km of D_+ fiber followed by $L/3$ km of D_- fiber, with pre-compensation of -1600 ps/nm and corresponding post-compensation.

Table 1. Fiber parameters.

Parameters	D_+ fiber	D_- fiber
Dispersion [ps/(nm km)]	20	-40
Dispersion Slope [ps/(nm ² km)]	0.06	-0.12
Effective Cross-sectional Area [μm^2]	110	50
Nonlinear refractive index [m^2/W]	$2.6 \cdot 10^{-20}$	$2.6 \cdot 10^{-20}$
Attenuation Coefficient [dB/km]	0.19	0.25

Fig. 5. BER performance of BI-LDPC-CM scheme for dispersion map from Fig. 4.

3. BEYOND 100-Gb/s OPTICAL TRANSMISSION USING MULTILEVEL CODING WITH LDPC COMPONENT CODES

In this Section we describe our second approach to achieve beyond 100-Gb/s optical transmission. It is based on multilevel coding (MLC), initially proposed by Imai and Hirakawa in 1977 [24]. The key idea behind the MLC is to protect individual bits using different binary codes and use M -ary signal constellations. The decoding is based on so-called *multistage decoding* (MSD) algorithm in which the decisions from prior (lower) decoding stages are passed to next (higher) stage [25]. Despite its attractiveness because of large coding gains, the MLC with the MSD algorithm has a serious limitation for use in high-speed applications which is due to inherently large delay of the MSD algorithm. One possible solution is to use the *parallel independent decoding* (PID) [25].

Fig. 6. Multilevel coding scheme with LDPC component codes: (a) transmitter configuration, and (b) direct detection receiver configuration.

A block diagram of MLC/PID scheme is shown in Fig. 6. Notice that we introduced this concept in [17], but in context of 80-Gb/s optical transmission. The source bit streams coming from $L(=\log_2 M)$ different sources u_i ($i=1,2,\dots,L$) are encoded using different (n, k_i) LDPC encoders (k_i is the information word length of i th data stream and n is the codeword length). The corresponding bits are then combined into a signal point (I_k, Q_k) using an appropriate mapping rule and differential encoding. Two orthogonal electrical streams (in-phase I_k and quadrature Q_k) are used as RF inputs of Mach-Zehnder Modulators (MZMs). At the receiver side, the LDPC decoders operate independently and in parallel. The main difference between this scheme and the scheme described in Section 2 is that LDPC codes with different parameters are used for different input streams, and there is no need for an interleaver. The number of bits per symbol R in MLC scheme is equal to the sum of the individual code rates $R_i=k_i/n$

$$R = \sum_{i=0}^{L-1} R_i = \frac{1}{n} \sum_{i=0}^{L-1} k_i = \frac{k}{n}, \quad k = \sum_{i=0}^{L-1} k_i \quad (7)$$

The use of different LDPC coders for different input bit streams allows us to allocate the code rates optimally, resulting in better BER performance compared to BI-LDPC-CM scheme. The LDPC decoders input LLRs are calculated from the symbol reliabilities in a fashion similar to that explained in Section 2. The only difference is that LLRs in Eqn. (4) correspond to different LDPC codes.

The aggregate bit rate of 100-Gb/s can be achieved on several different ways. For example, by selecting the symbol rate at 50 Giga symbols/s, and by employing 8-DPSK (8-PSK or 8-QAM) transmission with three different component systematic LDPC codes with code rates 0.75, 0.75 and 0.5 respectively, the resulting spectral efficiency would be 2 bits/s/Hz, and aggregate rate 100-Gb/s.

As an illustration of performance improvement achievable by the MLC, we performed Monte Carlo simulations on AWGN channel for 8-DPSK based MLC scheme carrying 2 bits/symbol by using the following LDPC component codes: (4320,3242), (4320,3242), and (4320,2160). The component LDPC codes belong to the class of block-circulant LDPC codes [20]. The results of simulation are given in Fig. 7 for Gray mapping rule. The operating symbol rate is 50 Giga symbol/s. For Gray mapping rule, the MLC scheme based on parallel independent decoding provides about 0.6 dB improvement at BER of 10^{-7} over corresponding BI-LDPC-CM scheme, at the expense of higher encoder/decoder complexity.

Fig. 7 BER performance of an MLC scheme operating at 100-Gb/s aggregate bit rate.

