# Alamouti-type polarization-time coding in coded-modulation schemes with coherent detection

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**Abstract:** We present the Almouti-type polarization-time (PT) coding scheme suitable for use in multilevel ( $M \ge 2$ ) block-coded modulation schemes with coherent detection. The PT-decoder is found to be similar to the Alamouti combiner. We also describe how to determine the symbols log-likelihood ratios in the presence of laser phase noise. We show that the proposed scheme is able to compensate even 800 ps of differential group delay, for the system operating at 10 Gb/s, with negligible penalty. The proposed scheme outperforms equal-gain combining polarization diversity OFDM scheme. However, the polarization diversity coded-OFDM and PT-coding based coded-OFDM schemes perform comparable. The proposed scheme has the potential of doubling the spectral efficiency compared to polarization diversity schemes.

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**OCIS codes**: (060.4510) Optical communications; (999.9999) Polarization mode dispersion (PMD); (999.9999) Low-density parity-check (LDPC) codes, (999.9999) Alamouti-scheme

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### 1. Introduction

The performance of fiber-optic communication systems operating at high data rates is degraded by intra-channel and inter-channel fiber nonlinearities, polarization mode dispersion (PMD), and chromatic dispersion [1]. To deal with PMD a number of methods have been proposed, three of them seem to be able successfully to tackle the PMD effects: (i) turbo equalization [2], (ii) orthogonal frequency division multiplexing (OFDM) [3], and (iii) digital FIR equalizer [4].

In this paper we propose an alternative scheme, which is based on the Alamouti-type [5] polarization-time (PT) coding [6] with low-density parity-check (LDPC) codes [1]. We show that this scheme outperforms the polarization-diversity OFDM, when compared for the same launch power. (We use the term diversity in the same fashion that is used in wireless communications [5], [10], [13], which is different from the polarization multiplexing [4].) The key idea of Alamouti-type PT coding is to use the channel twice during the symbol duration. In the first channel use, the transmitter sends symbol  $s_x$  using x-polarization channel and symbol  $s_y$  using y-polarization channel. In the second channel use, the transmitter sends symbol  $-s^*_v$  by using x-polarization channel, and symbol  $s^*_x$  by using y-polarization. With proper combining on a receiver side, the tolerance to PMD can be improved compared to the corresponding polarization diversity scheme. We discuss two possible schemes, one in which only one polarization at the receiver side is used, and the second one in which both polarizations are used. When the channel coefficients, representing the so called channel state information (CSI), are known at the receiver side, both schemes are able to compensate for DGD of 800 ps, with negligible penalty; however, in the first scheme one polarization is not used at the receiver side, although both polarizations are used on transmitter side, resulting in 3 dB penalty compared to the second scheme. Notice that Alamouti-type coding has already been considered for use in optical communications, but in different context: in [7] to deal with atmospheric turbulence present in free-space optical channel and in [6] to deal with fiber nonlinearities. We derive a PT-decoder scheme, which is similar to Alamouti's combiner [5]. We describe how to use this scheme in combination with multilevel modulation and forward error correction (FEC). The arbitrary FEC scheme can be used with proposed PT-coding. However, the use of LDPC codes leads to channel capacity achieving performance [7]. Given the current high interest in coherent OFDM systems, the proposed scheme is described using the coded-OFDM as an illustrative example. We also describe how to determine the symbols reliabilities in the presence of laser phase noise.

# 2. Description of Alamouti-type polarization-time coding scheme with LDPC codes as channel codes

For the first-order PMD study the Jones matrix, neglecting the polarization dependent loss and depolarization effects due to nonlinearity, can be represented in fashion similar to [8]

$$\boldsymbol{H}(\omega) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix} = \boldsymbol{R}^{-1} \boldsymbol{P}(\omega) \boldsymbol{R}, \quad \boldsymbol{P}(\omega) = \begin{bmatrix} e^{-j\omega\tau/2} & 0 \\ 0 & e^{j\omega\tau/2} \end{bmatrix}, \tag{1}$$

where  $\tau$  denotes DGD,  $R=R(\theta,\varepsilon)$  is the rotational matrix defined by

$$\mathbf{R} = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right)e^{j\varepsilon/2} & \sin\left(\frac{\theta}{2}\right)e^{-j\varepsilon/2} \\ -\sin\left(\frac{\theta}{2}\right)e^{j\varepsilon/2} & \cos\left(\frac{\theta}{2}\right)e^{-j\varepsilon/2} \end{bmatrix}.$$

