Energy-efficient spatial-domain-based hybrid multidimensional coded-modulations enabling multi-Tb/s optical transport

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Abstract: In addition to capacity, the future high-speed optical transport networks will also be constrained by energy consumption. In order to solve the capacity and energy constraints simultaneously, in this paper we propose the use of *energy-efficient hybrid D*-dimensional signaling (D>4) by employing all available degrees of freedom for conveyance of the information over a single carrier including amplitude, phase, polarization and orbital angular momentum (OAM). Given the fact that the OAM eigenstates, associated with the azimuthal phase dependence of the complex electric field, are orthogonal, they can be used as basis functions for multidimensional signaling. Since the information capacity is a *linear* function of number of dimensions, through D-dimensional signal constellations we can significantly improve the overall optical channel capacity. The energy-efficiency problem is solved, in this paper, by properly designing the D-dimensional signal constellation such that the mutual information is maximized, while taking the energy constraint into account. We demonstrate high-potential of proposed energy-efficient hybrid D-dimensional coded-modulation scheme by Monte Carlo simulations.

OCIS codes: (060.0060) Fiber optics and optical communications; (060.4080) Modulation.

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#149543 - $15.00 USD Received 20 Jun 2011; revised 20 Jul 2011; accepted 26 Jul 2011; published 15 Aug 2011

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1. Introduction

The exponential internet traffic growth projections require considerable increase of transmission rates at every level of the underlying information infrastructure, from core to access and data center networks. Higher volumes of traffic also increase the energy consumption of transmission and switching equipment needed to route this traffic. Recent studies indicate that the energy consumed by the Internet equipment is roughly 8% of the total energy consumed in the US with predictions that it can grow up to 50%, with current trend, by the end of this decade [1,2]. Therefore, the Internet is becoming constrained not only by capacity, but also by its energy consumption.

As a solution to never ending high-capacity demands, in a series of articles (e.g [3–6].) we proposed the use of multidimensional coded-modulation schemes. The key idea behind these papers is to exploit multiple degrees of freedom already available for the conveyance of information on a photon. We also know that photons can carry spin angular momentum (SAM), which is associated with polarization, and orbital angular momentum (OAM), which is associated with azimuthal phase dependence of the form $\exp(il\phi)$ ($l = 0, \pm 1, \pm 2,\ldots; i^2 = -1$). The ability to generate OAM modes in both free-space optical (FSO) [5] and multimode-fiber (MMF) links [6–8], might lead to future transparent heterogeneous optical networks, as discussed in [9]. The high-speed optical transmission over multimode/multi-core fiber links is becoming a hot research topic, which can be judged by number of recent publications [6–14].

In order to solve high-bandwidth demands and energy-efficiency problems simultaneously, in this paper, we propose an energy-efficient spatial-domain-based hybrid coded-modulation scheme. The proposed scheme is based on hybrid D-dimensional (D>4) signal constellations, and exploits all available degrees of freedom for conveyance of the information over a single carrier including amplitude, phase, polarization and OAM. From Shannon's theory we know that the channel capacity is a logarithmic function of signal-to-noise ratio, but *linear* function in number of dimensions. Therefore, by increasing the number of dimensions we can dramatically improve the overall channel capacity. On the other hand, the energy-efficiency problem can be solved by properly designing the D-dimensional signal constellation such that the mutual information is maximized, while taking the energy constraint into account. We demonstrate high-potential of proposed energy-efficient hybrid D-dimensional coded-modulation scheme by Monte Carlo simulations. The proposed coded-modulation scheme is very flexible, it can be used for various applications ranging from short-haul to long-haul, and can be used in both single-mode fiber (SMF) and MMF links.

The remainder of the paper is organized as follows. In Section 2, we describe the energy-efficient signal constellation design. In Section 3, we describe the proposed energy-efficient spatial-domain-based hybrid low-density parity-check (LDPC)-coded modulation scheme. We present our numerical results and discuss their significance in Section 4. Finally, some important concluding remarks are given in Section 5.

