LDPC-coded MIMO optical communication over the atmospheric turbulence channel using *Q*-ary pulse-position modulation

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Abstract: We describe a coded power-efficient transmission scheme based on *repetition* MIMO principle suitable for communication over the atmospheric turbulence channel, and determine its *channel capacity*. The proposed scheme employs the Q-ary pulse-position modulation. We further study how to approach the channel capacity limits using low-density parity-check (LDPC) codes. Component LDPC codes are designed using the concept of pairwise-balanced designs. Contrary to the several recent publications, bit-error rates and channel capacities are reported assuming that p.i.n. photodetectors are used instead of ideal photon-counting receivers. The atmospheric turbulence channel is modeled using the Gamma-Gamma distribution function due to Al-Habash *et al*. Excellent bit-error rate performance improvement, over uncoded case, is found.

OCIS codes: (060.4510) Optical communications; (010.1330) Atmospheric turbulence; (060.4080) Modulation; (999.9999) Multiple-input multiple-output (MIMO); (999.9999) Low-density parity-check (LDPC) codes

References and Links

1. H. Willebrand, and B.S. Ghuman, Free-Space Optics: Enabling Optical Connectivity in Today's Networks: Sams Publishing, 2002.
2. S. G. Wilson, M. Brandt-Pearce, Q. Cao, and J.J.H. Leveque, III, "Free-space optical MIMO transmission with Q-ary PPM," IEEE Trans. Commun. **53**, 1402-1412 (2005).
3. S. G. Wilson, M. Brandt-Pearce, Q. Cao, M. Baedke, "Optical repetition MIMO transmission with multipulse PPM," IEEE Selected Areas Comm. **23**, 1901-1910 (2005).
4. M. A. Al-Habash, L.C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," Opt. Eng. **40**, 1554-1562 (2001).
5. I. B. Djordjevic, B. Vasic, and M. A. Neifeld, "Power efficient LDPC-coded modulation for free-space optical communication over the atmospheric turbulence channel," in Proc. OFC 2007, Paper no. JThA46, March 25-29, 2007, Anaheim, CA, USA.
6. I. B. Djordjevic, B. Vasic, M. A. Neifeld, "Multilevel coding in free-space optical MIMO transmission with Q-ary PPM over the atmospheric turbulence channel," IEEE Photon. Tehnol. Lett. **18**, 1491-1493 (2006).
7. N. Cvijetic, S.G. Wilson, and M. Brandt-Pearce, "Receiver optimization in turbulent free-space optical MIMO channels with APDs and Q-ary PPM," IEEE Photon. Tehnol. Lett. **19**, 1491-1493 (2007).
8. G. Ungerboeck, "Channel coding with multilevel/phase signals," IEEE. Trans Inf. Theory **28**, 55-67 (1982).
9. I. B. Djordjevic, S. Sankaranarayanan, S. K. Chilappagari, and B. Vasic, "Low-density parity-check codes for 40 Gb/s optical transmission systems," IEEE J. Sel. Top. Quantum Electron. **12**, 555-562 (2006).
10. J. A. Anguita, I. B. Djordjevic, M.A. Neifeld, and B. V. Vasic, "Shannon capacities and error–correction codes for optical atmospheric turbulent channels," J. Opt. Networking **4**, 586-601 (2005).
11. I. Anderson, *Combinatorial Designs and Tournaments*: Oxford University Press, 1997.
12. G. Caire, G. Tarrico, E. Biglieri, "Bit-interleaved coded modulation," IEEE. Trans Inf. Theory **44**, 927-946 (1998).
13. H. Imai, and S. Hirakawa, "A new multilevel coding method using error correcting codes," IEEE. Trans Inf. Theory **IT-23**, 371-377 (1977).
14. K. Simon, and V.A Vilnrotter, "Alamouti-type space-time coding for free-space optical communication with direct detection," IEEE Trans. Wireless Comm. **4**, 35 – 39 (2005).
15. V. Tarokh, H. Jafarkani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," IEEE. Trans Inf. Theory **45**, 1456-1467 (1999).
16. I. B. Djordjevic, B. Vasic, and M. A. Neifeld, "LDPC coded OFDM over the atmospheric turbulence channel," Opt. Express **15**, 6332-6346 (2007).

