Suppression of Intrachannel Nonlinear Effects Using Pseudoternary Constrained Codes

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***Abstract*—In this paper, a novel approach for suppressing the intrachannel nonlinear effects using pseudoternary constrained codes is proposed. A significant Q-factor improvement of up to 9.75 dB is obtained. The eye opening penalty (EOP) is also significantly improved by more than 10.24 dB.**

***Index Terms*—Constrained codes, intrachannel fiber nonlinearities, optical communications.**

I. Introduction

In high-speed transmission (at 40 Gb/s and above), the major nonlinear penalties are due to intrachannel interactions, such as intrachannel four-wave-mixing (IFWM) and intrachannel cross-phase modulation (IXPM), rather than due to interchannel interactions [four-wave mixing (FWM) and cross-phase modulation (XPM)] [1]–[11]. IXPM is caused by the modulation of a pulse phase by nonlinear interaction with neighboring pulses within the channel [11], resulting in timing jitter. In IFWM, at sufficiently high dispersion, energy is transferred to the middle of a neighboring bit slot [1], causing either a ghost (fictitious) pulse in an empty bit slot or an amplitude jitter in a nonempty bit slot.

Intrachannel interactions may be reduced by proper dispersion map design [1], [9], but no method exists for complete cancellation of the intrachannel nonlinearities. A common approach used to tackle the problem of suppressing IFWM is to find a proper modulation format. As the creation of ghost pulses during IFWM interaction is a phase-sensitive effect, these solutions aim to reduce ghost pulses by removing phase coherence in the pulses emitted by the optical transmitter in a given neighborhood [2], [5], [7], [10].

A rather different approach based on constrained codes (also known as modulation codes or line codes) is proposed in our recent paper [11]. The key idea behind constrained codes is to deter the bit patterns that cause a ghost pulse effect. The constrained code we proposed [11] was binary, and in order to further improve system performance, we investigated codes with larger alphabets. In this paper, we present four pseudoternary codes with a very good performance: the pseudoternary code of rate 0.78 successfully removing asymmetric patterns like "1101" and "1011"; the pseudoternary code of rate 0.76, which in addition to removing the asymmetric patterns restricts the number of successive zeros to six; the pseudoternary code of rate 0.83; and the pseudoternary block code of rate 0.8. These codes provide significant performance improvement, as shown later in the text: up to 9.75 dB in Q-factor and more than 10.24 dB in eye opening penalty (EOP) improvement. (Since 1 and -1 in direct detection are recognized as a symbol "1," the constrained codes proposed here are called pseudoternary codes.)

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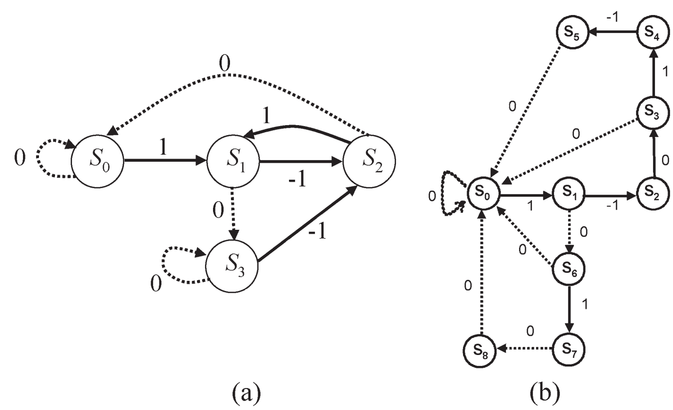
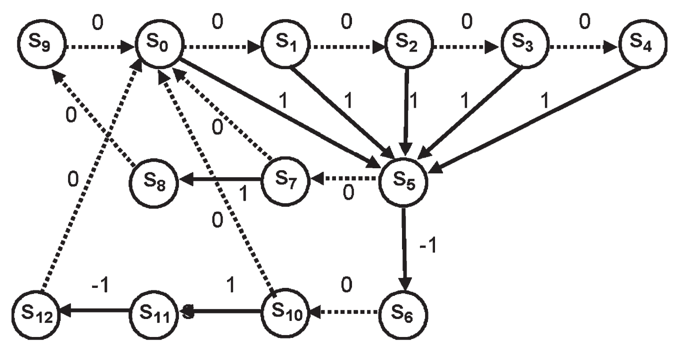