#149543 - $15.00 USD Received 20 Jun 2011; revised 20 Jul 2011; accepted 26 Jul 2011; published 15 Aug 2011

2. Energy-efficient signal constellation design

The basic energy-efficient optical communication problem can be formulated as follows. The set of symbols $X = \{x_1, x_2, \ldots, x_M\}$ that occur with *a priori* probabilities $p_1, \ldots, p_M$ [$p_i = \Pr(x_i)$]; with corresponding energies $E_1, \ldots, E_M$; are to be transmitted over the optical channel of interest. The symbols from the set X satisfy the following two constraints: (1) $\sum_i p_i = 1$ (probability constraint) and (2) $\sum_i p_i E_i \leq E$ (energy constraint). In the presence of amplified spontaneous emission (ASE) noise and various channel impairments (fiber nonlinearities, PMD, PDL and filtering effects), we can use the Lagranagian method in maximizing the mutual information $I(X,Y)$, defined as $I(X,Y) = H(X)\text{-}H(X|Y)$, where $H(X)$ is the entropy of the channel input X and $H(X|Y)$ is the conditional entropy of channel input X given the channel output Y. The corresponding Lagranagian, by taking energy and probability constraints into account, can be formulated as follows:

$$L = \overbrace{-\sum\nolimits_i p_i \log p_i}^{H(X)} - \overbrace{\left(-\sum\nolimits_i p_i \sum\nolimits_j P_{ij} \log Q_{ji}\right)}^{H(X|Y)} + \lambda\left(\sum\nolimits_i p_i - 1\right) + \mu\left(\sum\nolimits_i p_i E_i - E\right), \qquad (1)$$

where with P_{ij} we denoted the transition probabilities $P_{ij} = \Pr(y_j|x_i)$, which can be determined by channel estimation or for ASE noise dominated scenario we can use Gaussian approximation. In Eq. (1), with Q_{ji} we denoted $\Pr(x_i|y_j)$, which can be determined by Bayes' rule as $Q_{ji} = \Pr(x_i|y_j) = \Pr(x_i,y_j)/\Pr(y_j) = P_{ij}p_i/\sum_k P_{kj}p_k$. The optimum signal constellation coordinates cannot be found in analytical form, however, we can use Arimoto-Blahut-like algorithm, but now taking the energy constraint into account. The *energy-efficient Arimoto-Blahut algorithm* (EE-ABA) can be formulated as follows:

0) Initialization: Choose arbitrary input distribution, say uniform $p_i = 1/M$.

1) Q_{ji} update-rule: $Q_{ji}^{(t)} = P_{ij} p_i^{(t)} / \sum_k P_{kj} p_k^{(t)},\ P_{ij} = \Pr\left(y_j \mid x_i\right).$

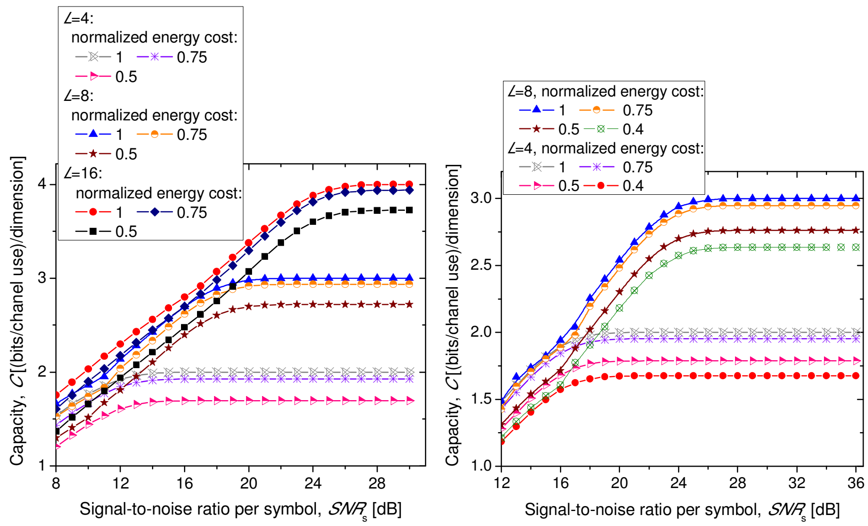
2) p_i update-rule: $p_i^{(t+1)} = \dfrac{\exp\left(-\mu E_i - H^{(t)}\left(x_i \mid Y\right)\right)}{\sum_k \exp\left(-\mu E_k - H^{(t)}\left(x_k \mid Y\right)\right)},$

where $H\left(x_i \mid Y\right) = -\sum_k P_{ik} \log Q_{ki}$ and the parameter μ is determined from energy constraint. Repeat the steps 1)-2) until convergence. (The superscript (t) denotes the iteration index.).