 $\theta$  denotes the polar angle,  $\varepsilon$  denotes the azimuth angle, and  $\omega$  is the angular frequency. For coherent detection OFDM, the received symbol vector  $\mathbf{r}_{i,k} = [r_{x,i,k} \ r_{y,i,k}]^T$  at ith OFDM symbol and kth subcarrier can be represented by

$$\mathbf{r}_{i,k} = \mathbf{H}(k)\mathbf{s}_{i,k}e^{j(\phi_T - \phi_{LO})} + \mathbf{n}_{i,k}, \tag{2}$$

where  $s_{i,k} = [s_{x,i,k} \ s_{y,i,k}]^T$  denotes the transmitted symbol vector,  $n_{i,k} = [n_{x,i,k} \ n_{y,i,k}]^T$  denotes the noise vector dominantly determined by the amplified spontaneous emission (ASE), and the Jones matrix H was introduced in Eq. (1) (we use index k to denote the kth subcarrier

frequency  $\omega_k$ ).  $\phi_\Gamma$  and  $\phi_{LO}$  denote the laser phase noise processes of transmitting and local lasers that are commonly modeled as the Wiener-Lévy processes [14], which are a zero-mean Gaussian processes with corresponding variances being  $2\pi\Delta v_T |t|$  and  $2\pi\Delta v_L |t|$ , where  $\Delta v_T$  and  $\Delta v_{LO}$  are the laser linewidths of transmitting and receiving laser, respectively. The transmitted/received symbols per subcarrier are complex-valued, with real part corresponding to the in-phase coordinate and imaginary part corresponding to the quadrature coordinate of corresponding constellation point. Fig. 1 shows the magnitude responses of  $h_{xx}$  and  $h_{xy}$ coefficients of Jones matrix against normalized frequency  $f\tau$  (the frequency is normalized with DGD  $\tau$  so that the conclusions are independent on the bit rate) for two different cases: (a)  $\theta=\pi/2$  and  $\varepsilon=0$ , and (b)  $\theta=\pi/3$  and  $\varepsilon=0$ . In the first case channel coefficient  $h_{xx}$  tends to zero for certain frequencies, while in the second case it never becomes zero; suggesting that the first case represents the worst case scenario. To avoid this problem, in direct detection OFDM systems someone can redistribute the transmitted power among subcarriers not being under fading, or use the polarization diversity coherent detection OFDM [3]. We propose an alternative approach that can be used for a number of modulation formats including M-ary phase-shift keying (PSK), M-ary quadrature-amplitude modulation (QAM) and OFDM as well. This method is based on space-time coding proposed to deal with fading in wireless communication systems, with Alamouti-type scheme [5] being a particular example that can be straightforwardly applied here. Once more, we would like to point out that Alamouti-type coding has already been proposed for use in optical communications (see [6,7]) but in different context: in [6] to deal with nonlinearities in fiber-optics channel, while in [7] to deal with scintillation for free-space optical channel.

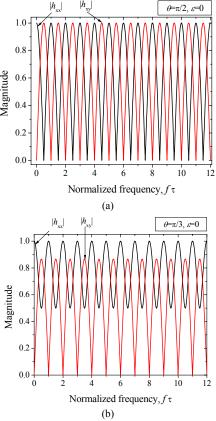


Fig. 1 Magnitude response of  $h_{xx}$  and  $h_{xy}$  Jones matrix coefficients against the normalized frequency for: (a)  $\theta = \pi/2$  and  $\varepsilon = 0$ , and (b)  $\theta = \pi/3$  and  $\varepsilon = 0$ .

