Four-dimensional Nonbinary LDPC-Coded Modulation Schemes for Ultra High-Speed Optical Fiber Communication

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Abstract—In this letter, we propose a four-dimensional (4D) nonbinary LDPC-coded modulation (NB-LDPC-CM) scheme suitable for beyond 100 Gb/s optical fiber communication. Incorporating spectrally-efficient modulation formats to achieve high aggregate bit rates and nonbinary LDPC codes for forward error correction (FEC), the proposed scheme offers a superior advanced FEC solution for optical fiber communication systems than the prior-art bit-interleaved LDPC-coded modulation (BI-LDPC-CM) scheme. Compared to the previously reported BER performance results of BI-LDPC-CM, the proposed scheme offers additional net coding gains (NCGs) of 0.29 dB, 1.17 dB, and 2.17 dB at the BER of $10^{-7}$ when 16-, 32-, and 64-point 4D constellations are used, respectively.

Index Terms—Coded modulation, forward error correction, optical fiber communication, low-density parity-check codes.

I. INTRODUCTION

IEEE 802.3ba standard on 40/100 Gigabit Ethernet (40/100 GbE) was ratified in June 2010 as a response to the ever-increasing demands for higher capacity transmission over optical fiber links. Discussions over the next upgrade for Ethernet have already started. While some argue that 1 TeraBit Ethernet (TbE) should be standardized next to meet the projected demand, some advocate a more conservative upgrade to 400 GbE first. Regardless of whose projections will come true, all agree that 100 GbE is yet another station but not the final destination in the evolution of Ethernet.

As the operating symbol rates increase, the deteriorating effects of fiber nonlinearities and polarization-mode dispersion (PMD) reach levels that inhibit reliable communication over the optical fiber network. Thus solutions for 100 GbE and beyond need to attain ultra high transmission speeds in terms of aggregate bit rates while keeping the operating symbol rates low to facilitate nonlinearity and PMD management. A promising solution employing coded modulation using low-density parity-check (LDPC) codes as component codes has already been discussed in our previous works [1], [2]. The underlying idea is to use spectrally-efficient modulation formats at low symbol rates along with strong LDPC codes for forward error correction (FEC) in order to accomplish reliable communication at high aggregate bit rates.

In addition to spectral efficiency, power efficiency of a modulation format plays an important role in communication system design. Coherent optical communication physically allows using four dimensions for modulation rather than only two dimensions over which conventional modulation formats, e.g., quadrature amplitude modulation, are defined. As a result, one can exploit this four-dimensional (4D) signal space to set up more power-efficient signal constellations than one could do using the 2D signal space—by increasing the Euclidean distance between constellation points for a given average signal power [3]. 4D bit-interleaved LDPC-coded modulation (4D BI-LDPC-CM) employing binary LDPC codes for FEC was discussed in [2]. In this paper, we employ nonbinary LDPC codes for FEC and show that the proposed 4D nonbinary LDPC-coded modulation (4D NB-LDPC-CM) scheme can provide larger coding gains than the corresponding 4D BI-LDPC-CM scheme. Furthermore, the proposed 4D NB-LDPC-CM scheme can reduce receiver latency by avoiding costly “turbo-like” iterations between the detection and decoding units, which is essential for acceptable error correction performance in any BI-LDPC-CM scheme.

The remainder of the paper is organized as follows. In Section II, we introduce the proposed 4D NB-LDPC-CM scheme and highlight its differences from the corresponding prior-art 4D BI-LDPC-CM scheme. Section III presents our results and discussion. We conclude our paper in Section IV.

II. FOUR-DIMENSIONAL NONBINARY LDPC-CODED MODULATION

Fig. 1 depicts the transmitter and receiver configurations for the proposed 4D NB-LDPC-CM scheme. Compared to prior-art 4D BI-LDPC-CM scheme presented in [2], we observe the following differences: 1) the set of $m$ binary LDPC encoders are replaced by a single 2$^m$-ary LDPC encoder, 2) the block interleaver unit, which distributes $N$ codeword bits of each binary LDPC encoder used in BI-LDPC-CM over all of $N$ transmitted symbols [2], is eliminated, 3) the feedback loop into the maximum a posteriori probability (MAP) detector from the decoding unit is eliminated, and 4) symbol-to-bit and
vice versa conversion interfaces are eliminated since the nonbinary decoder operates at the symbol level. We should highlight item 3 further. BI-LDPC-CM requires iterative detection and decoding for good performance [4]. Since both the MAP detection process and \( m \) parallel LDPC decoding processes contribute to computational complexity and latency of the receiver, multiple soft information exchanges between the detector and the decoding units can very fast cause large complexity and latency figures at the receiving ends. Eliminating this feedback loop, the proposed 4D NB-LDPC-CM scheme can help lower the latency and computational complexity. Moreover, as we will present in Section III, the proposed scheme improves the BER performance.

A 2\(^m\)-ary, \( m > 1 \), LDPC(N,K) code is an LDPC code of code rate \( R = K/N \) defined over the finite field, or the Galois field of, order \( 2^m \), denoted by GF(\( 2^m \)). Since every 2\(^m\)-ary symbol can be represented as a binary vector of length \( m \), we can call \( m \) parallel binary source channels as a single 2\(^m\)-ary channel as in Fig. 1. In NB-LDPC-CM, an incoming \( K \)-symbol-long input block from a 2\(^m\)-ary source channel is encoded into a 2\(^m\)-ary LDPC codeword of length \( N \). The mapper then maps each 2\(^m\)-ary symbol to a point in a signal constellation comprised of 2\(^m\) points. Note that the order of the field over which the component nonbinary LDPC code is designed and the size of the signal constellation are both equal to 2\(^m\). Since they are equal, there is no need for iterative detection and decoding operation in NB-LDPC-CM as mentioned previously. The mapper outputs are then modulated via a 4D optical modulator whose internal structure is depicted in Fig. 2.

