LDPC-Coded Optical Coherent State Quantum Communications

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***Abstract*—The problem of communication using optical coherent quantum states, in the presence of background radiation, is considered. Two modulation formats are studied, ON–OFF keying (OOK) and M-ary pulse-position modulation (PPM). The bit-error-rate performance improvement due to low-density parity-check coding is reported. For OOK, it is assumed that the coherent state signal has a random phase. For an average number of noise photons $N = 0.1$, the required number of signal photons per information bit is six in the case of coded quantum OOK. For the same level of noise $(N = 0.1)$ and assuming that signal phase is known, coded 16-ary PPM requires only 1.21 of signal photons per information bit.**

***Index Terms*—Coherent states, low-density parity-check (LDPC) codes, optical communications, quantum receiver.**

I. Introduction

THE optical coherent state discrimination by real-time quantum-feedback [1]–[3] is a promising approach that can be used instead of delicate quantum superposition [4] and entanglement measurement [5]. By virtue of uncertainty in quantum mechanics, the amplitude and phase of the received optical field cannot be determined simultaneously with arbitrary accuracy [6]. Moreover, the quantum mechanics postulates that an exact measurement of an observable gives as an outcome one of the eigenvalues, leaving the measured system in corresponding eigenstate. The repeated projection-valued probability-operator measurement, immediately afterwards, on the same quantum system would yield the same outcome [6]. The quantum hypothesis testing [6] is an excellent approach to maximize the information gain obtained from a quantum measurement, while minimizing the disturbance by the measurement. The recent experimental verification by Cook *et al.* [1] of long-standing theoretical prediction [6], opens up the optimism that optical coherent state communications might become reality soon.

The optical field provided by a laser is a convenient quantum system for transmission of information [2]. Unfortunately, the optical coherent states are not orthogonal, and someone has to use the quantum hypothesis testing to successfully separate the transmitted messages [1], [3], [5], [6]. Several coherent state receiver candidates, such as Dolinar, Sasaki-Hirota, and Kennedy receivers, have been compared by Geremia in [2]. Geremia has shown that Dolinar and Sassaki-Hirota receivers achieve the Helstrom bound, while the Kennedy receiver is shot-noise-limited. The Dolinar receiver has been shown to be the most robust against different receiver imperfections, and as such is the most promising candidate for future optical coherent state communications. A Dolinar-like receiver, based on coherent state discrimination using a closed-loop measurement, has recently been experimentally demonstrated in [1].

Motivated by this recent achievement, in this letter, we study the improvements that can be obtained by employing the high-rate structured low-density parity-check (LDPC) codes [7], [8], if Dolinar-like quantum receivers [9] are employed. We determine the bit-error-rate (BER) performance of LDPC-coded ON–OFF keying (OOK), and M-ary pulse-position modulation (PPM) for optical coherent quantum state communications. We assume that optimum-quantum and suboptimum photon counting receivers are affected by background radiation, and that the phase of coherent state signal is random. The LDPC codes are designed using the concept of balanced-incomplete block-design (BIBD) [10]. The possible applications include deep-space optical communications, short free-space optical links (such as those of interest for interchip communications), and quantum cryptography.

II. Optical Coherent State Quantum Communications[1]

The received optical field mode is said to be in a *coherent state*, when the state, described by a ket (vector) $|\alpha\rangle$, is right eigenvector of the annihilation operator a (the *annihilation operator* a decreases the number of photons by one)

$$a|\alpha\rangle = \alpha|\alpha\rangle \tag{1}$$

with α being a complex eigenvalue. The main complications arise because *coherent states are not orthogonal*. Namely, $\langle\alpha|\beta\rangle = \exp[\alpha^*\beta - |\alpha|^2/2 - |\beta|^2/2] \neq 0$.