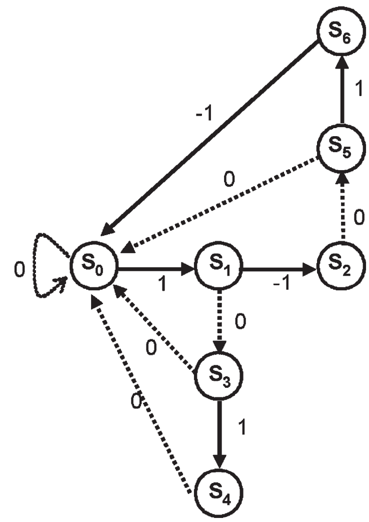
Fig. 1. Directed graph model of (a) AMI and (b) pseudoternary constrained code of rate 0.78.

II. Pseudoternary Constrained Codes

It was recently pointed out [10] that the double resonance pattern "11011," which is one of the most important patterns in the process of ghost pulse creation [11], does not contribute at all to ghost pulse creation in duobinary and alternate mark inversion (AMI) optical transmission. However, asymmetric patterns like "1101" and "1011" and long sequences of ones do not cancel each other in zero-bit positions, hence giving rise to a ghost pulse. Duobinary and AMI may be considered as trivial constrained codes of memory one and code rate one. For example, AMI may be described by the directed graph model as shown in Fig. 1(a). Graph states are associated with various types of sequences generated in the past. For example, the state S_2 is reached after two consecutive ones (the first sent with phase 0 and the second with phase π radians). Valid sequences can be obtained by reading off the edge labels while making transitions from one state to another. In AMI code, the contributions to the ghost pulse at zero-bit location in sequence "1,-1,0,1,-1" are symmetric and cancel



(a)



(b)

Fig. 2. (a) Pseudoternary constrained code of rate 0.76 and RLL constraint 6. (b) Directed graph model of pseudoternary constrained code of rate 0.83.

each other. However, in asymmetric sequences like "1,0,−1,1," "1,−1,0,1," "1,0,−1,1,−1," "1,−1,0,1,−1,1,−1," etc., there is no symmetry of ones around zero-bit position, and ones in the so-called resonant positions [11] contribute to the ghost pulse creation.

By keeping the symmetry of ones around the zero-bit positions and providing the alternate change of phase in ones, the contributions to ghost pulse creation completely cancel each other at a zero-bit position. The pseudoternary constrained code satisfying such a property is shown in Fig. 1(b). The symmetry is imposed by a deliberate addition of a single pulse on the left or right side of the "asymmetric" sequence (e.g., the sequence "1,−1,0,1" is translated into "1,−1,0,1,−1"). By imposing such a constraint, the code rate is reduced down to 0.78, but significant performance improvement is obtained as illustrated in Figs. 4–6. To further improve system performance, the symmetry should be maintained throughout the whole sequence. Unfortunately, such a pseudoternary constrained code will have an unacceptably low code rate (the interested reader is referred to [12] for the mathematical proof). For timing recovery, the number of successive zeros k should be limited. Such kind of constraint is well known as runlength limited (RLL) constraint [15]. The pseudoternary constrained code of code rate 0.76, with RLL constraint 6, is proposed in Fig. 2(a). For larger RLL constraints, the code rate gradually increases up to 0.78. To keep the code rate reasonably high, some asymmetry in ternary modulation code design must be tolerated. One such pseudoternary code of rate 0.83 is proposed in Fig. 2(b), in which the appearance of asymmetric sequence "1011" is not completely eliminated.

In the next section, a little different approach is proposed. Instead of deliberately adding ones around the asymmetric sequence, the pseudoternary sequences are generated in a way that the ghost pulse creation process is eliminated in any subsequence of given length.

