Activities and Findings

This section will serve as your report to your program officer of your project's activities and findings. Please describe what you have done and what you have learned, broken down into four categories:

1. Describe the major research and education activities of the project.

This project is primarily concerned with the study of various aspects and channel models of the free-space optics (FSO) communication channel, and the development of novel forward error-correction schemes for such channels. The motivation behind this project is that the FSO communication channel offers an excellent alternative for wireless communications for its wide bandwidth and relatively low cost, compared to RF wireless links. FSO communications also represent a promising technology to integrate a variety of interfaces and network elements. We are interested in developing the FSO systems that are robust in the presence of atmospheric turbulence. Another important goal of this project is to solve the incompatibility problem that arises from the bandwidth mismatch between RF/microwave and optical channels.

Over the past year, we have pursued several research problems related to the communication over hybrid and heterogeneous optical networks, study of mitigation techniques to simultaneously compensate different channel impairments over different link types in heterogeneous optical networks [FSO links, single mode fiber (SMF) and multimode fiber (MMF) links], adaptive modulation and coding enabling hybrid FSO- wireless communications, orbital angular momentum (OAM) FSO communications, FPGA implementation of binary LDPC decoders for FSO channels, design and implementation of nonbinary LDPC encoders and decoders for FSO channels, and multilevel multidimensional LDPC-coded modulation.

Specific accomplishments include:
1. Development of coding techniques for hybrid and heterogeneous optical networks,
2. Study of polarization-multiplexed coded orthogonal frequency division multiplexing (OFDM),
3. FPGA implementation of LDPC decoders,
4. Design and implementation of quasi-cyclic nonbinary LDPC codes,
5. Study of adaptive modulation for free-space optical communications,
6. Study of adaptive modulation and coding for communication over hybrid free-space optical – wireless communication channels,
7. We have quantified the crosstalk that results from OAM beam propagation through a turbulent channel,
8. We have computed the FSO channel capacity for OAM multiplexing,
9. Study of trapping sets and deriving conditions for guaranteed error-correction capability of LDPC codes,
10. Design and analysis of two-bit message passing decoders for LDPC codes,
11. Study and design of quantum LDPC codes,
12. Study of communication over quantum optical free-space optical channels,
13. Development of quantum LDPC encoders and decoders using integrated optics,
14. Joint message-passing decoding for partial response channels,
15. We proposed the multidimensional LDPC-coded modulation to achieve beyond 100 Gb/s optical transmission, and
16. Initial study of OAM quantum key distribution (QKD).

The educational activities include teaching the following graduate and undergraduate courses: digital communications I and II, optical communications, advanced optical communications, channel coding, neural networks, quantum error correction, signals and systems, and introduction to digital communications. We have presented several tutorial lectures to various groups on and off campus.

2. Describe the major findings resulting from these activities.

Adaptive Modulation and Coding for Communication over the Atmospheric Turbulence Channels

Free-space optics (FSO) communication is the technology that can address any connectivity needs in future optical networks, be in the core, edge or access. In metropolitan area networks (MANs), the FSO can be used to extend the existing MAN rings; in enterprise, the FSO can be used to enable local area network (LAN)-to-LAN connectivity and intercampus connectivity; and the FSO is an excellent candidate for the last-mile connectivity. However, an optical wave propagating through the atmosphere experiences the fluctuations in amplitude and phase due to scintillation.

In this section, which is based on our recent conference submission [1], we propose to use the adaptive modulation and coding as an efficient way to deal with strong atmospheric turbulence. The key idea behind our proposal is to estimate the channel conditions at the receiver side and feed this channel estimate back to the transmitter using an RF feedback channel, so that the transmitter can be adapted relative to the channel conditions. We study two
adaptive modulation scenarios: (i) variable-rate variable-power adaptation, (ii) truncated channel inversion with fixed rate. We also study the improvements that can be obtained by using adaptive low-density parity-check (LDPC)-coded modulation. We show that in the strong turbulence regime, even the deep fades in the order of 35 dB and above can be tolerated.