If the received optical field for *binary coherent state communications*, in the absence of background light, is in one pure state $|\psi_0\rangle$ under hypothesis H_0 and in another $|\psi_1\rangle$ under hypothesis H_1, the corresponding *density operators* (see [6] for definition) are

$$\rho_0 = |\psi_0\rangle\langle\psi_0|, \quad \rho_1 = |\psi_1\rangle\langle\psi_1|. \tag{2}$$

The expectation value of density operator ρ_1 under hypothesis H_1, *in the presence of background light*, assuming that *absolute*

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[1]We use Dirac notation in representing the vectors (known as kets), and their scalar products. The scalar product of two kets $|u\rangle$ and $|v\rangle$ is denoted by $\langle u|v\rangle$, while its complex conjugate by $\langle u|v\rangle^* = \langle v|u\rangle$. The kets, describing the states of a quantum-mechanical system, are transformed by *Hermitian* operators (see [6] for more details).

received phase is unknown, can be determined by [6]

$$\begin{aligned}\langle\rho_1\rangle &= \sum_k P_k^{(1)}|k\rangle\langle k| \\ P_k^{(1)} &= (1-v)v^k \exp\left[-(1-v)N_s\right] L_k\left[-(1-v)^2 N_s/v\right] \\ N_s &= |\alpha|^2, \quad v = (N+N_s)/(N+N_s+1)\end{aligned} \tag{3}$$

where N is the average number of thermal photons, N_s is the average number of signal photons, and $|k\rangle$ denotes the number or Fock eigenstate. $L_k(x)$ in (3) denotes the kth Laguerre polynomial. The corresponding density operator of ρ_0 is obtained from the previous expression by setting $N_s = 0$. The maximum *a posteriori* probability (MAP) rule is simply

$$P_k^{(1)}/P_k^{(0)} \underset{H_0}{\overset{H_1}{\gtrless}} \Lambda_0 = \frac{\zeta}{1-\zeta} \tag{4}$$

where ζ is *a priori* probability of hypothesis H_0. The corresponding *log-likelihood ratio* (LLR) required for LDPC decoding (assuming equal probable transmission, $\zeta = 1/2$) is

$$\begin{aligned}L(\alpha_k) &= \log\frac{P_k^{(0)}}{P_k^{(1)}} = (1-v_k)*N_s \\ &\quad -\log\left\{L_k\left[-(1-v_k)^2 N_s/v_k\right]\right\}, \quad v_k = \frac{N_k}{N_k+1}.\end{aligned} \tag{5}$$

In M-ary transmission, at every signaling interval $(0, T)$ one message out of M messages, denoted by $m_1, \ldots, m_M$, is transmitted. For example, in M-ary PPM, the symbol interval T_s is subdivided into M slots of duration $T = T_s/M$, and all symbols have the same energy but occupy different time slots. Corresponding received electromagnetic field is specified by one out of M density operators $\rho_1, \ldots, \rho_M$. The receiver decides among M hypothesis $H_1, \ldots, H_M$, which one H_j corresponds to the transmitted message m_j, so that probability of error is minimal. The density operator and state vector for orthogonal coherent signaling under each hypothesis H_k can be represented as [6]

$$\begin{aligned}\rho_k &= |k\rangle\langle k| \\ |k\rangle &= |0\rangle_1 \ldots |0\rangle_{k-1} \left|N_s^{1/2}\right\rangle_k |0\rangle_{k+1} \ldots |0\rangle_M\end{aligned} \tag{6}$$

where N_s is the average number of photons corresponding to the kth signal, $\langle k|k\rangle = 1$, and $\langle k|j\rangle = \exp(-N_s)$ $(j \neq k)$. The optimum receiver measures a set of commuting projectors [6]

$$\begin{aligned}\Pi_i &= |\alpha_i\rangle\langle\alpha_i|, \quad |\alpha_i\rangle = \sum_{j=1}^{M}\alpha_{ij}|j\rangle \\ \alpha_{ii} &= \frac{a+(M-2)b}{(a-b)\left[a+(M-1)b\right]} \\ \alpha_{ij} &= \frac{-b}{(a-b)\left[a+(M-1)b\right]}(j \neq i)\end{aligned} \tag{7}$$

where

$$\begin{aligned}a &= b + \sqrt{1-\xi}, \\ b &= \frac{-\sqrt{1-\xi}+\sqrt{1+(M-1)\xi}}{M}, \quad \xi = \exp(-N_s).\end{aligned}$$