The proposed PT coding scheme, with an LDPC code as channel code, when used in coded-OFDM, is shown in Fig. 2. The bit streams originating from m different information sources are encoded using different  $(n,k_i)$  LDPC codes of code rate  $r_i=k_i/n$ .  $k_i$  denotes the number of information bits of ith (i=1,2,...,m) component LDPC code, and n denotes the codeword length, which is the same for all LDPC codes. The use of different LDPC codes allows us to optimally allocate the code rates. The bit-interleaved coded modulation (BICM) scheme can be considered as a special multilevel coding (MLC) scheme in which all of the component codes are identical [1]. The outputs of m LDPC encoders are written row-wise into a block-interleaver block. The mapper accepts m bits at time instance i from the (mxn) interleaver column-wise and determines the corresponding M-ary  $(M=2^m)$  signal constellation point  $(\phi_{1,i}, \phi_{Q,i})$  in two-dimensional (2D) constellation diagram such as M-ary PSK or M-ary QAM. (The coordinates correspond to in-phase and quadrature components of M-ary 2D constellation.)

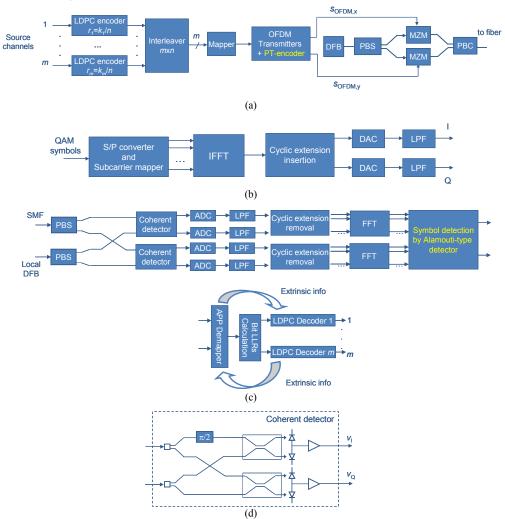


Fig. 2 The architecture of PT coding scheme based on OFDM concatenated with LDPC coding: (a) transmitter architecture, (b) OFDM transmitter architecture (x- or y-polarization), (c) receiver architecture, and (d) balanced coherent detector configuration. DFB: distributed feedback laser, PBS(C): polarization beam splitter (combiner), MZM: dual-drive Mach-Zehnder modulator, APP: *a posteriory* probability, LLRs: log-likelihood ratios.

The PT-encoder operates as follows. In the first half of ith time instance ("the first channel use") it sends symbol  $s_x$  to be transmitted using x-polarization channel and symbol  $s_y$  to be transmitted using y-polarization channel. In the second half of ith time instance ("the second channel use") it sends symbol  $-s*_y$  to be transmitted using x-polarization channel, and symbol  $s_x^*$  to be transmitted using y-polarization. Therefore, the PT-coding procedure is similar to the Alamouti-scheme [5]. Notice that Alamouti-type PT-coding scheme has the spectral efficiency comparable to coherent OFDM with polarization diversity scheme (once more we use the term diversity in the same sense it is used in wireless communications [5],[10] to denote that the same symbol was transmitted in both polarizations). When the channel is used twice during the same symbol period, the spectral efficiency of this scheme is twice higher than that of polarization diversity OFDM, but in that case the bandwidth usage is doubled. Notice that the hardware complexity of PT-encoder/decoder is trivial compared to that of maximum-likelihood sequence detection (MLSD). The transmitter complexity is higher than that required for MLC/BICM scheme we proposed in [1]; it requires additional PT-encoder, a polarization beam splitter (PBS), a polarization beam combiner (PBC), and two OFDM transmitters. On the receiver side, we have the option to use only one polarization or to use both polarizations. The receiver architecture employing both polarizations and OFDM is shown in Fig. 2(c). Compared to MLC/BICM scheme we proposed in [1] it requires the use of an additional coherent detector (whose configuration is shown in Fig. 2(c)), a PT-decoder, two PBSs, and two OFDM receivers.

The OFDM symbol is generated as described below.  $N_{\rm QAM}$  input QAM symbols are zero-padded to obtain  $N_{\rm FFT}$  input samples for inverse FFT (IFFT) (the zeros are added in the middle), and  $N_{\rm G}$  non-zero samples are inserted to create the guard interval. For efficient chromatic dispersion and PMD compensation, the length of cyclically extended guard interval should be longer than the total spread due to chromatic dispersion and maximum value of DGD. The cyclic extension is obtained by repeating the last  $N_{\rm G}/2$  samples of the effective OFDM symbol part ( $N_{\rm FFT}$  samples) as a prefix, and repeating the first  $N_{\rm G}/2$  samples as a suffix. After D/A conversion (DAC), the OFDM signal is converted into the optical domain using the dual-drive Mach-Zehnder modulator (MZM). Two MZMs are needed, one for each polarization. The outputs of MZMs are combined using the polarization beam combiner (PBC). The same DFB laser is used as CW source, with x- and y-polarization being separated by a polarization beam splitter (PBS).

