Statistical physics inspired energy-efficient coded-modulation for optical communications

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Because Shannon's entropy can be obtained by Stirling's approximation of thermodynamics entropy, the statistical physics energy minimization methods are directly applicable to the signal constellation design. We demonstrate that statistical physics inspired energy-efficient (EE) signal constellation designs, in combination with large-girth low-density parity-check (LDPC) codes, significantly outperform conventional LDPC-coded polarization-division multiplexed quadrature amplitude modulation schemes. We also describe an EE signal constellation design algorithm. Finally, we propose the discrete-time implementation of D-dimensional transceiver and corresponding EE polarization-division multiplexed system.

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The exponential Internet traffic growth projections place enormous transmission rate demands on the underlying information infrastructure at every level, from the core to access networks. The 100 Gbit/s Ethernet (100 GbitE) standard has been adopted recently (IEEE 802.3 ba), and 400 GbitE and 1 Tbit/s Ethernet (1 TbitE) are likely to be developed shortly. Recent studies indicate that the energy consumed by the Internet equipment is roughly 8% of the total energy consumed in the U.S. with predictions that it can grow up to 50% by 2020 if the current trend continues [1]. Therefore, the Internet is becoming constrained not only by capacity, but also by its energy consumption. In order to solve both problems simultaneously, we propose the use of statistical physics principles [2] in signal constellation design. Specifically, because Shannon's entropy can be obtained by Stirling's approximation of thermodynamics entropy, the statistical physics energy minimization methods are directly applicable to the signal constellation design. By using statistical physics concepts, we describe an energy-efficient signal constellation design algorithm (EE-SCDA). In the absence of noise, the optimum distribution follows the Gibbs–Boltzmann distribution from thermodynamics. In the presence of amplified spontaneous emission (ASE) noise and channel impairments, we use EE-SCDA to determine the optimum source distribution and represent the signal constellation design as a center of mass problem. We propose discrete-time implementation of a D-dimensional transceiver as well as a corresponding EE polarization-division multiplexed (PDM) system. We demonstrate by Monte Carlo simulations that statistical physics inspired EE signal constellation designs, in combination with large-girth low-density parity-check (LDPC) codes, significantly outperform conventional PDM coded-modulation schemes.

The EE optical communication problem can be formulated as follows. The set of symbols $X=\{x_1,x_2,\ldots,x_M\}$ that occurs with a priori probabilities $p_1,p_2,\ldots,p_M$ [$p_i=\Pr(x_i)$, $i=1,\ldots,M$]; with corresponding symbol energies $E_1,\ldots,E_M$; are to be transmitted over the optical channel of interest. The prior probabilities satisfy the following probability constraint: $\sum_i p_i=1$. By interpreting the symbols as states of a thermodynamic system, and their probabilities of occurrence as probabilities of a system being in a particular state $p_i=N_i/N$, where N_i is the number of subsystems being in state x_i and $N=N_1+\ldots+N_M$; we can establish a one-to-one correspondence between communications and thermodynamics systems. The number of states in which a particular set of occupation number N_i occurs is given by the multinomial coefficient $C(\{N_i\})=N!/(N_1!\ldots N_M!)$. Corresponding thermodynamics entropy is defined by [2]:

$$S_t=k\log C\left(\{N_i\}_{i=1}^M\right)=k\log[N!/(N_1!\ldots N_M!)],\qquad(1)$$

where k is the Boltzmann constant. For convenience, we will use the following definition of thermodynamic entropy $S=(S_t/k)/N$. By using Stirling's approximation, given by $\log n!=n\log n-n+O(\log n)$, the normalized thermodynamics entropy, based on (1), becomes

$$\begin{aligned}S(X)&\approx(N\log N-\sum_{i=1}^M N_i\log N_i)/N\\&=-\sum_{i=1}^M(N_i/N)\log(N_i/N)=-\sum_{i=1}^M p_i\log p_i=H(X).\end{aligned}\qquad(2)$$