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17. I. B. Djordjevic, S. Denic, J. Anguita, B. Vasic, and M. A. Neifeld, "LDPC-Coded MIMO optical communication over the atmospheric turbulence channel," accepted for presentation at Globecom 2007.

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1. Introduction

Free-space optical (FSO) communication has received significant attention recently, as a possible alternative to solve the bottleneck connectivity problem, and as a supplement to more conventional RF/microwave links [1]. FSO methods also represent an enabling technology to integrate a variety of interfaces and network elements. However, an optical wave propagating trough the air experiences fluctuations in amplitude and phase due to atmospheric turbulence [1-7]. This intensity fluctuation, also known as scintillation, is one of the most important factors that degrade the performance of an FSO communication link, even under the clear sky condition.

The incompatibility of RF/microwave and optical communication technologies arises from the large bandwidth mismatch between these two channel types and is another problem believed to be the limiting factor in efforts to further increase future transport capabilities [1]. For this reason RF/microwave-optical interface solutions that enable the aggregation of multiple RF/microwave channels into a single optical channel are becoming increasingly important.

In this paper we consider a coded power-efficient transmission scheme based on Q-ary pulse-position modulation (PPM). To enable the transmission under the strong atmospheric turbulence we propose the use of the *multi-laser multi-detector* (MLMD) concept [2,3],[5,6]. Although the MLMD concept is analogous to multiple-input multiple-output (MIMO) wireless concept, the underlying physics is different, and novel both optimal and sub-optimal FSO configurations are required. In several recent publications, MLMD concept itself [2,3] and different coding techniques [5,6] are studied assuming an *ideal photon-counting receiver*. The recent paper due to Cvijetic *et al*. [7], is an exception form this common practice. The simulation results reported in [5,6] indicate that LDPC-coded repetition MIMO is an excellent candidate that might enable the transmission over the strong atmospheric turbulence. This led us to a fundamental question: "What is the Shannon's capacity of a coded MIMO free-space optical communication system for *non-ideal photodetection* based on p.i.n. photodiodes?" The purpose of the paper is twofold: (i) to determine the channel capacity of a coded MIMO FSO system in the presence of atmospheric turbulence, and (ii) to see how much this limit can be approached using the best known coding techniques.

To determine the channel capacity we employed the concept proposed by Ungerboeck in [8], although in a different context (for multiamplitude/multiphase signaling). To see how close we can approach those limits with coded repetition MIMO, we employed the low-density parity-check (LDPC) codes [9,10]. The LDPC codes are designed using the concept of pairwise balanced designs (PBDs) [11]. The LDPC coded data stream is mapped into a stream of Q-ary PPM symbols using the following two options: (i) bit-interleaved coded modulation (BICM) [5,12], and (ii) multilevel coding (MLC) [6,13]. Notice that coded repetition MIMO might not be the best option. Once more, the channel capacity and bit-error rates are determined assuming that p.i.n. photodiodes are used, and the channel is modeled using Gamma-Gamma distribution [4].

2. MIMO concept, channel description, and channel capacity

In order to achieve MIMO FSO transmission, M laser sources and N photodetectors are employed [2,3],[5,6], as shown in Fig. 1. The laser sources and photodetectors are positioned so that different transmitted symbols from different channels experience different atmospheric turbulence conditions. MIMO processing may be combined with space-time coding as explained in [14,17]; however, the FSO propagation physics will require new space-time code development and optimization because the wireless space-time codes do not satisfy the orhtogonality principle [15]. Each receiver in a FSO MIMO system, from Fig. 1(c), measures an *incoherent* superposition of the transmitted signals.

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For aggregation of RF/microwave channels and a conversion into optical domain we have recently studied two options: (i) BICM [5], and (ii) MLC [6] both with LDPC codes as component codes. LDPC coded data streams in either BICM or MLC were mapped into Q-ary PPM symbols. The BER performances were assessed assuming an ideal photon-counting receiver. For the completeness of the presentation, here we described only the BICM scheme from our recent paper [5]. To avoid the repetition, for MLC scheme description an interested reader is referred to [6].