The step 1) is derived based on Bayes' rule: $Q_{ji} = \Pr(x_i|y_j) = \Pr(x_i,y_j)/\Pr(y_j) = P_{ij}p_i/\sum_k P_{kj}p_k$. In the absence of noise and channel impairments we set the conditional entropy term in step 2) to zero and obtain the Gibbs distribution $p_i = \exp(-\beta E_i)/[\sum_k \exp(-\beta E_k)]$. Notice that original Arimoto-Blahut algorithm [15] does not impose the energy constraint. By EE-ABA, we obtain the optimum source distribution, while taking the energy constraint into account. Unfortunately, it is not practical to use the signal constellation points with non-uniform distribution. To solve this problem, we propose to apply this algorithm for an auxiliary signal constellation with significantly larger number of constellation points M' than desired signal constellation size M, such that M'>>M, and then determine the desired constellation points as a *center of mass* of closest M'/M constellation points in this auxiliary signal constellation.

As an illustration, in Fig. 1 we report the information capacities for different normalized energy cost functions for number of amplitude levels per dimension L. In Fig. 1(a), we provide the results corresponding to coherent detection, while in Fig. 1(b) we provide the results corresponding to direct detection. It is clear from the Figure, that when the normalized energy cost function is lower than one, we are facing certain information capacity degradation. For channel model we use I-ary input J-ary output (J>I) channel model, which is a valid model for reasonably high signal-to-noise ratios (SNRs).

#149543 - $15.00 USD Received 20 Jun 2011; revised 20 Jul 2011; accepted 26 Jul 2011; published 15 Aug 2011



(a) (b)

Fig. 1. Information capacities per dimension for different normalized energy cost values and different number of amplitude levels per dimension L: (a) coherent detection case, and (b) direct detection case.

3. Energy-efficient hybrid coded-modulations schemes enabling multi-Tb/s optical transport

The energy-efficient signal constellation is obtained by using EE ABA, described in previous section. The coordinates of the EE signal constellation from D-dimensional mapper, implemented as a single look-up-table (LUT), are used as the inputs to the D-dimensional modulator, whose configuration is shown in Fig. 2(a). This modulator generates the signal constellation points by

$$s_i = \mathrm{C}_D \sum_{d=1}^{D} \phi_{i,d} \Phi_d, \tag{2}$$

where $\phi_{i,d}$ denotes the dth coordinate ($d = 1,\ldots,D$) of the ith signal-constellation point, the set $\{\Phi_1,\ldots,\Phi_D\}$ represents the set of $D = 2MN$ orthogonal bases functions, where factor two comes from two orthogonal polarization states, N denotes the number of orthogonal OAM eigenstates and M basis functions are defined as

$$\Phi_m(nT) = \exp\left[\mathrm{j}2\pi(m-1)nT/T_s\right] \ (m = 1,\ldots,M), \tag{3}$$

where T_s is the symbol duration, and T is the sampling interval, related to symbol duration by $T = T_s/U$, with U being the oversampling factor [see Fig. 2 part (c)]. (In Eq. (2), C_D denotes the normalization factor.) The EE signal constellation coordinates are split into N groups of $2M$-coordinates per each OAM mode. The $2M$-coordinates of each group are used as input of $2M$-dimesnional modulator, whose configuration is shown in Fig. 2(b). The $2M$-dimensional modulator is composed of two M-dimensional modulators, one for each polarization, whose configuration is shown in Fig. 2(c). The M signal-constellation point coordinates after up-sampling are passed through corresponding discrete-time (DT) pulse-shaping filters of impulse responses $h_m(n) = \Phi_m(nT)$, whose outputs are combined together into a single complex data stream. After separation of real and imaginary parts and digital-to-analog conversion (DAC), the corresponding Re- and Im-parts are used as inputs to I/Q modulator. The mode multiplexing scheme, shown in Fig. 2(a), operates as follows.