The hypothesis H_j is chosen when the eigenvalue of Π_j is equal to 1. In the presence of background light, the density operator ρ_k under hypothesis H_k has the form [6]

$$\begin{aligned}\rho_k &= \prod_{j=1}^{M}\int \exp\left[-\frac{|\alpha_j-\mu_{kj}|^2}{N_k}\right]|\alpha_j\rangle\langle\alpha_j|d^2\alpha_j/(\pi N_k) \\ \mu_{kj} &= N_s^{1/2}\delta_{kj}\end{aligned} \tag{8}$$

where the N_k is the mean number of noise photons. An optimum quantum receiver for arbitrary M still seems to be unknown, instead several suboptimum receivers have been proposed [6]. One such receiver uses the optimum receiver in the absence of background light. Unfortunately, it exhibits the error floor phenomena (see [6]), and as such is not considered here. Instead, the suboptimum photon counting receiver is considered because it does not exhibit error floor phenomena. For such a receiver the symbol LLRs, assuming $\zeta_k = 1/M$, can be determined by

$$\begin{aligned}L(\alpha_k) &= (1-v_k)*N_s - \log\left\{L_k\left[-\frac{(1-v_k)^2 N_s}{v_k}\right]\right\}, \\ v_k &= \frac{N_k}{N_k+1}.\end{aligned} \tag{9}$$

The bit reliabilities $L(b_j^{(k)})$, $(j = 1, 2, \ldots, \log_2 M)$ ($b_j^{(k)}$ is the jth bit in observed symbol m_k binary representation $\boldsymbol{b}^{(k)} = (b_1^{(k)}, b_2^{(k)}, \ldots, b_{\log M}^{(k)})$) are determined from symbol LLRs by

$$L\left(b_j^{(k)}\right) = \log\frac{\sum_{m_k:b_j^{(k)}=0}\exp\left[L(\alpha_k)\right]}{\sum_{m_k:b_j^{(k)}=1}\exp\left[L(\alpha_k)\right]} \tag{10}$$

and forwarded to the LDPC decoder.

III. LDPC-Coded Modulation

The source bit stream, for M-ary PPM, is encoded using an (n, k) LDPC code of code rate $r = k/n$ (k denotes the number of information bits, n denotes the codeword length). The $l \times n$ block-interleaver collects l code words in row-wise fashion. The mapper accepts l bits at a time from the interleaver column-wise and determines the corresponding symbol for Q-ary ($Q = 2^l$) PPM signaling using a Gray mapping rule. In OOK transmission, the block-interleaver and Gray mapper are omitted. The component girth-8 LDPC code is designed using the concept of BIBDs [10]. (The girth is the length of shortest cycle in corresponding bipartite representation of a parity-check matrix.) A balanced incomplete block design, denoted as $\mathrm{BIBD}(v, k, \{0, 1, \ldots, \lambda\})$ is a collection of subsets (blocks) of a v-set V with a size of each block k, so that each pair of elements occurs in *at most* λ of the blocks. Notice that we have relaxed the constraint in definition of BIBD from [10] by replacing the word *exact* with *at most*. The purpose of this relaxation is to increase the number of possible BIBDs that result in LDPC codes of high code rates. The LDPC codes from BIBDs naturally arise as girth-6 codes. To increase the girth from 6 to 8, certain blocks from a BIBD are to be removed. To improve the BER performance of the M-ary PPM scheme, we perform the iteration between *a posteriori* probability demapper and LDPC-decoder.

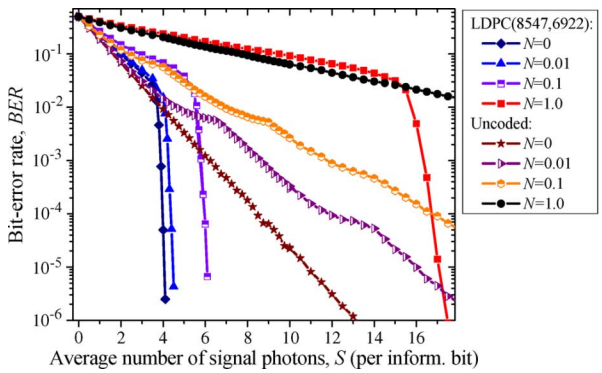

Fig. 1. BERs of binary coherent state communications (direct detection) with unknown phase. S denotes the average number of information signal photons per symbol one, and N denotes the average number of noise photons.