The adaptive FSO communication system, shown in Fig. 1, consists of a transmitter, propagation path through the atmosphere, and a receiver. The optical transmitter includes a semiconductor laser of high launch power, adaptive modulation and coding block, and power control block. To reduce the system cost, the direct modulation of laser diode is used. At the receiver side, an optical system collects the incoming light and focuses it onto a detector, which generates an electrical current proportional to the incoming power. The intensity channel estimate is transmitted back to the transmitter by using an RF feedback channel.

Several probability density functions (PDFs) have been proposed for the intensity variations at the receiver side of an FSO link. For example, Al-Habash et al. proposed a statistical model that factorizes the irradiance as the product of two independent random processes each with a Gamma PDF. This model is valid for wide range of atmospheric turbulence regimes and as such is adopted here.

There are many parameters that can be varied at the transmitter side relative to the FSO channel intensity gain; including data rate, power, coding rate, and combinations of different adaptation parameters. The transmitter power adaptation, similar to wireless communications, can be used to compensate for signal-to-noise (SNR) variation due to atmospheric turbulence, with the aim to maintain a target bit error probability $P_b$. The power adaptation therefore "inverts" the FSO channel scintillation so that the FSO channel behaves similarly as an AWGN channel to the receiver. In variable rate adaptation, we change the signal constellation size for the fixed symbol rate depending on FSO channel conditions. When the FSO channel conditions are favorable we increase the constellation size, decrease it when channel conditions are not favorable, and not transmit at all when the intensity channel coefficients are below the irradiance threshold.

In the rest of this section, we describe two adaptation policies (by observing the average power constraint): (i) truncated channel inversion with fixed rate, and (ii) adaptive-power adaptive-rate scheme. We apply the adaptation to $M$-ary pulse-amplitude modulation (MPAM) format, which is selected because negative amplitude signals cannot be transmitted over FSO channels with direct detection. Notice that $M$-ary pulse-position modulation (MPPM) can also be used. Because MPPM is highly spectrally inefficient, we restrict our attention to MPAM. The truncated channel inversion power adaptation policy for FSO channels is defined by
where $i_t$ is the channel irradiance at time instance $t$, $P(i_t)$ is the instantaneous power, $P$ is the average power, and $i_{tsh}>0$ is the threshold irradiance. ($E[]$ denotes mathematical expectation operator.) Therefore, the channel inversion is performed only when the irradiance is above the certain threshold $i_{tsh}>0$. The threshold irradiance is to be chosen on such a way to maximize the spectral efficiency, defined as $R/B$ (with $R$ being the data rate and $B$ being the channel bandwidth), which for MPAM is given below

\[
\frac{R}{B} = \frac{1}{2.19} \max_{i_{tsh}} \left\{ \log_2 \left( 1 + K \int_0^{i_{tsh}} \frac{1}{E_{i_{tsh}} \left[ 1/i_t^2 \right]} \right) P(i_t \geq i_{tsh}) \right\},
\]

where $K = K(P_b)$ is function of a target bit error probability $P_b$ and $P(i_t \geq i_{tsh}) = \int_{i_{tsh}}^{\infty} p(i_t) d(i_t)$, with $p(i_t)$ being the irradiance PDF.

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**Fig. 1** Block diagram of adaptive modulation and coding FSO system with RF feedback. LLRs: log-likelihood ratios, APP: a posteriori probability.

**To derive the optimum power adaptation policy, we define the Lagrangian, differentiate it with respect to $P(i_t)$ and set this derivative to be equal to zero. The following optimum power adaptation policy (that is known**
as “water-filling” in wireless literature) is obtained as the result of this derivation

\[ \frac{K \Gamma_0 P(i_t)}{p} = \begin{cases} \frac{1}{i_{tsh}^2} - \frac{1}{i_t^2}, & i_t \geq i_{tsh}, \\ 0, & i_t < i_{tsh} \end{cases}, \quad (3) \]

