# Multiple order coded aperture spectrometer

**S. D. Feller, Haojun Chen, and D. J. Brady**
*Duke University Fitzpatrick Institute for Photonics*
*Box 90291*
*Durham, NC 27708*
*sfeller@duke.edu*

**M. E. Gehm**
*University of Arizona ECE Department*
*Tucson, AZ 85721*
*gehm@ece.arizona.edu*

**Chaoray Hsieh, Omid Momtahan, and Ali Adibi**
*School of Electrical and Computer Engineering*
*Georgia Institute of Technology*
*Atlanta, GA 30332-0250*
*adibi@ece.gatech.edu*

**Abstract:** We introduce a multiple order coded aperture (MOCA) spectrometer. The MOCA is a system that uses a multiplex hologram and a coded aperture to increase the spectral range and throughput of the system over conventional spectrometers while maintaining spectral resolution. This results in an order of magnitude reduction in system volume with no loss in resolution.

**OCIS codes:** 090.1970, 300.6190

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## References and links

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## 1. Introduction

Spectroscopy is commonly used in many applications to determine the chemical composition of materials. While high resolution spectrometers are traditionally located in the laboratory, the proliferation of small form factor computing elements has led to increasingly compact and inexpensive spectrometers for portable and distributed sensing applications.

A traditional spectrometer consists of a slit aperture, a diffraction grating and a detector in the image plane of the slit along with relay optics. Light that passes through the aperture

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undergoes angular dispersion as a function of wavelength at the grating, producing a spatial-spectral response at the detector. In modern systems where the image plane is typically a digital detector array such as a CCD, the diffraction limited spot size is small relative to the area of a pixel which becomes the limiting factor in system resolution. Since the response on each row is nearly identical due to the vertical nature of the slit, the spectral range of the system is determined by wavelengths that are incident on the detector and the spectral resolution is approximately the number of detector elements in a row along the dispersion direction of the detector. This is shown in Eqs. 1 and 2 where $N$ is the number of channels, $L$ is the width of the CCD, $\Delta p$ is the width of the individual pixels, and $\Delta\lambda$ is the range of wavelengths incident upon the image plane.

$$\delta\lambda = \frac{\Delta p \Delta\lambda}{L} = \frac{\Delta\lambda}{N} \tag{1}$$

$$N = \frac{L}{\Delta p} \tag{2}$$

An improved sampling approach can increase the spectral resolution or the spectral range of the system by implementing a better mapping between the spectral channels of the source and the detector elements. This can be achieved by folding multiple spatially discrete spectral projections onto different rows on the CCD. In an ideal case where each row receives a non-overlapping spectral projection from the source, a one-to-one mapping between spectral states and detector elements is achieved. The spectral resolution and spectral range for this ideal case is shown in Eqs. 3 and 4 where $\Delta p^2$ represents the area of a pixel and $L^2$ represents the area of the CCD.

$$\delta\lambda = \frac{\Delta p^2 \Delta\lambda}{L^2} = \frac{\Delta\lambda}{N} \tag{3}$$

$$N = \frac{L^2}{\Delta p^2} \tag{4}$$

To directly implement a one-to-one mapping between the spectral channels of the source and the detector elements on the CCD the spatial extent of the aperture must be reduced to a pinhole to prevent spectral channel overlap. This reduction in the area of the aperture limits the amount of light available for detection and adversely affects system sensitivity. This is a common problem in spectroscopy applications that can be overcome by introducing a coded aperture at the input that can be computationally inverted to generate a unique response for each spectral channel [4].

The Multiple Order Coded Aperture (MOCA) spectrometer uses a hologram of three overlapping diffraction gratings recorded with a small angular shift along the vertical axis. Each grating is optimized for a different spectral range, producing shifted or scaled spectral projections onto the image plane. The number of diffractive elements recorded in the hologram can be increased to achieve the ideal mapping discussed above. In this paper we discuss the design of the MOCA and demonstrate the effectiveness of this approach for developing micro spectrometers.

## 2. System design

The MOCA is a 4-f optical system based on our earlier Dispersion Multiplexed Spectrometer (DMS) [1] with the optical design shown in Fig. 2. The input aperture is an order 37 Modified Uniformly Redundant Array (MURA)[2] with 28 micron square holes and 2.072 square millimeters in total area. The aperture is imaged onto the holographic grating using 9mm diameter

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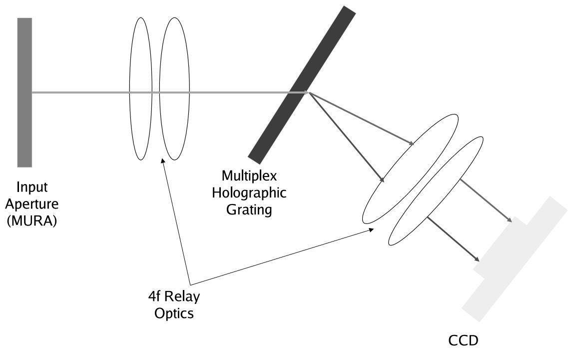

Fig. 1. MOCA Optical Design

relay lenses in a Petzval configuration. The output from the hologram is then imaged onto the detector array using the second set of matched relay lenses. The relay optics are optimized for low distortion using Zemax software. A monochrome Sony CCD with 5.6 micron pixel spacing embedded in an Unibrain Fire-i digital board camera is located at the detector plane. The total optical path for the system from the aperture to the CCD is 65 millimeters.