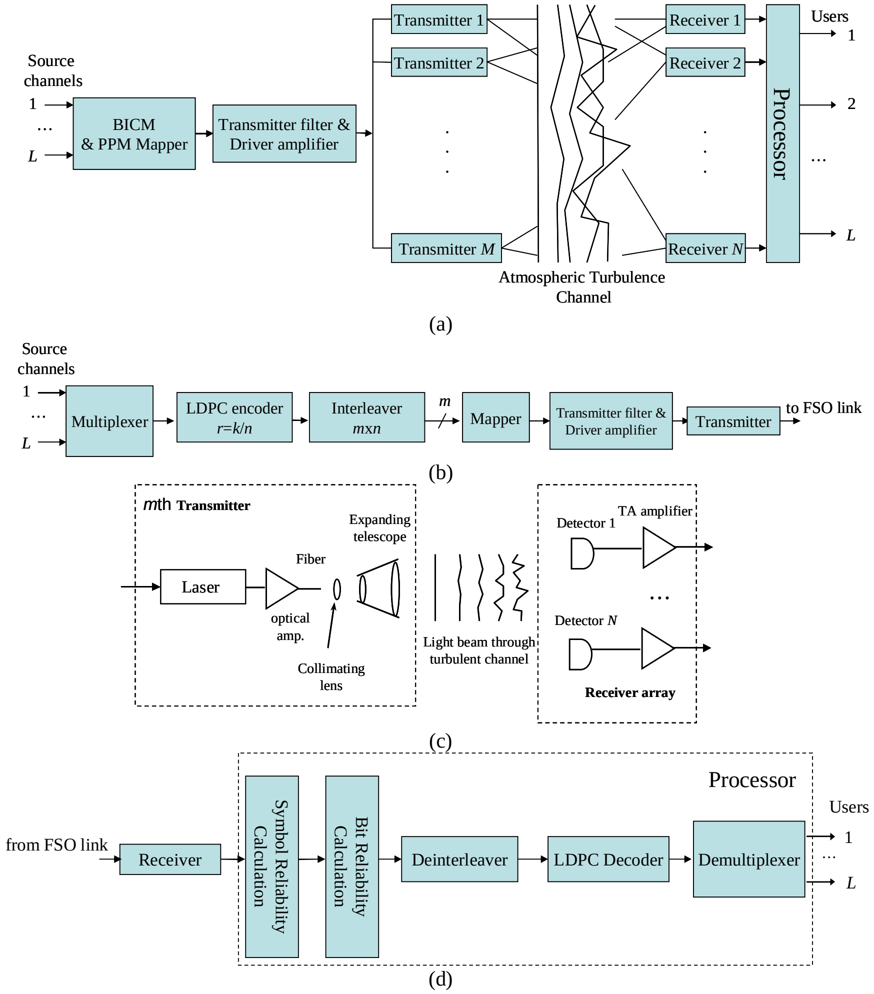

Fig. 1. (a) Atmospheric optical MLMD system with Q-ary PPM and BICM, (b) transmitter side, (c) *m*th transmitter – receiver array configuration, and (d) processor configuration.

The source bit streams coming from L RF/microwave sources are multiplexed together and encoded using an (n,k) LDPC code of code rate $r=k/n$ (k-the number of information bits, n-the codeword length). The $m\times n$ block-interleaver, collects m code-words written row-wise. The mapper accepts m bits at a time from the interleaver column-wise and determines the corresponding slot for Q-ary ($Q=2^m$) PPM signaling using a *Gray mapping* rule. With this BICM scheme, the neighboring information bits from the same source are allocated into different PPM symbols. In each signaling interval T_s a pulse of light of duration $T=T_s/Q$ is transmitted by a laser. (The signaling interval T_s is subdivided into Q slots of duration T.) The total transmitted power P_{tot} is fixed and independent of the number of lasers so that emitted power per laser is P_{tot}/M. This technique improves the tolerance to atmospheric turbulence, because different Q-ary PPM symbols experience different atmospheric turbulence conditions. The ith (i=1,2,…,M) laser modulated beam is projected toward the jth (j=1,2,…,N) receiver using the expanding telescope, and the receiver is implemented based on a p.i.n. photodetector in a trans-impedance amplifier (TA) configuration. Notice that the MLC scheme employs different (n,k_i) LDPC codes (k_i-dimensionality of ith component code), and it is able to carry $\sum k_i/n$ bits per symbol, which is generally smaller than m. Therefore the spectral efficiency (expressed in bits/symbol) of the BICM scheme is higher. Moreover, the BICM scheme employs only one LDPC code for all RF/microwave users, which simplifies the implementation. The use of only one LDPC code allows iterating between the *a posteriori* probability (APP) demapper and the LDPC decoder (we will call this step the *outer iteration*), further improving the BER performance.