With this adaptation policy more power and higher data rates are transmitted when the FSO channel conditions are good, less power and lower data rates are transmitted when FSO channel is bad, and nothing is transmitted when the FSO irradiance falls below the threshold \( i_{tsh} \). The optimum threshold can be obtained numerically by solving the following equation:

\[ \int_{i_{tsh}}^{\infty} \left( \frac{1}{i_{tsh}^2} - \frac{1}{i_t^2} \right) p(i_t) \, di_t = K \Gamma_0, \quad (4) \]

where \( \Gamma_0 \) is the signal-to-noise ratio (SNR) in the absence of turbulence. The corresponding spectral efficiency can be evaluated by

\[ \frac{R}{B} = \frac{1}{2.19} \int_{i_{tsh}}^{\infty} \log_2 \left( \frac{i_t^2}{i_{tsh}^2} \right) p(i_t) \, di_t \, [\text{bits/s/Hz}]. \quad (5) \]

We further propose to implement adaptive coding based on LDPC-coded modulation. The input data are LDPC encoded and written to a buffer. Based on FSO channel irradiance, \( i_t, m(i_t) \) bits are taken at a time from a buffer and used to select the corresponding point from MPAM signal constellation. The number of bits to be taken from the buffer are determined by using either (2) or (5).

In Fig. 2 we show spectral efficiency performance of adaptive LDPC(16935,13550)-coded MPAM for different adaptation scenarios and different turbulence strengths. The turbulence strength is characterized by the unitless Rytov variance \( \sigma_R^2 = 1.23 \, C_n^2 \, k^{7/6} L^{1/6} \), where \( k = 2\pi/\lambda \) (\( \lambda \) is the wavelength), \( L \) denotes the propagation distance, and \( C_n^2 \) is refractive index structure parameter. Two proposed schemes perform comparable in the weak turbulence regime \( (\sigma_R=0.2) \), while variable-power variable-rate significantly outperforms truncated channel inversion scheme in strong turbulence regime \( (\sigma_R=2.0) \). The coding gain over adaptive modulation at \( P_b = 10^{-6} \) for \( R/B = 4 \) bits/s/Hz is 7.2 dB in both (weak and strong) turbulence regimes. Larger coding gains are expected at lower BERs, and for higher spectral efficiencies. Further improvements can be obtained by increasing the girth (the shortest cycle in corresponding bipartite graph representation of parity-check matrix) of LDPC codes, and employing better modulation formats. The increase in codeword length to 100,515 does not improve \( R/B \) performance that much as shown in Fig. 2. It is interesting to notice that by employing adaptive coding, the communication under saturation regime is possible. Moreover, for variable-rate variable-power scheme there is no
degradation in saturation regime ($\sigma_R >> 1$) compared to strong turbulence regime. Overall improvement from adaptive modulation and coding for $R/B=4$ bits/s/Hz at $P_b=10^{-6}$ over non-adaptive uncoded modulation ranges from 10.5 dB (3.3 dB from adaptive modulation and 7.2 dB from coding) in the weak turbulence regime and 38.9 dB in the strong turbulence regime (31.7 dB from adaptive modulation and 7.2 dB from coding).

Fig. 2 Spectral efficiency vs. symbol SNR for adaptive LDPC-coded MPAM.


**Communication over Hybrid Free-Space Optical - Wireless Fading Channels**

In this section, which is based on our recent conference submission [2], we propose to use a hybrid FSO-RF system with adaptive modulation and coding as an efficient way to deal with strong atmospheric turbulence. Adaptive modulation and coding can enable robust and spectrally-efficient transmission over both $\alpha$-$\mu$ (or generalized Gamma) wireless fading channel and FSO channel. The key idea behind our proposal is to split the encoded sequence between FSO and wireless channels, estimate the channel conditions in both channels at the receiver side and feed this channel estimate back to both RF and FSO transmitters using an RF feedback channel, so that the transmitters can be adapted relative to the channel conditions. The power and data rate in both channels are adapted so that the total channel capacity in both channels is maximized. The optimum power adaptation policy is determined that maximizes channel capacity
simultaneously in both channels. We also study, similarly as in previous section, the improvements that can be obtained by using adaptive LDPC-coded modulation.