III. Rate 4/5 Pseudoternary Nonlinear Block Code

To keep the code rate reasonably high and the encoder complexity reasonably low, we propose a nonlinear pseudoternary block code of rate 4/5 $(= 0.8)$ in which any sequence of length 5, $c_i c_{i+1} c_{i+2} c_{i+3} c_{i+4}$, satisfies the following constraint: If for $k, l, m \in [i, i+4]$ and $k + l - m \in [i, i+4]$, k and l not necessarily distinct, $c_{\mathrm{k}} = c_{\mathrm{l}} = c_{\mathrm{m}} = 1$ or $c_k = c_l = c_m = -1$, then $c_{k+l-m} \neq 0$. Such a constraint ensures that the 0s in resonant positions [11] are modified to 1 or -1, thereby mitigating ghost pulse creation.

The encoder is described by a lookup table (Table I, generated by searching for allowable sequences), mapping the 4-bit binary input to a corresponding codeword of length 5. Any nonzero 4-bit sequence can be mapped to more than one codeword since the codewords with entries differing just in phase are considered as the same codeword upon photodetection (e.g., from receiver point of view the codewords 00001 and 0000-1 are identical). Such flexibility allows the selection of codewords so that the ghost pulse creation is eliminated in any subsequence of 5 bits. The capacity of the constraint is 0.96; therefore, a similar Q-factor improvement can be obtained by designing of more complex code of higher code rate.

IV. Numerical Results

The simulations were run on a realistic dispersion-managed 40 Gb/s single-channel model. Since the focus of this paper is on the suppression of intrachannel nonlinearities, to demonstrate the capability of pseudoternary codes to reduce IFWM and IXPM, the ASE noise was ignored in Figs. 4–6, but, for completeness of discussion, included in Fig. 7. The effects of Kerr nonlinearities (self-phase modulation, IXPM, IFWM), stimulated Raman scattering (SRS), dispersion (group velocity dispersion, GVD), second-order GVD, crosstalk, and intersymbol interference were taken into account. For light propagation through the fiber, the nonlinear Schrödinger equation was solved using the split step Fourier method [13], [14].

The dispersion map, shown in Fig. 3, is composed of N (25–60) spans of length $L = 48$ km, each span consisting of $2L/3$ km of D_+ fiber followed by $L/3$ km of D_- fiber. The fiber parameters are as follows: D_+ fiber: dispersion of 20 ps/(nm km), dispersion slope of 0.06 $\mathrm{ps/(nm^2\ km)}$, effective area equal to 110 $\mu\mathrm{m}^2$, and loss equal to 0.19 dB/km.

TABLE I
RATE 4/5 NONLINEAR PSEUDOTERNARY BLOCK CODE

Input	*Codewords*			
0000	00000			
0001	00001	0000-1		
0010	00010	000-10		
0011	00100	00-100		
0100	01000	0-1000		
0101	10000	-10000		
0110	10001	-1000-1	1000-1	-10001
0111	10010	-100-10	100-10	-10010
1000	01001	0-100-1	0100-1	0-1001
1001	10-101	-1010-1		
1010	10-100	-10100		
1011	00-101	0010-1		
1100	00-110	001-10		
1101	01-100	0-1100		
1110	10-110	-101-10		
1111	01-101	0-110-1		

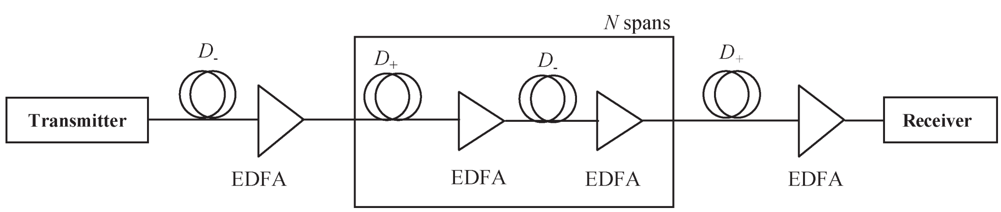


Fig. 3. Dispersion map under study.