The operations of all blocks, except the PT-decoder, are similar to those we reported in [1],[12]. The received symbol vectors for the first and second channel use can be written as follows

$$\mathbf{r}_{i,k}^{(m)} = \mathbf{H}(k) \mathbf{s}_{i,k}^{(m)} e^{i(\phi_T - \phi_{LO})} + \mathbf{n}_{i,k}^{(m)}, \quad m = 1, 2$$
(3)

where the Jones (channel) matrix H(k) is already introduced in (1) (we use again index k to denote the kth subcarrier frequency  $\omega_k$ ),  $\mathbf{r}_{i,k}^{(m)} = \left[\mathbf{r}_{x,i,k}^{(m)}, \mathbf{r}_{y,i,k}^{(m)}\right]^T$  denotes the received symbol vector in the mth (m=1,2) channel use of ith OFDM symbol and kth subcarrier, while  $\mathbf{n}_{i,k}^{(m)} = \left[\mathbf{n}_{x,i,k}^{(m)}, \mathbf{n}_{y,i,k}^{(m)}\right]^T$  denotes the corresponding noise vector. We use  $\mathbf{s}_{i,k}^{(1)} = \left[\mathbf{s}_{x,i,k}, \mathbf{s}_{y,i,k}^*\right]^T$  to denote the symbol transmitted in the first channel use, and  $\mathbf{s}_{i,k}^{(2)} = \left[-\mathbf{s}_{y,i,k}^*, \mathbf{s}_{x,i,k}^*\right]^T$  to denote the symbol transmitted in the second channel use (of the same symbol interval). The corresponding equivalent model is shown in Fig. 3, and is in agreement with channel model introduced by Shieh [3],[11]. Because the symbol vectors transmitted in the first and the second channel use of ith time instance are orthogonal,  $\mathbf{s}_{i,k}^{(1)} \mathbf{s}_{i,k}^{(2)} = \mathbf{0}$  (H denotes the Hermitian operation-simultaneous matrix transposition and complex conjugation), the equations (3) can be re-written by grouping separately x- and y-polarizations as follows

$$\begin{bmatrix} r_{x,i,k}^{(1)} \\ r_{x,i,k}^{(2)} \end{bmatrix} = \begin{bmatrix} h_{xx}e^{j\phi_{PN}} & h_{xy}e^{j\phi_{PN}} \\ h_{xy}^*e^{-j\phi_{PN}} & -h_{xx}^*e^{-j\phi_{PN}} \end{bmatrix} \begin{bmatrix} s_{x,i,k} \\ s_{y,i,k} \end{bmatrix} + \begin{bmatrix} n_{x,i,k}^{(1)} \\ *^{(2)} \\ n_{x,i,k}^{(2)} \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} r_{y,i,k}^{(1)} \\ r_{y,i,k}^{*(2)} \\ r_{y,i,k}^{*(2)} \end{bmatrix} = \begin{bmatrix} h_{yx}e^{j\phi_{PN}} & h_{yy}e^{j\phi_{PN}} \\ h_{yy}^*e^{-j\phi_{PN}} & -h_{yx}^*e^{-j\phi_{PN}} \end{bmatrix} \begin{bmatrix} s_{x,i,k} \\ s_{y,i,k} \end{bmatrix} + \begin{bmatrix} n_{y,i,k}^{(1)} \\ n_{y,i,k}^{*(2)} \end{bmatrix}.$$
 (5)