The adaptive hybrid FSO-RF communication system, shown in Fig. 3, consists of two parallel FSO and RF channels. The LDPC encoded data stream is partially transmitted over FSO and partially over RF channel. Operating symbol rate of FSO channel is commonly many times higher that that of RF channel. FSO channel comprises an FSO transmitter, propagation path through the atmosphere, and an FSO receiver. The optical transmitter includes a semiconductor laser of high launch power, adaptive mapper, and power control block. To reduce the system cost, the direct modulation of laser diode is used. The modulated beam is projected toward the distant receiver by using an expanding telescope assembly. Along the propagation path through the atmosphere, the light beam experiences absorption, scattering and atmospheric turbulence, which cause attenuation, and random variations in amplitude and phase. At the receiver side, an optical system collects the incoming light and focuses it onto a detector, which generates an electrical current proportional to the incoming power. The RF channel comprises adaptive RF mapper, RF power control, RF transmitter (Tx), transmitting antenna, wireless propagation path, receiver antenna, and RF receiver (Rx).

The RF channel estimates and FSO irradiance estimates are transmitted back to transmitters using the same RF feedback channel. Because the atmospheric turbulence changes slowly, with correlation time ranging from 10 $\mu$s to 10 ms, this is a plausible scenario for FSO channels with data rates in the order of Gb/s. Notice that erbium doped fiber amplifiers (EDFAs) cannot be used at all in this scenario because the fluorescence time is too long (about 10 ms). The semiconductor optical amplifiers (SOAs) should be used instead, if needed. The data rates and powers in both channels are varied in accordance with channel conditions. The symbol rates on both channels are kept fixed while the signal constellation diagrams sizes are varied based on channel conditions. When FSO (RF) channel condition is favorable larger constellation size is used, when FSO (RF) channel condition is poor smaller constellation size is used, and when the FSO (RF) channel signal-to-noise ratio (SNR) falls below threshold the signal is not transmitted at all. Both subsystems (FSO and RF) are designed to achieve the same target bit error probability ($P_b$). The RF subsystem employs $M$-ary quadrature amplitude modulation (MQAM), while FSO subsystem employs the $M$-ary pulse amplitude modulation (MPAM). MPAM is selected for FSO subsystem because negative amplitude signals cannot be transmitted over FSO channels with direct detection. These two modulation formats are selected as an illustrative example, the proposed scheme, however, is applicable to arbitrary multilevel modulations.
Fig. 3 System model. S/P: serial-to-parallel conversion, LD: laser diode, ADC: A/D converter, P/S parallel-to-serial converter, APP: a posteriori probability.

In Fig. 4 we report the spectral efficiencies for hybrid FSO-RF system shown in Fig. 3, with RF sub-system fading parameters $\alpha=3$, $\mu=2$ in both weak turbulence regime (Fig. 4(a)) and strong turbulence regime (Fig. 4(b)). We assume that FSO subsystem symbol rate is 10 times larger than RF subsystem data rate, that is $b=0.1$. For spectral efficiency of 2 bits/s/Hz the hybrid FSO-RF system outperforms the FSO system by 3.39 dB at BER of $10^{-6}$ and 3.49 dB at BER of $10^{-9}$. It is interesting to notice that even truncated channel inversion for hybrid system outperforms the optimum adaptation of FSO system by 0.8 dB at BER of $10^{-9}$ and spectral efficiency of 2 bits/s/Hz.

In Fig. 5 we report the spectral efficiencies for hybrid FSO-RF system, with RF sub-system fading parameters $\alpha=2$, $\mu=1$ (corresponding to Rayleigh fading) in both weak turbulence regime (Fig. 5(a)) and strong turbulence regime (Fig. 5(b)). This case corresponds to the situation where there is no...
line-of-site between transmit and receive antennas for RF subsystem. We again assume that $b=0.1$. For spectral efficiency of 2 bits/s/Hz the hybrid FSO-RF system outperforms the FSO system by 3.01 dB at BER of $10^{-6}$ and 3.11 dB at BER of $10^{-9}$. The truncated channel inversion for hybrid system performs comparable to the optimum adaptation of FSO system.

![Figure 5](image1.png)  
(a)  
![Figure 6](image2.png)  
(b)

Fig. 5 Spectral efficiencies of hybrid FSO-RF system with $\alpha=2$, $\mu=1$ (Rayleigh) fading against symbol SNR for different target bit probabilities of error: (a) in weak turbulence regime, and (b) in strong turbulence regime.