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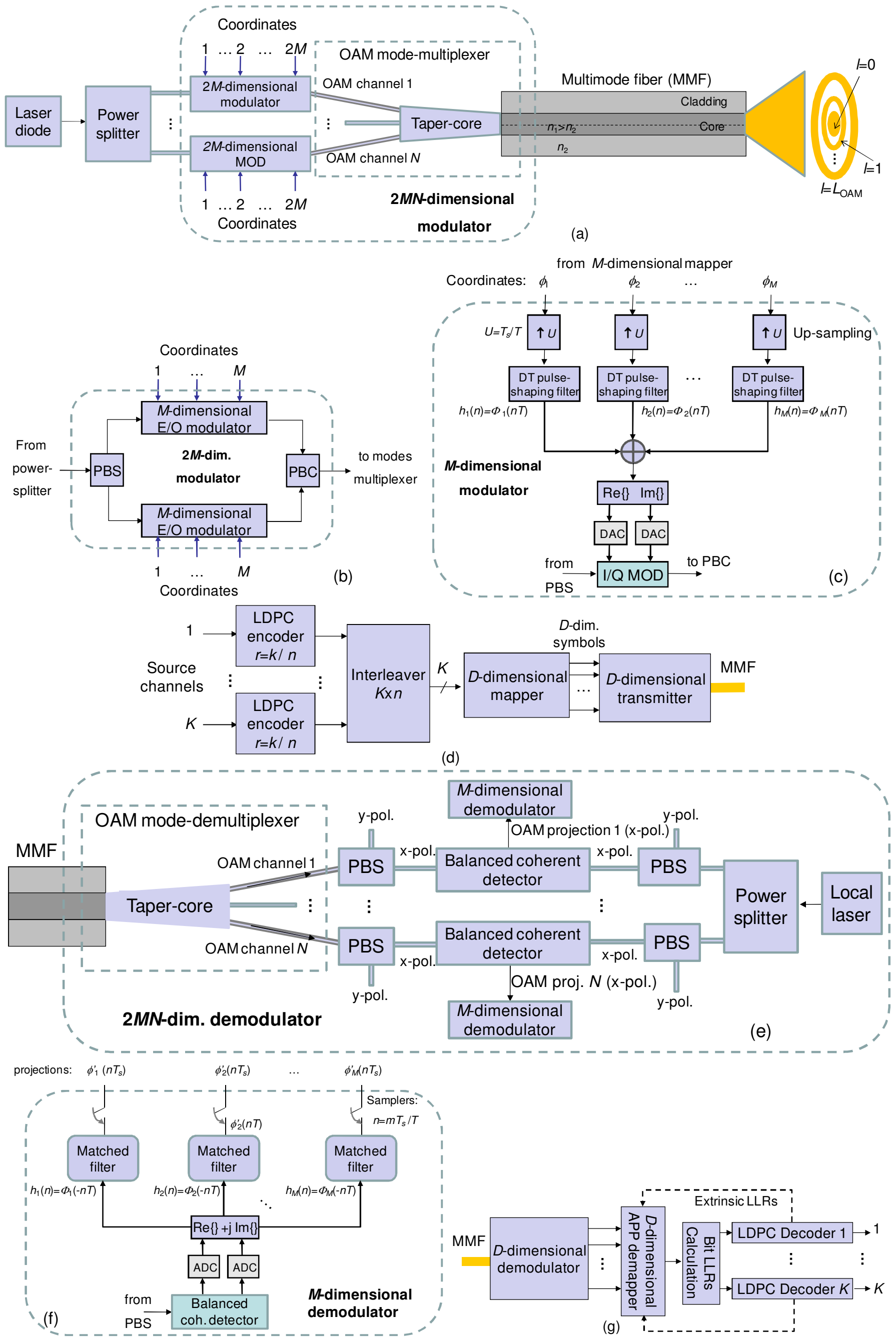

Fig. 2. Energy-efficient hybrid coded-modulation: (a) operation principle and D-dim. modulator configuration, (b) 2M-dim. modulator, (c) discrete-time (DT) M-dim. modulator, (d) overall Tx configuration, (e) D-dim. demodulator, (f) M-dim. demodulator, and (g) overall Rx configuration. PBS(C): polarization-beam splitter (combiner).