$D-$ fiber: dispersion of -40 ps/(nm km), dispersion slope of -0.12 ps/(nm^2 km), effective area equal to 30 μm^2, and loss equal to 0.25 dB/km. The nonlinear Kerr coefficient is 2.6×10^{-20} m^2/W. The precompensation of -320 ps/nm and corresponding postcompensation are also used. Twenty five to 60 spans of this map are observed, resulting in total length from 1200 to 2880 km. The erbium-doped fiber amplifiers (EDFAs) are deployed after every fiber section. The simulations were carried out with an average launched power of 0 dBm and a central wavelength of 1552.524 nm. An additional phase modulator is required to provide -1 modulation level.

The results of simulations are given in Figs. 4–7 (and are in excellent agreement with VPItransmissionMaker WDM 5.5). The proposed constrained codes are compared against the return to zero (RZ) modulation format (of duty cycle 33%). The pattern sequence used in simulations is of length $2^{15} - 1$. From Fig. 4, it is evident that the pseudoternary codes are successful in suppressing both IFWM (ghost pulse and amplitude jitter) and IXPM (timing jitter). After 60 spans, the eye diagram in RZ modulation format is completely closed, and the EOP tends to infinity while the eye diagrams of pseudoternary constrained codes are widely open.

The Q-factor improvement against the number of spans is given in Fig. 5 and the EOP improvement in Fig. 6. The influence of overhead is included in the calculations. Q-factor improvement is defined as

$$\Delta Q = 20 \log \left[\frac{Q_{\text{encoded}}}{Q_{\text{uncoded}}}\right] [\text{dB}] \tag{1}$$

where Q_{uncoded} is the calculated at bit rate R_b, and Q_{encoded} at line rate R_b/R, with R being the code rate. The EOP improvement is defined as

$$\Delta\text{EOP} = -[\text{EOP}_{\text{encoded}} - \text{EOP}_{\text{uncoded}}][\text{dB}]. \tag{2}$$

In (2), the EOP [1], [9] (in decibels) is defined by adapting 0.2 unit interval wide rectangular (20% of the bit period) of the maximum possible height h that can be fitted in the inner electrical eye diagram of the normalized signal; EOP $= -10\log(h/h_0)$ [dB], where h_0 is the maximum height of the rectangular in back-to-back configuration. [Similarly, as in (1), $\text{EOP}_{\text{encoded}}$ is calculated at line rate and $\text{EOP}_{\text{uncoded}}$ at bit rate.] The Q-factor and the EOP are calculated after optical filtering of bandwidth 80 GHz, photodetection, and electrical filtering of bandwidth 26 GHz. From Fig. 5, it is evident that even the limited presence of "1011" sequence may result in about 5-dB loss in Q-factor improvement. Fig. 5 shows that there exists an optimum number of spans (or equivalently optimum optical signal-to-noise ratio) for each particular code, and the optimum value is achieved around the point where an uncoded eye diagram is completely closed, suggesting that the EOP is not a good indicator of system performance.

For completeness of discussion, the proposed pseudoternary codes are also compared against RZ for the case when the ASE–noise interaction during transmission is included. The comparison in terms of Q-factor improvement is shown in Fig. 7. The noise figure (NF) of the corresponding EDFAs (deployed as described earlier in the text) is set to 6 dB.