4. BEYOND 100-Gb/s OPTICAL TRANSMISSION USING LDPC-CODED OFDM

In this Section we describe our third approach to achieve beyond 100-Gb/s optical transmission, which is based on LDPC-coded orthogonal frequency division multiplexing (OFDM), the concept recently proposed by author in [9]. OFDM [26], [27], [29]-[32] is a special case of a multicarrier transmission in which a single information-bearing stream is transmitted over many lower-rate sub-channels. It has been used in a number of applications such as digital audio broadcasting, high definition television terrestrial broadcasting, digital subscriber lines, wireless LANs, etc. OFDM offers a good spectral efficiency and efficient elimination of subchannel and symbol interference by using the fast Fourier transform (FFT) for modulation and demodulation, and requires only simple equalization to deal with dispersion effects [29]. The above features as well as its high immunity to chromatic dispersion, PMD, self-phase modulation, and burst-errors makes OFDM an intriguing candidate for high-speed optical transmission. Because the state-of-the-art optical communication systems essentially use the intensity modulation with direct detection (IM/DD), we consider the LDPC-coded optical OFDM with direct detection only. The coherent optical OFDM systems [31] require the use of an additional local laser, which increases the receiver complexity. At the same time coherent-OFDM systems are sensitive to the laser phase noise because the OFDM symbol rate becomes comparable to the DFB laser linewidth [29].

A basic variant of the system for beyond 100-Gb/s optical transmission is based on the OFDM transmitter and receiver configurations given in Fig. 8(a) and (b). Bits from b_i ($b_i > 1$) 1-Gb/s data streams are mapped into a two-dimensional signal point in a 2^{b_i} -point signal constellation such as QAM. The complex-valued signal points from all K subchannels are considered as values of the discrete Fourier transform (DFT) of a multicarrier OFDM signal. The symbol interval length in an OFDM system is $T = KT_s$, where T_s is the symbol-interval length in an equivalent single-carrier system. By selecting K , the number of sub-channels, sufficiently large, and by using the cyclic extension as shown in Fig. 8(c), the system tolerance against chromatic dispersion and PMD is very large compared to conventional systems [29]. The cyclic extension, is accomplished by repeating the last $N_G/2$ samples of the effective OFDM symbol part (N_{FFT} samples) as a prefix, and repeating the first $N_G/2$ samples as a suffix.

The transmitted OFDM signal in RF domain, upon D/A conversion and RF up-conversion, is defined by

$$s_{\text{OFDM}}(t) = \text{Re} \left\{ \sum_{k=-\infty}^{\infty} w(t-kT) \sum_{i=-(N_{FFT}/2)}^{(N_{FFT}/2)-1} X_{i,k} \cdot e^{j2\pi \frac{i}{T_{FFT}}(t-kT)} e^{j2\pi f_{RF}t} \right\} \quad (8)$$

$$kT - \left(T_G/2 \right) - T_{\text{win}} \leq t \leq kT + T_{FFT} + \left(T_G/2 \right) + T_{\text{win}}$$

In (8) $X_{i,k}$ denotes the i -th sub-carrier QAM symbol of the k -th OFDM symbol, T denotes the OFDM symbol duration, T_{FFT} is the FFT part duration, T_G is the guard interval (cyclic extension) duration, T_{win} denotes the windowing interval duration, $w(t)$ is the window function, and f_{RF} denotes the RF carrier frequency. After D/A conversion and RF up-conversion, the RF signal is mapped to the optical domain using Mach-Zehnder modulator (MZM). A sufficient *DC bias component* is added to the OFDM signal in order to enable recovery of the QAM symbols incoherently. The optimum bias is $\sim 50\%$ of the total electrical signal energy allocated for transmission of an RF carrier. In such a way both positive and negative portions of the electrical OFDM signal are transmitted to the photodetector. Distortion introduced by the photodetector, caused by squaring, is successfully eliminated by proper filtering, and recovered signal does not exhibit significant distortion. This scheme is sometimes called unclipped-OFDM scheme [26], [29]. The PIN photodiode output current can be written as

$$i(t) = R_{\text{PIN}} \left| s_{\text{OFDM}}(t) + b \right|^2 = R_{\text{PIN}} \left[\left| s_{\text{OFDM}}(t) * h(t) \right|^2 + |b * h(t)|^2 + 2R_e \left\{ \left(s_{\text{OFDM}}(t) * h(t) \right) (b * h(t)) \right\} \right], \quad (9)$$

where $s_{\text{OFDM}}(t)$ denotes the transmitted OFDM signal introduced in Eqn. (8). b is the DC bias component, R_{PIN} denotes the photodiode responsivity, and $h(t)$ denotes impulse response of the optical channel.

Fig. 8. LDPC-coded OFDM system with direct detection for beyond 100-Gb/s optical transmission: (a) transmitter architecture, (b) receiver configuration, and (c) OFDM symbol upon cyclic extension.