A continuous wave laser diode signal is split into N branches by using a power splitter (such as 1:N star coupler) to feed the corresponding 2M-dimensional modulators, each corresponding to one of the N OAM modes. The OAM mode multiplexer is composed of N waveguides, taper-core fiber and MMF, properly designed to excite orthogonal OAM modes in MMF (see [7,8] for more details and fabrication issues). The D-dimensional modulated

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signal is then transmitted over multi-mode/multi-core fiber link of interest. The overall transmitter configuration is shown in Fig. 2(d). The K binary data streams are encoded using K (n,k) LDPC codes. The codewords generated by LDPC encoders are written row-wise into corresponding block-interleaver. The K bits at time instance i are taken from block-interleaver column-wise and used as the input of corresponding D-dimensional mapper, implemented as an LUT, as already described above. The D-coordinates from D-dimensional mapper are used as input to the D-dimensional modulator, shown in Fig. 1(a). Coherent detection based D-dimensional demodulator is shown in Fig. 2(e). We first perform OAM mode-demultiplexing in OAM-demux block, whose outputs are projections along N OAM states. The nth OAM projection, which is a $2M$-dimensional signal, is used as input to the polarization-beam splitter (PBS). The x-polarization (y-polarization) signal, being itself an M-dimensional signal, is used as input to the balanced coherent detector (the second input is the corresponding output that originates from local laser, as shown in Fig. 2e). After coherent detection we recover Re- and Im-parts, which are after analog-to-digital conversion (ADC) combined into a single complex data stream. The same complex data stream is applied to the inputs of D matched filters of impulse responses $h_m(n) = \Phi_m(-nT)$. The corresponding outputs after re-sampling (see Fig. 2f) represent projections along basis functions Φ_m. At this point all $2MN$-coordinates of EE D-dimensional signal constellations are estimated, and corresponding coordinate estimates (representing the projections along D-basis functions) are forwarded to the D-dimensional *a posteriori* probability (APP) demapper, as shown in Fig. 2(g), which calculates symbol log-likelihood ratios (LLRs). In bit-LLRs calculation block, we calculate the bit likelihoods needed for LDPC decoding. After LDPC decoding, the extrinsic information is passed back to APP demapper. We iterate the extrinsic information between LDPC decoders and APP demapper until convergence or until pre-determined number of iterations has been reached. This procedure is similar to that we described in [6].

The spectral efficiency of the proposed hybrid D-dimensional scheme, where $D = 2MN$, is

$$\frac{S_E^{D\text{-dim. constellation}}}{S_E^{\text{PDM-QAM}}} = \frac{\log_2 L^D}{2\log_2 M_{\text{QAM}}} = \frac{D\log_2 L}{2\log_2 M_{\text{QAM}}} \tag{4}$$

times better than that of polarization-division-multiplexed (PDM) quadrature amplitude modulation (QAM) scheme. In (4), with M_{QAM} we denoted the QAM signal constellation size. Therefore, for the same number of amplitude levels per dimension ($M_{\text{QAM}} = L^2$), the spectral efficiency of the proposed scheme is (D/4)-times better than that of PDM-QAM. The aggregate data rate (per single wavelength) is determined by

$$\log_2(L^{2MN})\ \frac{\text{ch. bits}}{\text{ch. sym.}} \times R_s\ \frac{\text{ch. sym.}}{\text{s}} \times r\ \frac{\text{info. bits}}{\text{ch. bits}}, \tag{5}$$

where r is the code rate, which is assumed to be equal for LDPC codes at each level, and R_s is the symbol rate.

4. Performance analysis

We evaluate the bit-error rate (BER) performance of the proposed energy-efficient spatial-domain-based hybrid D-dimensional LDPC-coded modulation and compare it against the performance of the corresponding coded PDM-QAM. We performed Monte Carlo simulations for ASE noise scenario for three APP demapper-LDPC decoder iterations and 25 LDPC decoder inner iterations. The results of simulations are shown in Fig. 3, where we compare BER performance of energy-efficient polarization-division multiplexed (EE-PDM) (column-weight-3, girth-10) (16935,13550) LDPC coded modulation (CM) against that of PDM-QAM (for the same LDPC code). It is clear that for fixed L, the increase in the number of dimensions leads to small performance degradation as long as orthogonality of basis functions is preserved. The aggregate data rate of EE PDM coded-modulation scheme, per single OAM mode, is determined by $2 \times R_s \times \log_2(L^M) \times r$ / OAM mode, where R_s is the symbol rate and r