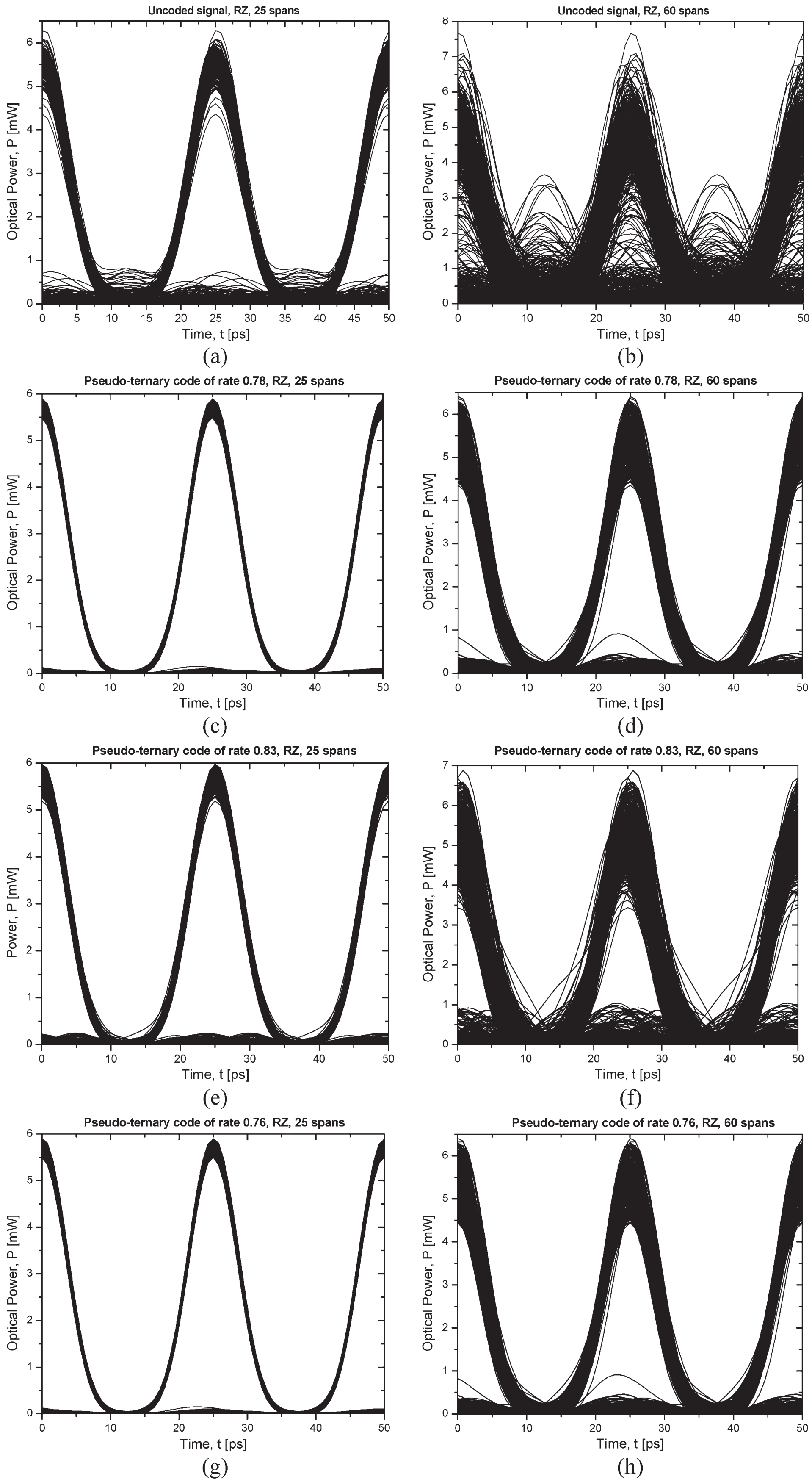


Fig. 4. Eye diagrams after 1200 km (left column) and 2880 km (right column) for an average power of 0 dBm with a precompensation of −320 ps/nm and corresponding postcompensation. (a) and (b) RZ format: uncoded signal eye diagrams. (c) and (d) Pseudoternary constrained code of rate 0.78. (e) and (f) Pseudoternary constrained code of rate 0.83. (g) and (h) Pseudoternary constrained code of rate 0.76.

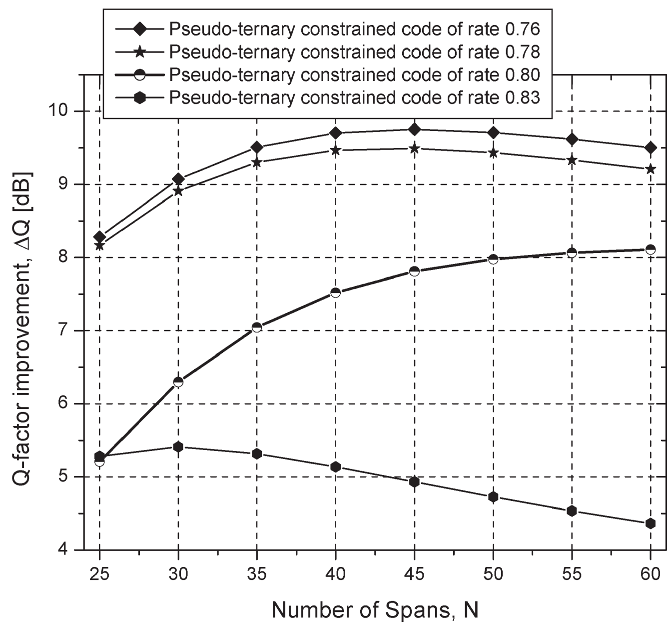


Fig. 5. Q-factor improvement of pseudoternary codes over RZ for different number of spans in the absence of ASE noise.