Therefore, Shannon's entropy $H(X)$ is a Stirling's approximation of the normalized thermodynamics entropy $S(X)$, indicating that different statistical physics energy minimization methods are directly applicable to communication systems. The received symbols are affected by ASE noise and various channel impairments and distortions, including fiber nonlinearities, dispersion, and filtering effects. The mutual information, or the amount of information transmitted over the channel, can be determined as $I(X,Y)=S(X)-S(X|Y)$, where Y is the output of the channel. For energy-efficient communication systems, we impose the following energy constraint: $\sum_i p_iE_i\le E$, where E denotes the available energy. In the absence of ASE noise and channel impairments, clearly $S(X|Y)=0$. In the presence of channel impairments, the maximization of (1), leads to the maximum mutual information, also known as information capacity, and can be performed by using the Lagrangian method:

$$L_m = S(X) - S(X|Y) + \alpha\left(1 - \sum_i p_i\right) + \beta\left(E - \sum_i p_i E_i\right) \quad \text{(3a)}$$

$$\cong H(X) - H(X|Y) + \alpha\left(1 - \sum_i p_i\right) + \beta\left(E - \sum_i p_i E_i\right), \quad \text{(3b)}$$

where $H(X|Y) = -\sum_i p_i \sum_j P_{ij} \log Q_{ji}$ and P_{ij} denotes the transition probability $P_{ij} = \Pr(y_j|x_i)$, which can be determined by channel estimation, by propagating a sufficiently long training sequence. In (3), with Q_{ji}, we denoted the $\Pr(x_i|y_j)$, which can be determined by Bayes' rule: $Q_{ji} = \Pr(x_i|y_j) = \Pr(x_i, y_j)/\Pr(y_j) = P_{ij}p_i/\sum_k P_{kj}p_k$. The second term in (3) is the conditional entropy. In the absence of ASE noise and channel impairments, the solution can be found in closed form as $p_i = \exp(-\beta E_i)/\sum_j \exp(-\beta E_j)$, which is clearly Gibbs distribution. Note that when the energy constraint is removed, by setting $\beta = 0$, the Gibbs distribution becomes uniform. In the presence of ASE noise and channel impairments, the mutual information optimization problem, with respect to input distribution and corresponding signal constellation, cannot be solved analytically. However, we can use the following algorithm, which we will refer to as the EE-SCDA:

0. *Initialization*: Choose an arbitrary auxiliary input distribution and signal constellation, with a number of constellation points M_a much larger that the target signal constellation M.
1. Q_{ji} *update-rule*: $Q_{ji}^{(t)} = P_{ij}p_i^{(t)}/\sum_k P_{kj}p_k^{(t)}$. (With superscript (t) we denoted the index of iteration.)
2. p_i update-rule: $p_i^{(t+1)} = e^{-\beta E_i - S^{(t)}(x_i|Y)}/\sum_k e^{-\beta E_k - S^{(t)}(x_k|Y)}$, where the Lagrange multiplier β is determined from energy constraint.
3. *Signal constellation design*: Determine the constellation points of target constellation as a center of mass of the closest M_a/M constellation points in the auxiliary signal constellation as follows: $s_i^{(t)} = \sum_{j \in n(i)} p_j^{(t)} s_{a,j}^{(t)}$, where $s_i^{(t)}$ is i-th target signal constellation point. The $s_{a,j}^{(t)}$ denotes j-th auxiliary signal constellation point in the neighborhood of s_i, denoted as $n(i)$.

Step (1) is based on Bayes' rule. Step (2) can be interpreted as being derived from Gibbs distribution by introducing the correction factor, which is the conditional entropy originating from channel impairments and noise. Notice that the original Arimoto–Blahut algorithm [3] does not impose the energy constraint and yields to the optimum source distribution only. By EE-SCDA, we obtain the optimized signal constellation by taking the energy constraint into account. Both optimum source distribution and energy-efficient signal constellation are obtained as the result of the proposed algorithm. Note the 2D-constellation design has been discussed in [4], however, the signal constellation is designed to minimize the symbol error probability instead. Another approach, discussed in [5] in context of multimode fibers, is information theory based signal constellation design, which represents the special case of statistical physics inspired SCDA. As an illustration, in Fig. 1 we report the information capacities (normalized per dimension) for different normalized energy cost functions, defined as E/E_s, where E_s is the average symbol energy of a given constellation and E is the energy used in Eq. (3). A number of amplitude levels per dimension, denoted by L, is used as parameter. It is evident, that when the normalized energy cost function is lower than one, we are facing information capacity degradation.