The outputs of the N receivers in response to symbol q, denoted as $Z_{n,q}$ (n=1,2,…,N; q=1,2,…,Q), are processed to determine the symbol reliabilities $\lambda(q)$ (q=1,2,…,Q) by

$$\lambda(q) = -\frac{\sum_{n=1}^{N}\left(Z_{n,q} - \frac{\sqrt{E_s}}{M}\sum_{m=1}^{M} I_{n,m}\right)^2}{\sigma^2} - \frac{\sum_{n=1}^{N}\sum_{l=1,l\neq q}^{Q} Z_{n,l}}{\sigma^2}, \tag{1}$$

where E_s is the symbol energy of uncoded symbol in electrical domain (in the absence of scintillation), which is related to the bit energy E_b by $E_s=E_b\log_2 Q$. σ^2 is the variance of TA thermal noise (that is modeled as additive white Gaussian noise (AWGN)), and it is related to the double-side power spectral density N_0 by $\sigma^2=N_0/2$. With I_{nm} we denoted the intensity of the light incident to nth photodetector (n=1,2,…,N), originated from mth (m=1,2,…,M) laser source, which is described by the Gamma-Gamma probability density function (PDF) [4]

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right),\ I > 0. \tag{2}$$

I is the signal intensity, $\Gamma(\cdot)$ is the gamma function, and $K_{\alpha-\beta}(\cdot)$ is the modified Bessel function of the second kind and order $\alpha-\beta$. α and β are PDF parameters describing the scintillation experienced by plane waves, and in the case of zero-inner scale are given by [4]

$$\alpha = \frac{1}{\exp\left[\frac{0.49\sigma_R^2}{(1+1.11\sigma_R^{12/5})^{7/6}}\right]-1}, \quad \beta = \frac{1}{\exp\left[\frac{0.51\sigma_R^2}{(1+0.69\sigma_R^{12/5})^{5/6}}\right]-1}, \tag{3}$$

where σ_R^2 is the Rytov variance given by

$$\sigma_R^2 = 1.23\ C_n^2\ k^{7/6} L^{11/6}. \tag{4}$$

$k = 2\pi/\lambda$ is the optical wave number, L is propagation distance, and C_n^2 is the refractive index structure parameter, which we assume to be constant for horizontal paths.

The bit reliabilities $L(c_j)$, (j=1,2,…,m) (c_j is the jth bit in observed symbol q binary representation $\boldsymbol{c}=(c_1,c_2,\ldots,c_m)$) are determined from symbol reliabilities by

$$L\left(c_j\right) = \log\frac{\sum_{\boldsymbol{c}:c_j=0}\exp\left[\lambda(q)\right]}{\sum_{\boldsymbol{c}:c_j=1}\exp\left[\lambda(q)\right]}, \tag{5}$$

and forwarded to the LDPC decoder. Hard decision from the LDPC decoder are demultiplexed and delivered to different RF/microwave users.