![Figure 6](image3.png)

Fig. 6 Spectral efficiencies against symbol SNR for adaptive LDPC-coded modulation in hybrid FSO-RF system.
In Fig. 6 we show $R/B$ performance of hybrid FSO-RF system with adaptive LDPC(16935,13550)-coded modulation (MPAM is used in FSO subsystem and MQAM in RF subsystem) for different adaptation scenarios. The symbol rate in FSO subsystem is set to be 10 times larger than that in RF subsystem ($b=0.1$). For spectral efficiency of 4 bits/s/Hz at BER of $10^{-6}$, the improvement of hybrid FSO-RF system over FSO system is 5.25 dB in Rayleigh fading ($\alpha=2$, $\mu=1$), 5.51 dB in Nakagami $m=2$ fading ($\alpha=2$, $\mu=2$) and 5.63 dB in $\alpha=3$, $\mu=2$ fading. For spectral efficiency of 2 bits/s/Hz at the same BER, the improvement of hybrid FSO-RF system over FSO system is 3.32 dB in Rayleigh fading, 3.72 dB in Nakagami $m=2$ fading and 3.86 dB in $\alpha=3$, $\mu=2$ fading.


**Polarization-Multiplexed Coded-OFDM**

In this sub-section we describe how to combine coded modulation with OFDM, which is illustrated in Fig. 7. This section is based on our journal paper [J5]. The two-dimensional (2D) signal constellation points (see Fig. 7(b)) are split into two streams for OFDM transmitters corresponding to the $x$- and $y$-polarizations. The QAM constellation points are considered to be the values of the fast Fourier transform (FFT) of a multi-carrier OFDM signal. The OFDM symbol is generated as follows: $N_{QAM}$ input QAM symbols are zero-padded to obtain $N_{FFT}$ input samples for inverse FFT (IFFT), $N_g$ non-zero samples are inserted to create the guard interval, and the OFDM symbol is multiplied by the Blackman-Harris window function. For efficient chromatic dispersion and PMD compensation, the length of cyclically extended guard interval should be longer than the total spread due to chromatic dispersion and DGD.

The cyclic extension is accomplished by repeating the last $N_g/2$ samples of the effective OFDM symbol part ($N_{FFT}$ samples) as a prefix, and repeating the first $N_g/2$ samples as a suffix. After D/A conversion (DAC), the RF OFDM signal is converted into the optical domain using the dual-drive Mach-Zehnder modulator (MZM). Two MZMs are needed, one for each polarization. The outputs of MZMs are combined using the polarization beam combiner (PBC). One DFB laser is used as CW source, with $x$- and $y$-polarization separated by polarization beam splitter (PBS). The polarization-detector soft estimates of symbols carried by the $k$th subcarrier in the $i$th OFDM symbol, $s_{i,k,x(y)}$, are forwarded to the APP demapper, which determines the symbol LLRs $\lambda_{x(y)}(q)$ ($q=0,1,\ldots,2^b-1$) of $x$- ($y$-) polarization by
\[
\lambda_{x(y)}(q) = -\left( \text{Re}\left[ \bar{s}_{i,k,x(y)} \right] - \text{Re}\left[ \text{QAM}(\text{map}(q)) \right] \right)^2 / (2\sigma^2) \\
- \left( \text{Im}\left[ \bar{s}_{i,k,x(y)} \right] - \text{Im}\left[ \text{QAM}(\text{map}(q)) \right] \right)^2 / (2\sigma^2),
\]

where \( \text{Re}[\cdot] \) and \( \text{Im}[\cdot] \) denote the real and imaginary part of a complex number, \( \text{QAM} \) denotes the QAM-constellation diagram, \( \sigma^2 \) denotes the variance of an equivalent Gaussian noise process originating from ASE noise, and \( \text{map}(q) \) denotes a corresponding mapping rule.

Let us denote by \( v_{j,x(y)} \) the \( j \)th bit in an observed symbol \( q \) binary representation \( v=(v_1, v_2, ..., v_b) \) for \( x-(y-) \) polarization (\( b \) denotes the number of bits per constellation point). The bit LLRs needed for LDPC decoding are calculated from symbol LLRs as described in [J5]. The extrinsic LLRs are iterated backward and forward until convergence or pre-determined number of iterations has been reached. The polarization-detector soft estimates can be obtained by employing: (i) polarization-time coding [J7] similar to space-
time coding proposed for use in MIMO wireless communication systems, (ii) using BLAST algorithm [J6], (iii) by polarization interference cancellation scheme [J6], or (iv) carefully performed channel matrix inversion [J5].