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is the code rate. By setting R_s = 31.25 Giga symbols/s (GS/s)/OAM mode, r = 0.8, L = 4, and M = 4 the aggregate data rate is 400 Gb/s, which is compatible with 400 Gb/s Ethernet when one OAM mode is used. As another example, by setting L = 4, M = 8, R_s = 50 GS/s, and r = 0.8, the aggregate data rate is 1.28 Tb/s/OAM mode, which is compatible with Tb/s Ethernet when one OAM mode is used. On the other hand, the aggregate data rate of PDM 256-QAM, for R_s = 50 GS/s and r = 0.8, is $2 \times R_s \times \log_2(256) \times r$ = 640 Gb/s, which is not sufficient for Tb/s Ethernet. Let us now compare the performance of EE PDM L = 4, M = 4 coded-modulation with PDM 256-QAM, having the same number of constellation points. At BER of 2.5×10^{-7}, the L = 4, M = 4 EE-PDM coded modulation scheme outperforms corresponding PDM 256-QAM by even 9.98 dB! In the same Figure, we provide the curve for three-dimensional signal-constellation with 64 points, obtained by sphere packing method due to Sloane [16]. The EE-PDM scheme with L = 4, M = 4 outperforms the sphere packing scheme by 0.98 dB at BER of 5.7×10^{-7}. Notice that aggregate data rate of EE-PDM scheme for L = M = 4 with R_s = 31.25 GS/s is 400 Gb/s, while the aggregate data rate of PDM sphere packing constellation is only 300 Gb/s. The proposed EE spatial-domain-based hybrid coded-modulation scheme is, therefore, a promising candidate for both 400 Gb/s and Tb/s Ethernet technologies, while significantly outperforming the conventional PDM-QAM. By using several OAM modes (two modes are used in EE CM schemes shown in Fig. 3), we can clearly achieve multi-Tb/s serial optical transport.

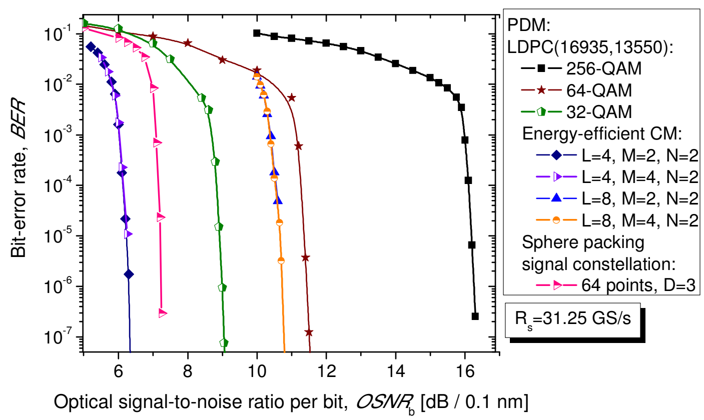

Fig. 3. Energy-efficient hybrid LDPC-coded modulation.

5. Concluding remarks

Inspired by high potential of multidimensional signal constellations and recent demonstrations in which OAM modes are successfully excited in MMFs, we proposed the use of energy-efficient spatial-domain-based hybrid coded D-dimensional modulation as an enabling technology for multi-Tb/s serial optical transport. We demonstrated by simulations that proposed energy-efficient coded modulation scheme significantly outperforms conventional PDM-QAM scheme. As an example, the L = 4, M = 4 EE-PDM coded modulation scheme outperforms corresponding PDM 256-QAM by even 9.98 dB at BER of 2.5×10^{-7}. We also have presented an energy-efficient Arimoto-Blahut algorithm and described how it can be used in energy-efficient signal constellation design. Finally, we demonstrated that proposed energy-efficient signal constellations can be used to enable 400 Gb/s Ethernet, Tb/s Ethernet, and multi-Tb/s serial optical transport.

Acknowledgments

This work was supported in part by the National Science Foundation (NSF) under Grant CCF-0952711, through NSF CIAN ERC under grant EEC-0812072, and in part by NEC Labs.

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