After this general description of coded repetition MIMO scheme, in the rest of this Section we turn our attention to the calculation of channel capacity using the Eq. (5) in [8]. Assuming equally probable transmission, and organizing the receiver outputs in a form of matrix

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$\boldsymbol{Z}=(Z_{n,q})_{N\times Q}$ ($Z_{n,q}$-the nth receiver response to qth symbol) the channel capacity can be determined by

$$C=\log_2 Q-\underset{\bar{I}_n}{E}\ \underset{\boldsymbol{Z}|\bar{I}_n}{E}\left\{\log_2\sum_{q=1}^{Q}\exp\left[-\sum_{n=1}^{N}\frac{\left(Z_{n,q}-\bar{I}_n\sqrt{E_s}\right)^2-\left(Z_{n,q_0}-\bar{I}_n\sqrt{E_s}\right)^2}{2\sigma^2}\right]\right\}, \quad (6)$$

where $\bar{I}_n=\frac{1}{M}\sum_{m=1}^{M}I_{m,n}$, q_0 is an arbitrary symbol, and with E[·] we denoted the operator of ensemble averaging (other parameters are introduced earlier). Because we assumed equally probable transmission (Pr(q)=1/Q, q=1,..,Q), the averaging over different symbols in (6) will not affect the result. Notice that ensemble averaging is to be done for different channel conditions ($\bar{I}_n$) and for different thermal noise realizations ($\boldsymbol{Z}\,|\,\bar{I}_n$) by using Monte Carlo simulations, and taking into account the fact that conditional probability density function $p\left(Z_{n,q}\,|\,\bar{I}_n\right)$ is Gaussian

$$p\left(Z_{n,q}\,|\,\bar{I}_n\right)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{\left(Z_{n,q}-\bar{I}_n\right)^2}{2\sigma^2}\right]. \quad (7)$$

3. PBD based LDPC codes

In this Section, we turn our attention to the design of component LDPC codes to be used in either BICM or MLC scheme. The LDPC codes are designed using the concept of PBDs [11]. A pairwise balanced design, denoted as PBD(v,K,{0,1,…,λ}) is a collection of subsets (blocks) of a v-set V with a size of each block $k_i\in K$ ($k_i\leq v$), so that each pair of elements occurs in *at most* λ of the blocks. (Notice that we have relaxed the constraint in definition of PBD from [11] by replacing the word *exact* with *at most*.) For example, the following blocks: {1,6,9}, {2,7,10}, {3,8,11}, {4,12}, {5,13}, {1,7,11}, {2,8,12},{3,13}, {1,8,13}, {2,9}, {3,10}, {4,6,11}, {5, 7,12}, {1,10}, {2,11}, {3,6,12}, {4,7,13}, {5,8,9}, {1,12}, {2,6,13}, {3,7,9}, {4,8,10}, and {5,11} create an PBD(13,{2,3},{0,1}) (having 9 blocks of size 2, and 14 blocks of size 3), with parameter λ=1. By considering elements of blocks as position of ones in corresponding element-block incidence matrix, a parity-check matrix of an equivalent *irregular* LDPC code of girth-6 is obtained.

4. Numerical results

The results of channel capacity calculation in strong turbulence regime (σ_R=3.0, α=5.485, β=1.1156) for different number of lasers, photodetectors, and number of slots are given in Fig. 2. The significant spectral efficiency improvement is possible by using the multi-level schemes, such as Q-ary pulse position modulation.

The BER results of simulations for strong turbulence regime (σ_R=3.0, α=5.485, β=1.1156) are shown in Fig. 3, for different number of lasers, photodetectors and number of slots, by employing an (6419,4794) irregular girth-6 LDPC code of rate 0.747 designed using the concept of PBD introduced in Section 3. The MLC scheme with spectral efficiency of 2.241 bits/symbol combined with MLMD scheme employing 2 lasers and 4 photodetectors provides about 21dB improvement over LDPC coded binary PPM employing one laser and one photodetector. Corresponding BICM scheme of higher spectral efficiency (3bits/symbol) provides about 20dB improvement. The number of inner iteration in message-passing LDPC decoder is set to 25 in both schemes, while the number of outer iterations in BICM scheme is set to 10. MLC employs a parallel-independent LDPC decoding as explained in [6]. Therefore, the bit-interleaved LDPC-coded modulation scheme, although simpler to implement than MLC scheme, performs slightly worse in BER performance, but provides higher spectral efficiency. Although excellent coding gains are obtained, from Fig. 2 we can conclude that both schemes are still several dBs away from the theoretical limit. This suggests that repetition MIMO is not the best MIMO approach. We have recently shown in [17] that space-time coding [14,15] from wireless communication is not a channel capacity approaching technique either. In order to approach the channel capacity closer, novel space-time codes taking the underlying FSO physics into account are needed.