We turn our attention now to the EE coded-modulation proposal. The coordinates of the EE signal constellation, stored in a look-up table (LUT), are used as the inputs to the D-dimensional modulator, whose discrete-time (DT) implementation is shown in Fig. 2(a). The DT D-dimensional modulator generates the signal constellation point as follows:

$$s_i = C_D \sum_{m=1}^{D} \phi_{im} \Phi_m, \quad (4)$$

where ϕ_{im} denotes the m-th coordinate ($m = 1, \ldots, D$) of the ith signal-constellation point, the set $\{\Phi_1, \ldots, \Phi_D\}$ represents the set of D orthogonal basis functions, such as orthogonal subcarriers and various classes of orthogonal polynomials, and C_D denotes the normalization factor. As an alternative solution, discussed in [5], someone can use two orthogonal polarization states, in-phase and quadrature components, and orbital angular momentum (OAM) states (in few-mode fibers) as basis functions. The key difference with respect to [5] is that a D-dimensional constellation is generated in an electrical domain, while in [5] both electrical and optical basis functions are employed. The signal-constellation point coordinates after upsampling are passed through corresponding DT pulse-shaping filters of impulse responses $h_m(n) = \Phi_m(nT)$, whose outputs are combined together into a single complex data stream. After separation of real and imaginary parts and digital-to-analog conversion (DAC), the corresponding real and imaginary parts are used as inputs to the I/Q modulator. Two I/Q modulators are used for two orthogonal polarizations. The D-dimensional demodulator is shown in Fig. 2(b). After separation of two D-dimensional data streams carried in two polarizations by a polarization beam splitter (PBS), we perform coherent detection to recover Re- and Im-parts, which are after analog-to-digital conversion (ADC) combined into a single complex data stream. The same complex data stream is applied to the inputs of D matched filters of impulse responses $h_m(n) = \Phi_m(-nT)$. The corresponding outputs after resampling [see Fig. 2(b)] represent projections along basis functions Φ_m.

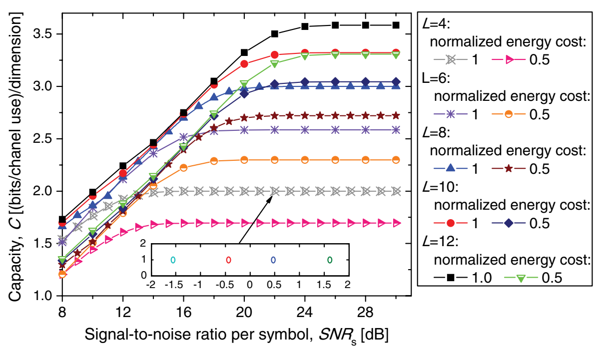

Fig. 1. (Color online) Information capacities per dimension. For channel model, we use I-ary input J-ary output ($J > I$) channel model, which is a valid model for reasonably high signal-to-noise ratios. The inset figure shows optimum constellation for $L = 4$ and $D = 1$.

The overall transmitter architecture is shown in Fig. 2(c). The B binary data streams per single polarization are encoded using $B(n, k)$ LDPC codes. The code words generated by LDPC encoders are written row-wise into a corresponding block-interleaver. The B bits at time instance i are taken from block-interleaver column-wise and used as the input of a corresponding D-dimensional mapper, implemented as a look-up table, which passes coordinates to the D-dimensional modulator [see Fig. 2(a)]. The receiver architecture per single polarization is depicted in Fig. 2(d). The D-dimensional signal constellation point, transmitted over a given polarization, is reconstructed in D-dimensional demodulator [see Fig. 2(b)], which provides projections along basis functions. The reconstructed coordinates are used as input to D-dimensional *a posteriori* probability (APP) demapper, which calculates symbol log-likelihood ratios (LLRs). The spectral efficiency of this EE scheme is $D \log_2 L / \log_2 M_{\mathrm{QAM}}$ times better than that of PDM-QAM (quadrature amplitude modulation), where L is the number of amplitude levels per dimension and M_{QAM} is QAM signal-constellation size.

In order to illustrate the high potential of proposed energy-efficient coded-modulation, we perform Monte Carlo simulations for the ASE noise dominated scenario, with results summarized in Fig. 3. This scenario is applicable to fiber-optics communications when the coarse digital back propagation is combined with the sliding-window turbo equalization scheme, as we described in [6]. We compare bit error rate (BER) performance of EE-PDM (16935,13550) LDPC-coded modulation (CM) against that of PDM-QAM, and previous IPQ-based signal constellation [6].