In Eqs. (4)-(5) we used  $\phi_{PN}$  to denote  $\phi_T$  -  $\phi_{LO}$ . If only one polarization is to be used we can solve either equation (4) or (5). However, the use of only one polarization results in 3 dB penalty with respect to the case when both polarizations are used. Following the derivation similar to that performed by Alamouti, it can be shown that the estimates of transmitted symbols at the output of PT-decoder (for ASE noise dominated scenario) can be obtain as follows

$$\tilde{s}_{x,i,k} = h_{xx}^* r_{x,j,k}^{(1)} e^{-j\phi_{PN}} + h_{xy} r_{x,i,k}^{*(2)} e^{j\phi_{PN}} + h_{yx}^* r_{y,j,k}^{(1)} e^{-j\phi_{PN}} + h_{yy} r_{y,i,k}^{*(2)} e^{j\phi_{PN}},$$

$$(6)$$

$$\tilde{s}_{y,i,k} = h_{xy}^* r_{x,i,k}^{(1)} e^{-j\phi_{PN}} - h_{xx} r_{x,i,k}^{*(2)} e^{j\phi_{PN}} + h_{yy}^* r_{y,i,k}^{(1)} e^{-j\phi_{PN}} - h_{yx} r_{y,i,k}^{*(2)} e^{j\phi_{PN}},$$

$$(7)$$

where  $\tilde{s}_{x,i}$  and  $\tilde{s}_{y,i}$  denote the PT-decoder estimates of symbols  $s_{x,i}$  and  $s_{y,i}$  transmitted in *i*th time instance. In case that only one polarization is to be used, say x-polarization, then the last two terms in equations (6) and (7) are to be omitted. The PT-decoder estimates are forwarded to the a posteriori probability (APP) demapper, which determines the symbol log-likelihood ratios (LLRs) in a fashion similar to that we reported in [1]. The bit LLRs are calculated from symbol LLRs as explained in [1], and forwarded to the LDPC decoders. The LDPC decoders employ the sum-product-with-correction term algorithm and provide the extrinsic LLRs to be used in the APP demapper, as explained in [1]. The extrinsic LLRs are iterated backward and forward, as illustrated in Fig. 2(c), until convergence or pre-determined number of iterations has been reached. The LDPC code used in this paper belong to the class of quasi-cyclic (array) codes of large girth ( $g \ge 10$ ) [9], so that the corresponding decoder complexity is low compared to random LDPC codes, and do not exhibit the error floor phenomena in the region of interest in fiber-optics communications ( $\le 10^{-15}$ ).

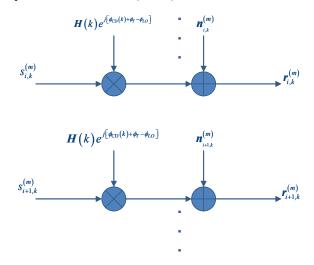


Fig. 3 Equivalent OFDM channel model.  $\phi_{CD}(k)$  denotes the phase distortion of kth subcarrier due to chromatic dispersion.

For the OFDM scheme with polarization diversity, assuming that x-polarization is used on a transmitter side and equal-gain combining on a receiver side, the transmitted symbol  $s_{i,k}$  at *i*th OFDM symbol and *k*th subcarrier can be estimated by:

$$\tilde{S}_{i,k} = \frac{r_{x,i,k} h_{xx}^* + r_{y,i,k} h_{xy}^*}{\left(\left|h_{xx}\right|^2 + \left|h_{xy}\right|^2\right) e^{j\phi_{PN}}},$$
(8)

where  $h_{xx}$  and  $h_{xy}$  are the channel coefficients introduced by Eq. (1),  $r_{x,i,k}$  and  $r_{y,i,k}$  represent the corresponding samples in x- and y-polarization branches, respectively.

The Alamouti-type detector soft estimates of symbols carried by kth subcarrier in ith OFDM symbol,  $\tilde{s}_{x(y)i,k}$ , are forwarded to the a posteriori probability (APP) demapper, which determines the symbol log-likelihood ratios (LLRs)  $\lambda_{x(y)}(s)$  of x- (y-) polarization by

$$\lambda_{x(y)}(s \mid \phi_{PN}) = -\frac{\left(\text{Re}\left[\tilde{s}_{i,k,x(y)}(\phi_{PN})\right] - \text{Re}\left[\text{QAM}\left(\text{map}(s)\right)\right]\right)^{2}}{2\sigma^{2}}$$

$$-\frac{\left(\text{Im}\left[\tilde{s}_{i,k,x(y)}(\phi_{PN})\right] - \text{Im}\left[\text{QAM}\left(\text{map}(s)\right)\right]\right)^{2}}{2\sigma^{2}}; \quad s = 0,1,...,2^{n_{b}} - 1$$
(9)