For example, in the presence of PMD, the PIN photodiode output current can be written as follows

$$i(t) = R \left\{ \left| \sqrt{k} \left(s_{\text{OFDM}}(t) + b \right) * h_V(t) \right|^2 + \left| \sqrt{1-k} \left(s_{\text{OFDM}}(t) + b \right) * h_H(t) \right|^2 \right\}, \quad (10)$$

where k denotes the power-splitting ratio between two principal states of polarizations (PSPs). For the first order PMD, the optical channel responses $h_H(t)$ and $h_V(t)$ of horizontal and vertical PSPs are given as [29] $h_H(t) = \delta(t + \Delta\tau/2)$ and $h_V(t) = \delta(t - \Delta\tau/2)$, respectively, where $\Delta\tau$ is the DGD of two PSPs. In the presence of chromatic dispersion the impulse response of optical fiber of length z can be found by $h(t) = \text{FT}^{-1}[H(\omega)]$ (FT^{-1} is the inverse Fourier transform), where

$H(\omega) = e^{-\frac{\alpha}{2}z} e^{j\left[\frac{\beta_2}{2}\omega^2 + \frac{\beta_3}{6}\omega^3\right]z}$. α denotes the fiber attenuation coefficient, β_2 denotes the group-velocity dispersion (GVD) parameter, and β_3 is the second order GVD parameter. In the presence of fiber nonlinearities we have to solve the nonlinear Schrödinger equation [33]

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + i\gamma \left(|A|^2 - T_R \frac{\partial |A|^2}{\partial T} \right) A, \quad (11)$$

numerically using split-step Fourier method as described in [33]. In (11) z is the propagation distance along the fiber, relative time, $T = t - z/v_g$, gives a frame of reference moving at the group velocity v_g , $A(z, T)$ is the complex field amplitude of the pulse, α , β_2 and β_3 are introduced above, and γ is the nonlinearity coefficient giving rise to Kerr effect nonlinearities: self-phase modulation, intrachannel four-wave mixing, intrachannel cross-phase modulation, cross-phase modulation, and four-wave mixing. T_R denotes the Raman coefficient describing the stimulated Raman scattering (SRS).

The aggregate 100-Gb/s rate can be achieved as follows. Let the number of sub-channels be set to $N_{\text{QAM}}=64$, FFT/IFFT calculated in $N_{\text{FFT}}=128$ points, RF carrier frequency set to 60 GHz, and the bandwidth of optical filter set to 120 GHz. The 100 data strings carrying 1-Gb/s Ethernet traffic are 16-QAM modulated, and transmitted on 50 sub-carriers. Remaining 14 sub-carriers are used for the transmission of pilots and FEC overhead. The guard interval is obtained by cyclic extension of $N_{\text{guard}}=2 \times 16$ samples.

Fig. 9 The received signal constellation for 16-QAM SSB transmission after 10 spans of dispersion map from Fig. 2: (a) before phase correction, and (b) after phase correction. (c) Signal constellation diagram before PMD compensation for DGD of 1600 ps, (d) signal constellation diagram after PMD compensation (other effects, except PMD, have been ignored).

As an illustration of performance improvements that can be obtained by using OFDM, let us consider the effects of Kerr nonlinearities and ASE noise for the dispersion map from Fig. 3. Fig. 9(a,b) shows the influence of Kerr nonlinearities for 16-QAM single side band (SSB) transmission. The phase noise introduced by the self-phase modulation (SPM) causes the rotation of the constellation diagram. By using phase-correction based on 4 pilot tones, the phase rotation due to SPM can be completely eliminated, as shown in Fig. 9(a,b). The simple channel estimation technique [29] based on short training sequence can be used to compensate for DGD in excess of 1500 ps. The signal constellation diagrams, after the demapper from Fig. 8, and before and after applying the channel estimation are shown in Fig. 9(c,d). They correspond to the worst case scenario (the power splitting ratio between two principle state of polarizations is $k=1/2$) and 10-Gb/s aggregate data rate. Therefore, the channel estimation based OFDM is able to compensate for DGD of 1600ps.

The BER curves for the uncoded 100-Gb/s OFDM SSB transmission using QPSK or 16-QAM are shown in Fig. 10(a) and (b), respectively. From Fig. 10(a) (for launch power of -3 dBm) it can be concluded that 100-Gb/s transmission over 3840 km is possible using OFDM and LDPC codes with threshold BER (the channel BER for which the decoder output BER is below 10^{-12}) of 10^{-2} , which cannot be achieved using the state-of-the art ETDM high-speed electronics operating at 100-Gb/s [5].

Fig. 10 Uncoded BER versus number of spans for 100-Gb/s SSB OFDM transmission and: (a) QPSK, (b) 16-QAM modulation; for different launch powers (P).

Different coded modulation schemes enabling beyond 100-Gb/s optical transmission described above employ LDPC codes. For completeness of presentation we briefly describe the LDPC codes employed in simulations. For more details on LDPC codes suitable for use in optical communications an interested reader is referred to [19]-[21], and references therein.