![Graph](image)

Fig. 8 BER performance of polarization multiplexed coded-OFDM, for DGD of 1200 ps. $R_D$ denotes the aggregate data rate.

In Fig. 8 we show both the uncoded and LDPC-coded BER performance of the polarization multiplexed LDPC-coded OFDM, against the polarization diversity OFDM scheme, for different constellations sizes. For DGD of 1200 ps, the polarization multiplexed scheme performs comparable to the polarization-diversity OFDM scheme in terms of BER (the corresponding curves overlap each other), but it has two times higher spectral efficiency. The net effective coding gain increases as the constellation size grows. For $M=4$ QAM based polarization multiplexed coded-OFDM the net effective coding gain is 8.36 dB at BER of $10^{-7}$, while for $M=32$ QAM based LDPC-coded OFDM (of aggregate data rate 100 Gb/s) the coding gain is 9.53 dB at the same BER.

**Coded-OFDM in Hybrid Optical Networks**

Future internet should be able to support wide range of services containing large amount of multimedia over different network types at high-speed. The future optical networks will therefore be hybrid, composed of different single-mode fiber (SMF), multi-mode fiber (MMF) and free-space optical (FSO) links. In these networks, novel modulation and coding techniques are needed capable of dealing with different channel impairments, be in SMF,
MMF or FSO links. We propose a coded-modulation scheme suitable for use in hybrid FSO – fiber-optics networks, which is based on our recent journal article [J18]. The proposed scheme is based on polarization-multiplexing and coded - OFDM with large girth quasi-cyclic LDPC codes as channel codes. The proposed scheme is able simultaneously to deal with atmospheric turbulence, chromatic dispersion and PMD. With a proper design for 16- QAM based polarization-multiplexed coded OFDM, the aggregate data rate of 100 Gb/s can be achieved for OFDM signal bandwidth of only 12.5 GHz, which represents a scheme compatible with 100 Gb/s per wavelength channel transmission and 100 Gb/s Ethernet.

An example of a hybrid FSO – fiber-optic network is shown in Fig. 9(a). This particular example includes inter-satellite links and connection to aircrafts. The fiber-optic portion of network could be a part of already installed MAN or WAN. The FSO network portion should be used whenever the pulling the ground fiber is expensive and/or takes too much long time for deployment, such as urban and rural areas, where the optical fiber links are not already installed. The corresponding hybrid optical networking architecture is shown in Fig. 9(b). We can identify three ellipses representing the core network, the edge network and the access network. The FSO links can be used in both edge and access networks. The hybrid optical networks imposes a big challenge to the engineers, because the novel signal processing techniques should be developed, which would be able simultaneously to deal atmospheric turbulence in FSO links; and with chromatic dispersion, PMD and fiber nonlinearities in fiber-optic links. One such coded-modulation technique is described in the rest of this section. By using the retro-reflectors the FSO systems can be applied even when there is no line of sight between transmitter and receiver.

The proposed coded-modulation scheme employs the coded-OFDM scheme with coherent detection. Notice that coded-OFDM scheme with direct detection has already been proposed by authors in [J9], as a scheme that in combination with interleaving is able to operate under the strong atmospheric turbulence. The use of coherent detection offers the potential of even 24 dB improvement over uncoded direct detection counterpart. One portion of improvement (10-13 dB) is coming from the fact that coherent detection can approach quantum-detection limit easier than direct detection. The second portion (about 11 dB) is coming from the use of large-girth LDPC codes [C10]. Let us now describe the operation principle of coded-OFDM scheme with coherent detection employing both polarizations. Given the fact the signal from Fig. 8 is going to be transmitted over the FSO links and over the fiber-optic links, we use a particular polarization-multiplexing capable to eliminate the influence of PMD. The transmitter and receiver shown in Fig. 10, to be used in hybrid optical network from Fig. 9, are able simultaneously to deal with atmospheric turbulence, residual chromatic dispersion and PMD.
Fig. 9 (a) A hybrid FSO – fiber-optic network example, and (b) a hybrid optical networking architecture.