where Re[] and Im[] denote the real and imaginary part of a complex number, QAM denotes the QAM-constellation diagram,  $\sigma^2$  denotes the variance of an equivalent Gaussian noise process originating from ASE noise, and map(s) denotes a corresponding mapping rule (Gray mapping rule is applied here). ( $n_b$  denotes the number of bits carried by symbol.) Notice that symbol LLRs in Eq. (9) are conditioned on the laser phase noise sample  $\phi_{PN} = \phi_{\Gamma} - \phi_{LO}$ , which is a zero-mean Gaussian process (the Wiener-Lévy process [14]) with variance  $\sigma^2_{PN} = 2\pi(\Delta v_T + \Delta v_{LO})|t|$  ( $\Delta v_T$  and  $\Delta v_{LO}$  are the corresponding laser linewidths introduced earlier). This come from the fact that estimated symbols  $\tilde{s}_{x(y)i,k}$  are functions of  $\phi_{PN}$ . To remove the dependence on  $\phi_{PN}$  we have to average the likelihood function (not its logarithm), over all possible values of  $\phi_{PN}$ :

$$\lambda_{x(y)}(s) = \log \left\{ \int_{-\infty}^{\infty} \exp\left[\lambda_{x(y)}(s \mid \phi_{PN})\right] \frac{1}{\sigma_{PN}\sqrt{2\pi}} \exp\left(-\frac{\phi_{PN}^2}{2\sigma_{PN}^2}\right) d\phi_{PN} \right\}. \tag{10}$$

The calculation of LLRs in eq. (10) can be performed by numerical integration. For the laser linewidths considered in this paper it is sufficient to use the trapezoidal rule, with samples of  $\phi_{PN}$  obtained by pilot-aided channel estimation as explained in [11].

Let us denote by  $b_{j,x(y)}$  the *j*th bit in an observed symbol *s* binary representation  $b=(b_1,b_2,...,b_{nb})$  for x- (y-) polarization. The bit LLRs required for LDPC decoding are calculated from symbol LLRs by

$$L\left(\hat{b}_{j,x(y)}\right) = \log \frac{\sum_{s:b_j=0} \exp\left[\lambda_{x(y)}(s)\right]}{\sum_{s:b_j=1} \exp\left[\lambda_{x(y)}(s)\right]}.$$
 (11)

Therefore, the *j*th bit LLR in Eq. (11) is calculated as the logarithm of the ratio of a probability that  $b_j=0$  and probability that  $b_j=1$ . In the nominator, the summation is done over all symbols *s* having 0 at the position *j*. Similarly, in the denominator summation is performed over all symbols *s* having 1 at the position *j*. The extrinsic LLRs are iterated backward and forward until convergence or pre-determined number of iterations has been reached, as explained above (see also Fig. 2(c)).

## 3. Evaluation of the proposed coded-modulation scheme

We are turning our attention to the BER performance evaluation of the proposed scheme. In Fig. 4(a) we show the uncoded BER performance of Alamouti-type PT-coding schemes and

coherent optical OFDM, QPSK based, schemes with either polarization diversity or PT-coding, when the ASE noise dominated scenario is observed. In Fig. 4(b) we report the results when Alamouti PT-coding scheme is concatenated with girth-10 LDPC(16935,13550) code.

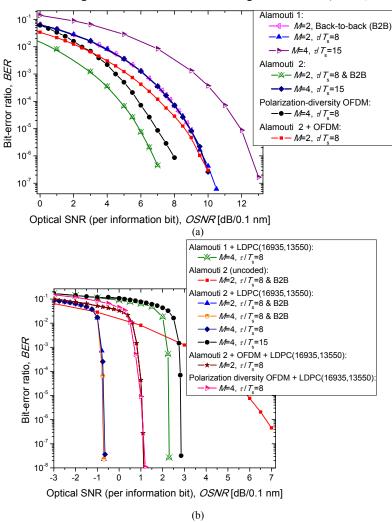


Fig. 4 BER performance of Alamouti-type polarization-time coding scheme: (a) uncoded case, and (b) LDPC-coded case. B2B: back-to-back.