5. BLOCK-CIRCULANT LDPC CODES

LDPC codes, invented by Gallager in 1960 [35], are linear block codes for which the parity check matrix has low density of ones. LDPC codes have generated great interests in the coding community recently, which resulted in a great deal of understanding of the different aspects of LDPC codes, and the decoding process. The inherent low-complexity of sum-product LDPC decoder opens up avenues for its use in different high-speed applications, including optical communications.

The most obvious way to the construction of an LDPC code is via the semi-random construction of a parity-check matrix H with prescribed properties [36]. To facilitate the implementation at high speed we prefer the use of *structured* LDPC codes [19]-[21]. For example, the parity-check matrix of a block-circulant LDPC code can be written as [20],

$$\mathbf{H} = \begin{bmatrix} p^{i_1} & p^{i_2} & p^{i_3} & \dots & p^{i_q} \\ p^{i_q} & p^{i_1} & p^{i_2} & \dots & p^{i_{q-1}} \\ \dots & \dots & \dots & \dots & \dots \\ p^{i_{q-r+2}} & p^{i_{q-r+3}} & p^{i_{q-r+4}} & \dots & p^{i_{q-r+1}} \end{bmatrix}, \quad (12)$$

with P being the permutation matrix $P=(p_{ij})_{n \times n}$, $p_{i,i+1}=p_{n,1}=1$ (zero otherwise). The exponents $i_1, i_2, \dots, i_q$ in (12) are carefully chosen to avoid the cycles of length four in corresponding bipartite graph of a parity-check matrix. In [20] we have shown that when exponents are selected as elements from the following set

$$L = \left\{ i : 0 \leq i \leq p^2 - 1, \theta^i + \theta \in GF(p) \right\}$$

where p is a prime, and θ is the primitive element of the finite field $GF(p^2)$; the resulting LDPC code is of girth-6. The girth is the shortest cycle in corresponding bipartite representation of a parity-check matrix. To increase the girth we carefully converted some of the permutation blocks in (12) into all-zeros blocks.

6. CONCLUSION

Three different bandwidth-efficient LDPC-coded modulation schemes enabling beyond 100-Gb/s optical transmission are described: (i) bit-interleaved LDPC-coded modulation scheme, (ii) multilevel coding scheme with LDPC component codes, and (iii) LDPC-coded OFDM scheme. In those schemes the aggregate bit rate above 100-Gb/s is maintained while modulation, coding, signal processing and transmission are done at 40 Giga symbols/s were dealing with intrachannel nonlinearities and PMD is more manageable and the implementation is easier. From standardization perspective point of view, and per ITU-T nomenclature, everything is based on ODU-3 and OTU-3, while future ODU/OTU- x ($x > 3$) are effectively supported. It was found that the coherent detection scheme brings an additional benefit of at least 2.5 dB in power margin, which can be effectively used either to extend the transmission distance, or to compensate for penalties due to effects expected in an all optical networking. Moreover, once the ETDM technology at 100-Gb/s reaches maturity of today's 40 Gb/s optical transmission systems, the schemes considered in this paper can be used to achieve transmission at much higher rates than 100-Gb/s. For example, by employing 1024-QAM LDPC-coded scheme operating at 100 Giga symbols/s we can achieve, at least in principle, 1-Tb/s optical transmission, the technology that might become important for 1-Tb/s Ethernet.

Since the operating speed of different approaches described in this paper is ~ 40 Giga symbol/s, the intrachannel nonlinearities still dominate over interchannel nonlinearities, and PMD is still a major transmission impairment. Several techniques mitigating the effects of intrachannel nonlinearities were recently proposed by author [12]-[16]. They can be classified into two broad categories. The constrained coding based mitigation techniques are described in [13], [14], while the LDPC-coded turbo equalization is described in [12], [15]. The role of constrained coding is to avoid those waveforms in the transmitted signal that are most likely to be received incorrectly. LDPC-coded turbo equalization scheme is a universal scheme that can be used: (i) to suppress fiber nonlinearities, the concept we demonstrated in [12], [15], (ii) for PMD compensation, our recent articles [28], [37], and for chromatic dispersion compensation, as explained in our recent conference paper [38]. The LDPC-coded turbo equalizer, is composed of two ingredients: (a) the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [39] based equalizer, and (b) the LDPC decoder. BCJR equalizer serves as nonlinear intersymbol interference (ISI) canceller, reduces the bit-error rate (BER) down to $\sim 10^{-3}$, and provides accurate estimates of the LLRs for LDPC decoder. We have shown in [12]-[16], [28], [29], [37] that those techniques provide excellent BER performance improvement, and believe that in a combination with LDPC-coded modulation techniques described in Sections 2-4 represent enabling technology for beyond 100-Gb/s optical transmission.

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