Irregular Low-Density Parity-Check Codes for Long-Haul Optical Communications

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***Abstract*—A novel forward error correction (FEC) scheme for long-haul optical communication systems based on irregular low-density parity check codes and iterative decoding is proposed. The proposed FEC scheme for long-haul optical transmission significantly outperforms all previously reported FEC schemes, and it has a potential for achieving theoretical limits on reliable transmission through an optical communication channel.**

***Index Terms*—Forward error correction, irregular low-density parity check codes, long-haul transmission, optical communications.**

I. Introduction

In TODAY'S ultralong-haul WDM transmission systems, the usage of forward error correction (FEC) is unavoidable. The coding gain allows in effectively increasing the amplifier spacing, the transmission distance and/or system capacity. The first generation of FEC codes based on Reed–Solomon (RS) codes, complying with ITU-T Rec. G.975, has been widely used in transoceanic submarine systems [1]. The second generation of FEC codes was based on the concatenation of two RS codes [1]. Sab and Lemaire proposed the use of turbo codes for Alcatel long-haul submarine transmission system [2]. Unfortunately, the turbo decoding algorithm is incompatible with optical transmission technology due to its high complexity. In our recent paper [3], we showed that the error performance and decoder hardware complexity can be greatly improved by using low-density parity-check (LDPC) codes along with an iteratively decodable scheme. Richardson *et al.* [4] showed that LDPC codes from irregular bipartite graphs with carefully chosen node degree distributions perform very close to the Shannon limit on the erasure channel. We present, in this paper, a systematic method to construct an irregular LDPC code from a regular one. In this method, an irregular LDPC code with desired node degree distribution pair is constructed by systematically splitting columns and rows of a regular LDPC code. We demonstrate further that irregular LDPC codes perform very well in optical communication channel too.

Although irregular LDPC codes created stochastically perform the best, the encoder structure becomes impractical for high-speed applications. By creating irregular codes in a structured manner allows to preserve low-complexity encoder structures (no permuter is necessary), and to achieve a significant coding gain.

Contrary to the common practice of considering the error control schemes using the AWGN channel assumption [1], [2], we study the performance of the proposed irregular LDPC schemes by taking into account all major impairments in a long-haul optical transmission such as amplified spontaneous emission noise, pulse distortion due to fiber nonlinearities, dispersion, crosstalk effects, intersymbol-interference, etc.

A significant performance improvement over conventional RS and Turbo product codes [1], [2] is demonstrated.

II. Irregular LDPC Codes Construction

As originally suggested by Tanner [5] an LDPC code can be represented by a *bipartite graph* consisting of *variable* (*bit*) nodes and *check* (*function*) nodes. (The number of edges emanating from a observed node is defined as its *degree*.) Any regular LDPC code with a parity-check matrix H^{reg}, having a column weight d_v and row-weight d_c is referred to as (d_v, d_c)-regular LDPC code, and can be constructed using concepts of combinatorics and finite geometries [3]. Any LDPC code not satisfying the above property is referred to as an *irregular* LDPC code. An irregular bipartite graph with maximum variable degree d_v and maximum check degree d_c can be specified by the sequences $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{d_v})$ and $\Omega = (\omega_1, \omega_2, \ldots, \omega_{d_c})$, where $\lambda_i(\omega_i)$ is the fraction of variable nodes (check nodes) with degree i.

In this letter, a systematic method to achieve an irregular LDPC code with any given node degree distribution pair is presented. This method is based on combinatorial arguments given in our recent paper [6]. The algorithm involves splitting columns and rows of parity-check matrix, H^{reg}, of the regular code in a systematic manner in order to achieve a desired degree distribution pair (Λ^{req}, Ω^{req}).

The important constraint on algorithm is to preserve the number of edges during the transformation from a regular graph to an irregular one. The process of splitting columns or rows of H^{reg} is equivalent to the creation of new variable or check nodes in a graph respectively. Since the column splitting does not affect the row distribution, and vice versa, the process of achieving column-weight distribution may be considered independent of the process of achieving the row-weight distribution, and as a consequence, just the problem of achieving a desired Λ^{req} will be formulated here.

Let us define a nonnegative integer vector $\boldsymbol{U}^{\mathrm{int}}$ whose elements $u_j^{\mathrm{int}}(\mathrm{j} = 1, 2, \ldots, k = \mathrm{d_v})\}$ is the number of columns of weight j required in the resulting irregular code, to achieve

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Λ^{req}. An integer vector $\boldsymbol{U}^{\text{int}}$ can be obtained by solving the constrained integer optimization problem in (1) and (2).

$$U^{\text{int}} = \arg_U \min \sum_{j=1}^{k} \left| \frac{u_j}{\sum_{m=1}^{k} u_m} - \lambda_j^{\text{req}} \right| \tag{1}$$

where the constraint is

$$\sum_{j=1}^{k} j u_j = bk \tag{2}$$

with b being the number of columns in H^{reg} matrix.