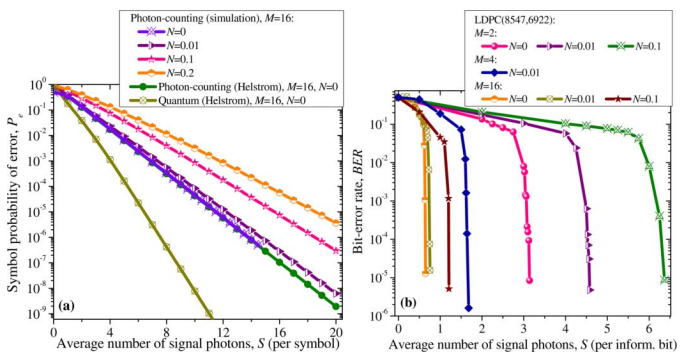

Fig. 2. Performance of M-ary PPM coherent state communications (for 5 outer, and 25 sum-product inner iterations) with known phase: (a) uncoded symbol error probability for $M = 16$; (b) LDPC-coded BERs.

IV. Numerical Results

The results of simulations for OOK optical coherent state communications in the presence of background light, and assuming that the phase is random, are given in Fig. 1. A girth-8 LDPC(8547,6922) code of code rate 0.81, designed using the concepts of BIBDs, is employed in simulations. In the absence of background radiation LDPC code provides 5.2-dB improvement at BER of 10^{-6}, and in the presence of background radiation coding gain improvement gradually grows as the level of background radiation increases. For example, for an average number of noise photons $N = 0.1$, the coding gain of 6.35 dB is obtained for the same BER. Larger coding gains are expected at lower BERs.

The results of simulations for M-ary PPM coherent state communications, in the presence of background radiation and assuming that the signal phase is known, are given in Fig. 2. In Fig. 2(a), we show the results for uncoded case by observing 16-ary PPM in terms of symbol error probability (against average number of photons per symbol) so that someone can compare the simulation results with those reported by Helstrom [6].

Notice that the quantum-optimum receiver in the presence of background radiation for arbitrary M is not known; instead we report the simulation results for photon-counting receiver. The results of simulation, in terms of BER versus average number of photons per information bit, for LDPC-coded case are reported in Fig. 2(b), for $M = 2, 4$, or 16 and different numbers of noise photons. The schemes with larger constellation size, as expected, are less sensitive to background radiation (because the symbol signal-to-noise ratio was kept constant for different M values).

V. Conclusion

We consider the problem of optical coherent state quantum communication in the presence of background radiation. Two modulation formats, OOK and M-ary PPM, are studied in combination with LDPC codes. For OOK transmission, we assume that coherent state signal has a random phase. We have found that for average number of noise photons 0.1, the required number of signal photons per information bit is 6 in the case of coded quantum OOK, and 6.37 for coded PPM (both observed at BER of 10^{-6}). In the absence of background radiation, for random signal phase, the required number of photons is 4.1 for coded OOK. The required number of photons for coded PPM in the absence of background noise (and assuming that phase is known) is 3.15. The required number of photons per information bit for coded 16-ary PPM is 1.21, for an average number of noise photons 0.1. The possible applications include deep-space optical communications, interchip free-space optical links, and quantum cryptography.

Notice that dual-containing LDPC codes [8] belong to the class of Calderbank–Shor–Steane (CSS) codes [4], and as such are compatible to modified Lo-Chau quantum key distribution (QKD) protocol via CSS codes [4]. By using the best known codes-LDPC codes (with error-correction capability t), we can increase the number of positions t in which qubits disagree before aborting the QKD protocol.

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