The results of simulations are obtained assuming that the CSI is known on a receiver side. We consider the PT-encoder as being inner encoder for LDPC code that represents an outer code in an equivalent concatenated coding scheme. The PT-coding based OFDM system parameters were chosen as follows: the number of QAM symbols  $N_{\rm QAM}$ =512, the oversampling is two times, OFDM signal bandwidth is set to 10 GHz, and the number of samples used cyclic extension  $N_{\rm G}$ =256. The 4 pilots were sufficient to estimate this level of laser phase noise. For the fair comparison of different M-ary schemes the OSNR on x-axis is given per information bit, which is also consistent with digital communication literature [5],[10],[13]. The code rate influence is included in Fig. 4 so that the corresponding coding gains are net effective coding gains. The results of single-carrier simulations correspond to M-ary RZ-PSK (M=2 or 4) transmission (with duty cycle of 33%). Although the results of simulations are given for 10 Gb/s transmission they are reported in terms of normalized DGD (DGD is normalized with the symbol duration), so that the conclusions are applicable for 40

Gb/s and 100 Gb/s transmissions as well. The average launch power per *symbol* is set to 0 dBm (and similarly as in wireless communications [5],[10],[13] represents the power per information symbol), and the Gray mapping rule is employed. The laser linewidths of transmitting and local laser are set to 200 kHz. The results of simulations, for coded case, are obtained for 30 iterations in LDPC decoder, and 3 outer (APP demapper-LDPC decoder) iterations (resulting in total 90 iterations). For coded-OFDM, 1 outer and 30 inner iterations are sufficient. We have found that, if the CSI is known at the receiver side, the normalized DGDs up to 8 can be compensated with negligible penalty when PT-decoder described by equations (6)-(7) is used.

For normalized DGD 8, the proposed scheme outperforms the polarization-diversity OFDM (with 512 subcarriers, oversampling being two times, 64 samples for cyclic extension, 16 samples for windowing, 4 pilots for laser phase noise cancellation, and OPSK used for modulation) by 1.2 dB (at BER of 10<sup>-6</sup>). (Notice that comparison is done for the same launch powers per constellation symbols or per OFDM symbols.) The scheme employing only one polarization on receiver side, instead of both, faces 3 dB performance degradation, as expected. The LDPC-coded case provides a significant BER performance improvement over PT-coded scheme alone. When the CSI is known at receiver side it is sufficient to implement PT-decoder and APP demapper separately, as shown in Fig. 2(c). Notice that for corresponding turbo equalization [2],[15] or MLSD schemes, the detection complexity grows exponentially as DGD increases [mostly due to complexity of Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [2],[15]], and for normalized DGD of 8 it would require the trellis description (see [2],[15]) with 2<sup>17</sup> states, which is too high for practical implementation. The proposed scheme also outperforms the scheme implemented by Nortel Networks researchers [4], capable of compensating the rapidly varying first order PMD with peak DGD of 150 ps [4]. Notice that complexity of PT-coding based LDPC-coded OFDM is comparable to that of the LDPC-coded OFDM with polarization diversity. In simulations, for the polarization diversity OFDM both polarizations were used on a transmitter side.

It is interesting to notice that PT-coding based OFDM without LDPC code performs worse than polarization diversity OFDM for BERs below 10<sup>-3</sup> (see Fig. 4(a)), but better at BERs above 10<sup>-2</sup> where is the BER threshold region of large-girth LDPC codes. The Alamouti-type PT coding based LDPC-coded OFDM performs comparable to the equal-gain combining polarization diversity LDPC-coded OFDM (see Fig. 4(b)).

## 4. Conclusion

We proposed the Almouti-type polarization-time coding scheme suitable for use in coded-modulation schemes with coherent detection, as an alternative to turbo equalization, PMD equalization scheme with FIR filters and polarization diversity OFDM. In contrast to the PMD turbo equalization scheme whose complexity grows exponentially as DGD increases (due to exponential increase in complexity of BCJR algorithm [2],[15]), the complexity of the Alamouti-type PT encoder/decoder stays the same. The proposed scheme outperforms the OFDM with polarization diversity. However, PT-coding based LDPC-coded OFDM performs comparable to polarization diversity LDPC-coded OFDM (for DGD of 800 ps and aggregate rate of 10 Gb/s). On the other hand, the proposed PT-coding scheme has the potential of doubling the spectral efficiency compared to polarization diversity schemes. We also describe how to determine the symbols log-likelihood ratios in the presence of laser phase noise.

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