A solution (say $\boldsymbol{U}^{\text{int}}$) for (1), has the minimum (absolute) error sum from that of Λ^{req} and satisfies the constraint of preserving the total number of edges. The idea underlying the method to determine such a solution, is to determine a solution (say $\boldsymbol{U}^{\text{real}}$) in $\Re^k$ space and then search for $\boldsymbol{U}^{\text{int}}$ in the neighborhood of $\boldsymbol{U}^{\text{real}}$. It is straightforward to find $\boldsymbol{U}^{\text{real}}$ from (3) and (4).

$$\left(\sum_{j=1}^{k} u_j^{\text{real}}\right)\left(\sum_{j=1}^{k} j\lambda_j^{\text{req}}\right) = bk \tag{3}$$

$$u_j^{\text{real}} = \lambda_j^{\text{req}} \sum_{m=1}^{k} u_m^{\text{real}} \tag{4}$$

An initial estimate of $\boldsymbol{U}^{\text{int}}$ obtained by rounding all components of $\boldsymbol{U}^{\text{real}}$, may result in an error (say e) as shown in (5). This error can be corrected for by solving the *linear diophantine equation* given in (6). A solution of (6) is considered *best* if it results in $\boldsymbol{U}^{\text{int}}$ that satisfies (1).

$$e = bk - \sum_{j=1}^{k} j \left[u_j^{\text{real}}\right] \tag{5}$$

$$\sum_{\forall j, u_j^{\text{real}} \neq 0} j x_j = e, \quad j = 1, 2, \ldots, k \tag{6}$$

$$u_j^{\text{int}} = \left[u_j^{\text{real}}\right] + x_j, \quad u_j^{\text{real}} \neq 0 \tag{7}$$

A column with k ones can be partitioned in p_k different ways as given by the well known Euler's formula, see (8). Let us define a *partition distribution matrix* W^{T}, such that its elements w_{ij} $(i = 1, 2, \ldots, p_k;\ j = 1, 2, \ldots, k)$ denote the number of columns of weight j, in the partition of the ith-type (the partition of the ith-type is one of the p_k different ways to partition k). The number of columns, h_j^{column}, of H^{reg} that require a partition of ith-type to achieve u_i is given by a nonnegative vector $\boldsymbol{h}^{\text{column}}$ called the *design partition profile* for columns. The problem of finding $\boldsymbol{h}^{\text{column}}$ can be formulated as a system of linear diophantine equations in (9).

$$\begin{aligned} p_k = &\sum_{j=1}^{j(j-1)<k} (-1)^{j+1} p_{k-\frac{j(3j-1)}{2}} \\ &+ \sum_{j=1}^{j(j-1)<k} (-1)^{j+1} p_{k-\frac{j(3j+1)}{2}} \end{aligned} \tag{8}$$

$$W\boldsymbol{h}^{\text{column}} = \boldsymbol{U}^{\text{int}} \tag{9}$$

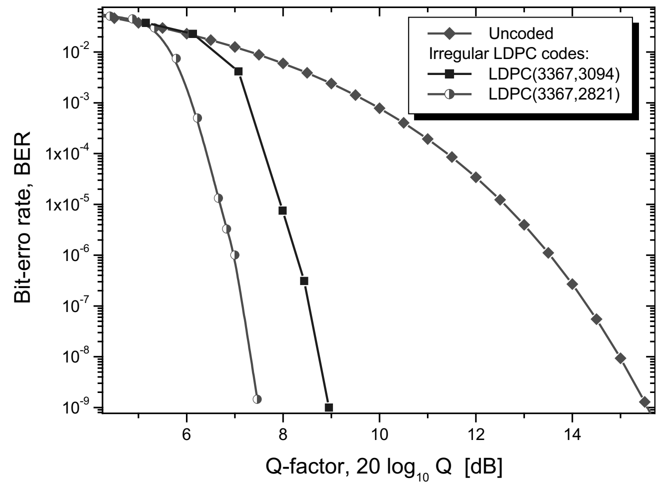

Fig. 1. Irregular LDPC codes performance at 40 Gb/s (in 4th iteration).

An efficient incremental algorithm to solve a system of linear diophantine equations proposed by Contejean *et al.* [7] is implemented here. This algorithm can be used to obtain a design partition profile for columns $\boldsymbol{h}^{\text{column}}$. Similarly, to achieve a desired row-weight distribution Ω^{req}, a design partition profile for rows, $\boldsymbol{h}^{\text{row}}$, using the method described above has to be determined. The columns and rows of H^{reg} are split using $\boldsymbol{h}^{\text{column}}$ and $\boldsymbol{h}^{\text{row}}$ to obtain the parity-check matrix of the irregular code. This irregular code H^{irreg} will have a degree distribution pair ($\boldsymbol{\Lambda}^{\text{achieved}}$, Ω^{achieved}) closest to the desired (Λ^{req}, Ω^{req}).