The dispersive element in the MOCA system is a hologram of three diffraction gratings. The gratings were designed to have center wavelengths of 675nm, 555nm, and 465nm respectively so that each grating would generate a unique spectral response on the CCD. The gratings are recorded with slight vertical tilts of $-2.5^\circ$,$0^\circ$, and $2.5^\circ$ to provide a vertical offset in their responses on the CCD, making it possible to disambiguate between each projection. The method used to generate the hologram is similar to the one used to generate the DMS spectrometer.

The MOCA is designed to produce shifted copies of the input aperture on the CCD based on the spectral content of the source. For a single wavelength, there will be up to three copies of the aperture, one for each grating recorded in the hologram as shown in the response of a 532nm laser in Fig. 2. Each input wavelength produces a unique response due to the horizontal dispersion of each grating. In the spectral range of interest, a 1 nm shift in the wavelength at the source results in an approximate four pixel shift in the response on the CCD for each grating. For polychromatic sources, a copy of the aperture for each spectral channel in the source and each grating is imaged onto the detector.

## 3. Inversion algorithm

The system response consists of multiple overlapping copies of the aperture mask that are shifted as a function of wavelength and scaled as a function of spectral intensity. This measured image must be inverted to extract the spectral content of the source. This is achieved using an Expectation-Maximization technique based on Richardson-Lucy deconvolution [3].

This expectation-maximization algorithm is a two-step iterative process that finds the maximum likelihood estimate between a measured image and a set of image parameters. In the case of the MOCA, the image parameters are the spectral response of the system for a monochromatic source at each wavelength across the spectral range of interest. The first step of the algorithm predicts the system response by scaling each wavelength based image response by its previously estimated intensity. This is shown in Eq. 5 where $I$ is the predicted system re-

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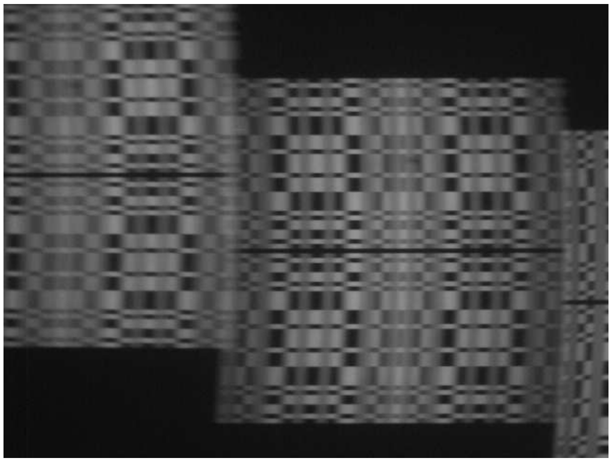

Fig. 2. CCD response of a 532 nm monochromatic source

sponse, $C_\lambda$ represents calibration images at wavelength $\lambda$, and $\hat{s}$ is the current spectral estimate. The second step updates the spectral estimate to minimize the difference between the estimated system response and the measured system response as shown in Eq. 6 where $M$ is the measured image and the $\bullet$ operator represents point-wise multiplication. The process repeats with the estimated intensity of the previous iteration being used as the initial guess in the next. The inversion process continues until the difference between the measured response and the estimated response becomes sufficiently small or a maximum number of iterations is reached.

$$I = \sum_{\lambda} C_{\lambda,i,j} \hat{s}_{m_\lambda} \tag{5}$$

$$\hat{s}_{m+1_\lambda} = \hat{s}_{m_\lambda} \sum C_{\lambda,i,j} \bullet \frac{M_{i,j}}{I_{i,j}} \tag{6}$$

The resolution of the system is determined by the number of unique image parameters used in the EM algorithm. The MOCA has been used to estimate the spectrum of a source in increments as small as .25nm in a range from 350nm to 800nm using this approach.

## 4. Experimental results

The system response of the MOCA was measured at one nanometer increments in the range from 350nm to 800nm. This results in a horizontal shift of approximately 4 pixels across all of the gratings for a 1nm change in wavelength making it possible to interpolate between calibration images to estimate the system response at every .25 nm. To ensure correct output from the monochromator, the spectral response of a 532.2nm green laser and a 633.2nm red laser were used as a reference as shown in Fig. 3. The reconstructed spectra of both sources have very sharp peaks at the expected wavelengths and were used to verify that the alignment and scaling of the calibration data is correct. The small peaks to the right of the signals are artifacts of the reconstruction process and would go away with further iterations.