At the receiver, the coherent detector outputs on the two quadratures of the two polarizations are passed to a MAP detector. The MAP detector, which implements the multilevel extension of the well-known Bahl-Cocke-Jelinek-Raviv algorithm as detailed in [5], produces the log-likelihood ratio (LLR) matrix of length \( N \times 2^m \). Each row \( k \), \( 0 \leq k < N \), of the LLR matrix is comprised of 2\(^m\) entries each corresponding to

\[
\lambda(s_k^{(a)}) = \log \left[ \frac{P(s_k^{(a)} | r)}{P(s_k^{(0)} | r)} \right],
\]

i.e., the logarithm of the ratio of the conditional probability that the \( k \)-th symbol sent by the transmitter being the 4D constellation point \( s_k^{(a)} \) corresponding to the 2\(^m\)-ary symbol \( a \in \text{GF}(2^m) = \{0, 1, \ldots, 2^m - 1\} \) over the probability that it is

Fig. 1. (a) Transmitter and (b) receiver configurations of a system using 4D NB-LDPC-CM scheme.

Fig. 2. Internal structure of a 4D modulator.
OSNR<sub>m</sub>.) There are three constellation sizes considered, namely 16, 32, and 64. In order to compare against the 4D BI-LDPC-CM performance curves presented previously in [2], we use the same signal constellations. Using Fig. 3, we computed the net coding gains (NCGs) for NB-LDPC-CM at the BER of $10^{-7}$ as 8.52 dB, 8.21 dB, and 9.14 dB for 16-, 32-, and 64-point constellations, respectively. As Fig. 3 shows, the proposed scheme outperforms its binary counterpart at all constellation sizes. To elaborate, at the BER of $10^{-7}$, 4D NB-LDPC-CM outperforms 4D BI-LDPC-CM by 0.29 dB, 1.17 dB, and 2.17 dB in terms of net coding gain (NCG) when 16-, 32-, and 64-point constellations are used, respectively. As the slopes of the performance curves indicate, the gaps between nonbinary and the corresponding binary curves are expected to increase for lower BERs such as $10^{-12}$ and $10^{-15}$, signifying larger NCGs and larger additional coding gains at lower BERs when using the proposed NB-LDPC-CM scheme. We should note that any 4D constellations other than the ones in [2] can be used. For example, the optimum constellations presented by Sloane et al. [8] may provide an additional 0.5 dB coding gain at the BER of $10^{-7}$ at the expense of increased implementation complexity since the optimum constellation points require 12-digit accuracy, which is much larger compared to accuracy required by the 4D regular polytopes used in [2]. Since the code rate is fixed at 0.8, the aggregate information bit rate reaches 160 Gb/s, 200 Gb/s, and 240 Gb/s for 16-, 32-, and 64-point constellations, respectively. At these beyond 100 Gb/s transmission rates, NB-LDPC-CM stands out as a stronger advanced FEC scheme.

As a final remark, we observe that additional NCGs provided by the proposed 4D NB-LDPC-CM scheme over its binary counterpart increases as the underlying constellation size increases. This stems from two reasons. First one is the suboptimality of BI-LDPC-CM [4]. In BI-LDPC-CM, m bits taken from the outputs of m parallel LDPC encoders are mapped to the same transmitted symbol, which forms a dependency between these bits. At the receiver side, however, these dependent bits are decoded independently from one another. To exploit the dependency and to lessen the effects of suboptimality, iterative detection and decoding is employed as alluded to in Section II. As the constellations, and hence the number of bits per symbol, i.e. m, grow, the effect of suboptimality gets harder to compensate for. The second reason is related to the “adaptive” nature of NB-LDPC-CM. To elucidate, NB-LDPC-CM matches (or “adapts” the size of the field order to the constellation size; thus, establishes a one-to-one correspondence between $2^m$-ary coded bits and the $2^n$-ary constellation points. A $2^m$-ary LDPC decoder provides its variable nodes and check nodes with a $2^n$-ary state space during decoding. (BI-LDPC-CM provides the binary state space to its variable and check nodes regardless of the number of bits per symbol used during transmission.) In other words, NB-LDPC-CM counteracts the growth in the constellation size by increasing the state space of the nodes used in decoding. As a result, NB-LDPC-CM scheme is able to provide increasing performance margins over the corresponding BI-LDPC-CM scheme as the constellations get larger.

IV. CONCLUSION

We proposed a nonbinary LDPC-coded modulation scheme, and showed its superior performance over the prior-art BI-LDPC-CM scheme. The superiority of the proposed scheme becomes more pronounced as the underlying constellation sizes and hence the aggregate information bit rates increase. Also, by omitting the iterative detection and decoding loops, which are crucial for good performance in BI-LDPC-CM, the proposed scheme can lower latency and computational complexity at receivers. With these important advantages, we believe the proposed scheme is an outstanding candidate for advanced FEC in future optical fiber communication systems.

REFERENCES