It is clear that for fixed L, the increase in the number of dimensions leads to small performance degradation as long as orthogonality of basis functions is preserved. The aggregate data rate of the EE PDM coded-modulation scheme is determined by $2 \times R_s \times \log_2(L^D) \times r$, where R_s is the symbol rate and r is the code rate. The comparisons are performed for fixed bandwidth equal to the symbol rate, R_s. By setting $R_s = 31.25$ Giga symbols/s (GS/s), $r = 0.8$, $L = 4$, and $D = 4$, the aggregate data rate is 400 Gb/s, which is compatible with 400 Gb/s Ethernet. As another example, by setting $L = 4$, $D = 10$, $R_s = 31.25$ GS/s, and $r = 0.8$, the aggregate data rate is 1 Tb/s, which is compatible with 1 Tb/s Ethernet. Let us now compare the performance of EE PDM $L = 4$, $D = 4$ coded-modulation with PDM 256-QAM ($D = 2$ in x-pol. and $D = 2$ in y-pol.), having the same number of constellation points. At a BER of 2.5×10^{-7}, the $L = 4$, $D = 4$ EE-PDM coded modulation scheme outperforms the corresponding PDM 256-QAM by even 9.98 dB. In Fig. 3, we also provide the comparison among proposed EE-SCDA in two-dimensions against conventional QAM and IPQ-based 2D constellations. The 8-ary EE-SCDA outperforms 8-QAM by 0.7 dB at BER of 10^{-8}. The 16-ary EE-SCDA outperforms 16-QAM by 0.74 dB at BER of 2×10^{-8}. As the coordinates obtained by EE-SCDA are stored in LUT, the complexity of the proposed scheme is comparable to that of PDM-QAM.

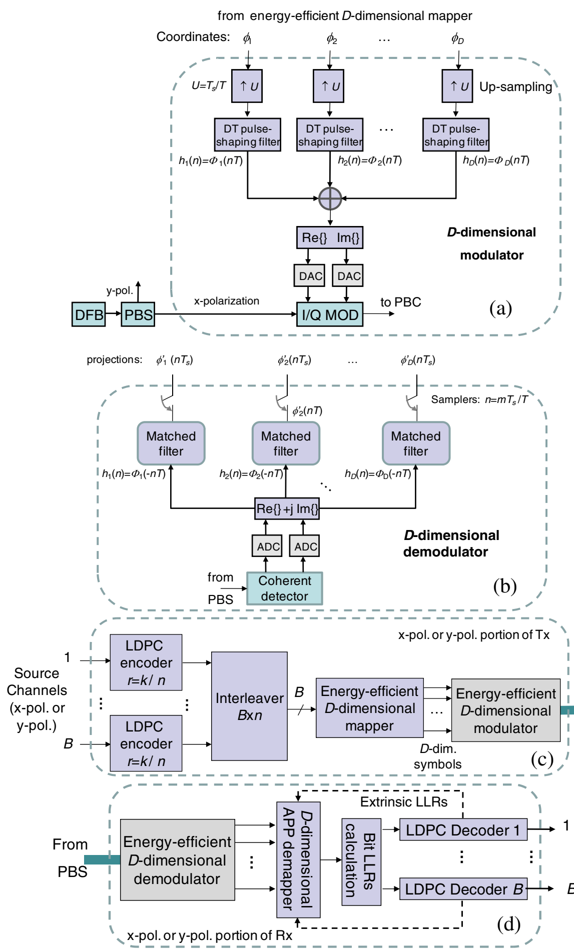

Fig. 2. (Color online) D-dimensional EE coded modulation (CM) scheme: (a) DT D-dim. modulator, (b) DT D-dim. demodulator, (c) Tx configuration, and (d) Rx configuration. PBS(C), polarization-beam splitter (combiner); T_s, symbol duration; T, sampling interval, related to T_s by $T = T_s/U$, where U is the oversampling factor; DFB, distributed feedback laser.

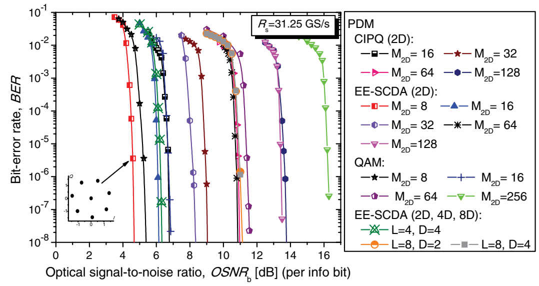

Fig. 3. (Color online) BER performance of EE-PDM LDPC-coded modulation. A given CIPQ signal constellation is obtained by placing one point in the origin and then running the IPQ procedure. Inset, optimum 8-ary constellation for $D = 2$.

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