To verify the accuracy of the MOCA the spectrum of a neon pen lamp and a typical phosphor-fluorescent lamp were measured with the MOCA and compared with the spectrum measured with an Ocean Optics USB 2000 spectrometer. A neon lamp spectrum has many sharp peaks

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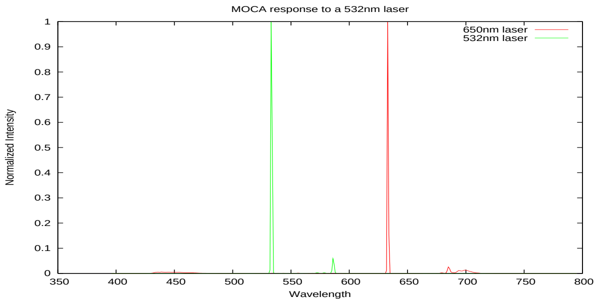


Fig. 3. Overlayed spectrum of a 532nm and a 632nm laser measured with the MOCA.

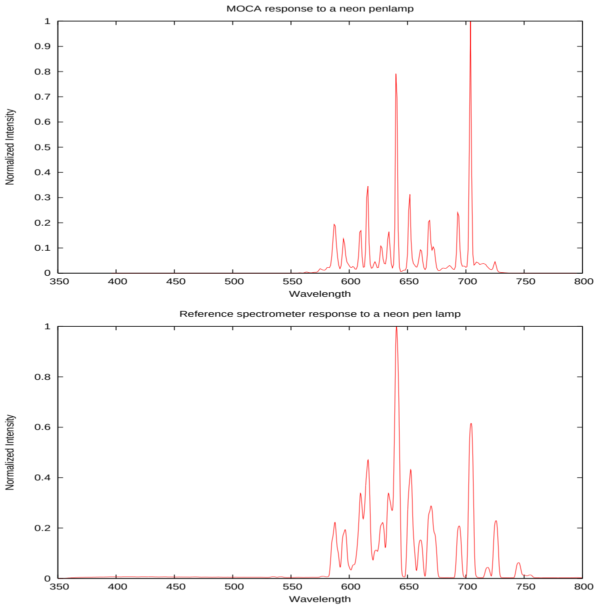
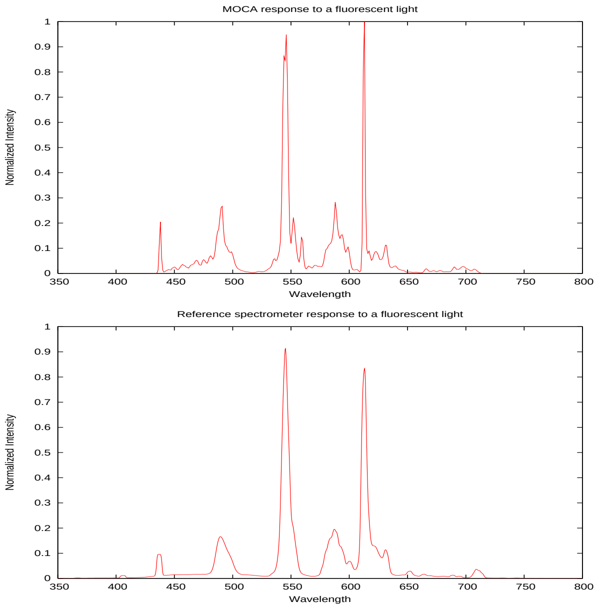


Fig. 4. Comparison of the spectrum of a neon pen lamp measured with the MOCA and the Ocean Optics reference spectrometer

within the spectral range of the MOCA while a fluorescent lamp spectrum has a more continuous distribution. Figure 4 shows the spectral estimate of a neon pen lamp measured with the MOCA compared Ocean Optics spectrometer. The reconstruction of the spectrum of a fluorescent lamp is shown in Fig. 5. Since neither system has been calibrated with respect to intensity, the comparison between these systems is respect to relative peak location and width.

## 5. Conclusion

The MOCA spectrometer demonstrates the increased spectral range and resolution that can be achieved by using multiple order diffractive elements in a dispersive spectrometer. The folding is accomplished by recording three slightly tilted diffraction gratings optimized for different frequencies on a single hologram. A MURA coded aperture is used at the input to improve the throughput of the system. An expectation-maximization approach is used to estimate the spec-


MOCA response to a fluorescent light

Reference spectrometer response to a fluorescent light

Fig. 5. Comparison of the spectrum of a fluorescent lamp measured with the MOCA and the Ocean Optics reference spectrometer

trum of the source from the system response. The performance of the MOCA is comparable to that of commercially available spectrometers. These experimental results indicate the viability of this approach.

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