III. Code Performance

In this section, we present the performance of irregular LDPC codes in the presence of dispersion, fiber nonlinearities, ISI and receiver noise resulting from signal-noise and noise-noise interaction in the PIN photodiode, using the transmission system model we proposed in [10] (for corresponding derivations reader is referred to our previous paper [9]) The influence of the transfer functions of the optical and electrical filters is taken into account as well. A WDM system with 40-Gb/s bit-rate per channel and a channel spacing of 100 GHz is considered. It is assumed that the observed channel is located at 1552.524 nm (193.1 THz) and that there exists a nonnegligible interaction with six neighboring channels. The extinction ratio is set to 13 dB. The transmitter and receiver imperfection is described through a back-to-back Q-factor, which is set to 23 dB. The optical filter is modeled as a super-Gaussian filter of order eight and bandwidth 1.5R_b (R_b-bit rate over code rate), while the electrical filter is modeled as a Gaussian filter of bandwidth 0.65R_b. The transmission system considered has a dispersion map composed of SMF section, followed by an EDFA to compensate the fiber losses in SMF section, and DCF section to compensate both GVD and second-order GVD, as well as another EDFA to compensate fiber losses in DCF section. An average power per channel of 0 dBm and a CSRZ signal format are assumed. Fig. 1 shows the BER results of a Monte Carlo simulation for the irregular LDPC (3367, 3094) (of redundancy 8.8%) (distribution pair: [($\lambda_3 = 0.4054$, $\lambda_6 = 0.0541$, $\lambda_7 = 0.027$, $\lambda_{10} = 0.2162$), ($\omega_{81} = 1$)]) and LDPC(3367,2821) (distri-

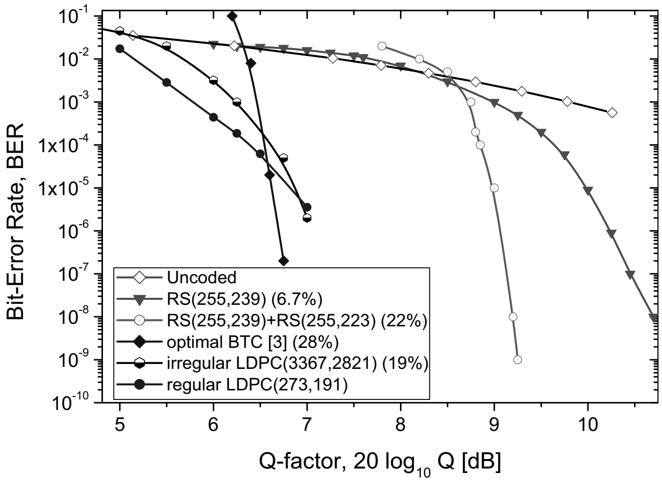

Fig. 2. Irregular LDPC codes performance against the conventional codes for AWGN channel.

bution pair: [($\lambda_3 = 0.4054$,$\lambda_6 = 0.0541$,$\lambda_7 = 0.0270$,$\lambda_9 = 0.2973$,$\lambda_{10} = 0.2162$),($\omega_{40} = 0.5$,$\omega_{41} = 0.5$)]) codes, constructed using the algorithm described in Section II, after the fourth iteration. An iterative soft-decision min-sum algorithm was used in decoding LDPC codes. The column and row weight distributions of codes were obtained by trial-and-error with a computer program. Hence, the distribution presented in Fig. 1 may not be the optimum degree distribution. (Since the code rate influence is included in Q-factor, the reported coding gain is equivalent to the net effective coding gain of AWGN channel). For BER of 10^{-9} the coding gains are 6.7 dB and 8.1 dB, 2–3.2 dB better than RS(255 239) [2].

The comparison of the proposed irregular LDPC codes with previously reported RS(255 239), the RS(255 223) + RS(255 239) concatenation scheme and block turbo code based on a product of two BCH (128, 113, 6) codes for AWGN channel is shown in Fig. 2. The bit-error rate curve of the initial regular (273, 191)-LDPC code is shown as well. The sum-product decoding algorithm was used. Although the regular code performs better at very low Q-factors, it may exhibit a BER floor because it is very short (the error floor is not shown in Fig. 2. (See [3] for more details on the acceptable codeword lengths for given code rates.) Also, notice that the redundancy of the regular code is 42.9%. The irregular LDPC (3367,2821) gives comparable results with Turbo block code $\mathrm{BCH}(128, 113, 6)^2$ [2] (of redundancy 28%) although it has smaller redundancy (∼19%). Much higher coding gains are expected for lower BER's. It is important to notice that the decoding algorithm is much simpler than that for RS or turbo codes.

IV. Conclusion

A novel error control scheme for long-haul optical communication systems based on irregular LDPC codes and iterative decoding is presented in this paper. As opposed to recent papers [1], [2] where the AWGN assumption is applied, we consider the performance of irregular LDPC codes in the presence of ASE noise, pulse distortion due to fiber nonlinearities, residual dispersion, crosstalk effects, intersymbol-interference, etc. The iterative decoding based on normalized min-sum algorithm has been demonstrated to give significant performance improvement over RS and block Turbo codes.

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