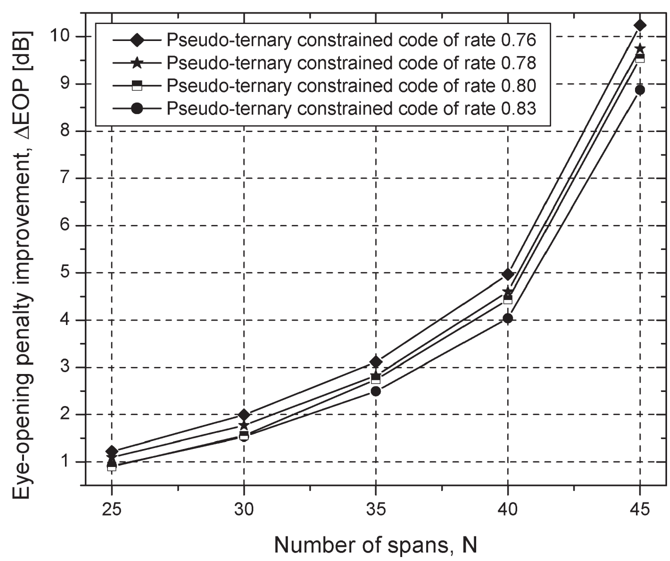


Fig. 6. EOP improvement of pseudoternary codes over RZ.

V. Conclusion

In conclusion, the use of pseudoternary constrained codes to counter the effects of IFWM and IXPM is proposed. Significant Q-factor improvement of up to 9.75 dB and significant EOP improvement of more than 10.24 dB, depending on code rate and number of spans, are demonstrated. At 40 Gb/s and above, constrained codes can significantly improve the transmission distance and system capacity. The constrained codes are capable of improving the FEC threshold in systems with severely degraded performance due to intrachannel nonlinearities. Moreover, at 40 Gb/s, distances of several thousand kilometers may be reached without employing any FEC. Notice that the complexity of the constrained encoder/decoder [15] is significantly simpler than any state of the art FEC scheme employed in long-haul optical transmission.

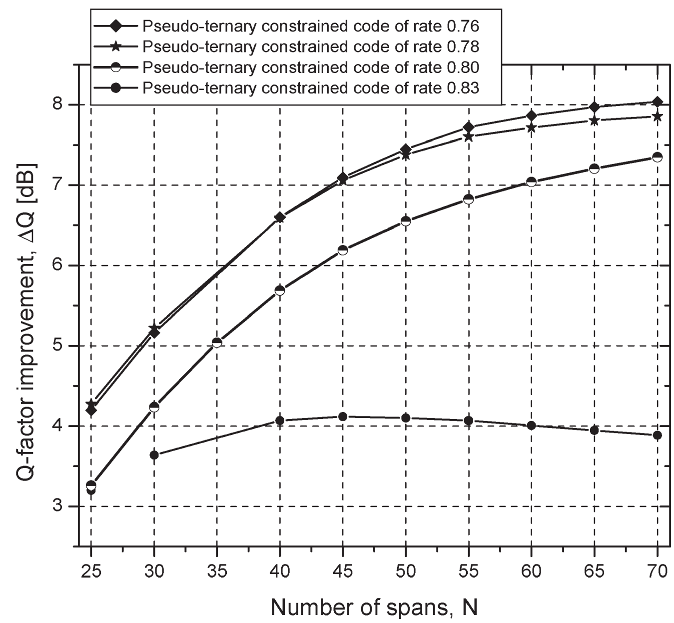


Fig. 7. Q-factor improvement of pseudoternary codes over RZ for different number of spans in the presence of ASE noise.

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