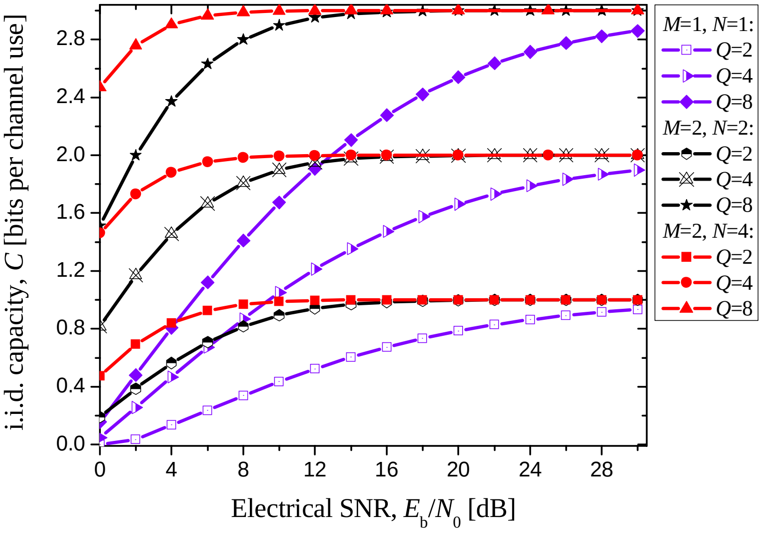

Fig. 2. Channel capacity for different number of lasers (M), photodetectors (N) and number of slots (Q) in strong turbulence regime (σ_R=3.0).

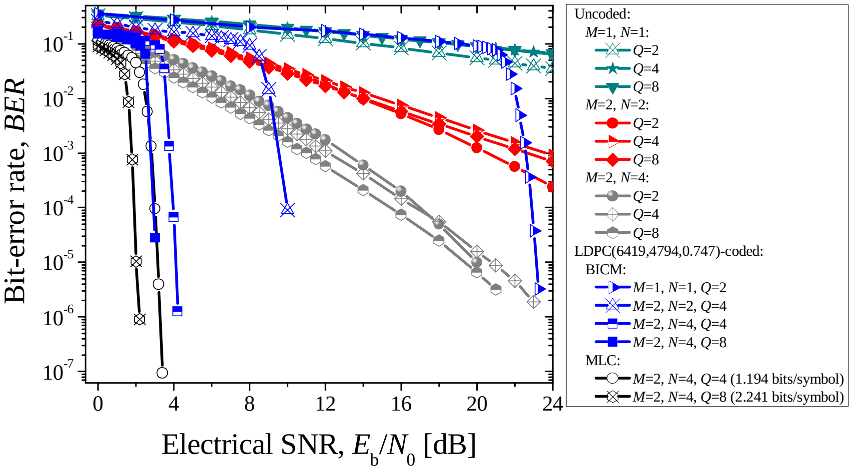

Fig. 3. BER performance of bit-interleaved LDPC-coded modulation against MLC for different MLMD configurations.

Notice that in simulation results reported here (in [2,3], and [5-7] as well) we assume that the channel is uncorrelated. In reality, at high bit rates the channel has temporal correlation and consecutive bits propagate through the similar channel conditions. Therefore, the channel capacities reported here can be considered as lower bound. This approach is valid when temporal correlation can be overcome by means of long interleavers. Unfortunately, the temporal correlation is difficult to simulate, especially under the strong turbulence regime, see [16] for more details about this problem.

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5. Conclusion

In this paper we describe a power-efficient coded repetition MIMO scheme suitable for communication over the atmospheric turbulence channel. We determine the channel capacity of this scheme assuming that p.i.n. photodetectors are employed, and evaluate how much we can approach this theoretical limit using the best known codes-LDPC codes. We have found that LDPC codes provide excellent coding gains, even 21 dB when 2 lasers and 4 photodetectors are used. Nevertheless we are still several dBs away from the Shannon limit. We have recently found in [17] that wireless space-time codes perform even worse than repetition MIMO. This suggests that novel space-time coding approaches are needed that take the physics of a free-space optical channel into account, which was left for future research.

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