The bit streams originating from $m$ different information sources are encoded using different $(n,k_i)$ LDPC codes of code rate $r_i=k_i/n$. $k_i$ denotes the number of information bits of $i$th ($i=1,2,...,m$) component LDPC code, and $n$ denotes the codeword length, which is the same for all LDPC codes. The use of different LDPC codes allows us to optimally allocate the code rates. If all component LDPC codes are identical, the corresponding scheme is commonly referred to as the bit-interleaved coded modulation (BICM). The outputs of $m$ LDPC encoders are written row-wise into a block-interleaver block. The mapper accepts $m$ bits at time instance $i$ from the $(m \times n)$ interleaver column-wise and determines the corresponding $M$-ary ($M=2^m$) signal constellation point $(\phi_{I,i},\phi_{Q,i})$ in two-dimensional (2D) constellation diagram such as $M$-ary phase-shift keying (PSK) or $M$-ary QAM. (The coordinates correspond to in-phase and quadrature components of $M$-ary two-dimensional constellation.)
The OFDM symbol is generated in similar fashion as described in previous section. After D/A conversion (DAC), the OFDM signal is converted into the optical domain using the dual-drive Mach-Zehnder modulators (MZMs). Two dual-drive MZMs are needed, one for each polarization. The outputs of MZMs are combined using the polarization beam combiner (PBC). The same distributed feedback (DFB) laser is used as CW source, with x- and y-polarizations being separated by a polarization beam splitter (PBS).

The key idea of this proposal is to use the OFDM with a large number of subcarriers (in order of thousands) so that the OFDM symbol duration becomes in order of $\mu$s, and by means of interleavers in order of thousands overcome the atmospheric turbulence with temporal correlation in the order of 10 ms. For the OFDM scheme to be capable to simultaneously compensate for chromatic dispersion and PMD, in addition to the atmospheric turbulence, the cyclic extension guard interval should be longer than total delay spread due to chromatic dispersion and DGD, as indicated above.

The OFDM is also an excellent candidate to be used for multi-user access [known as orthogonal frequency-division multiple access (OFDMA)]. In OFDMA, subsets of subcarriers are assigned to individual users. OFDMA enables the time, and frequency domain resource partitioning. In time domain, it can accommodate for the burst traffic (packet data) and enables the multi-user diversity. In frequency domain, it provides further granularity, and channel dependent scheduling. In OFDMA, different number of subcarriers can be assigned to different users, in order to support differentiated Quality of service (QoS). Each subset of sub-carriers can have different kinds of modulation formats, and can carry different types of data. The differentiated QoS can be achieved by employing the LDPC codes of different error correction capabilities. The OFDMA, therefore, represents an excellent interface between wireless/wireline and optical technologies.

Because for high-speed signals a longer sequence of bits is affected by the deep fade in the ms range due to atmospheric turbulence, we propose to employ the polarization-multiplexing and large QAM constellations in order to achieve the aggregate data rate of $R_D=100$ Gb/s, while keeping the OFDM signal bandwidth in the order of 10 GHz. For example, by using the polarization-multiplexing and 16-QAM we can achieve $R_D=100$ Gb/s for OFDM signal bandwidth of 12.5 GHz, resulting in bandwidth efficiency of 8 bits/s/Hz. Similarly, by using the polarization-multiplexing and 32-QAM we can achieve the same data rate ($R_D=100$ Gb/s) for OFDM signal bandwidth of 10 GHz, with bandwidth efficiency of 10 bits/s/Hz.

The receiver description requires certain knowledge of the channel. In what follows, we assume that fiber-optic channel characteristics are known on receiver side, because the fiber-optics channel coefficients can easily be determined by pilot-aided channel estimation. On the other hand, the hybrid optical network may contain different FSO and fiber-optic sections, while the channel characteristics of FSO link can change rapidly even during the day,
so it is reasonable to assume that FSO channel characteristics are not known on receiver side. The FSO transmitter can use a retro-reflector and a training sequence to sense the FSO channel.

Fig. 10 The transmitter and receiver configurations for LDPC-coded OFDM hybrid optical system with polarization multiplexing and coherent detection: (a) transmitter architecture, (b) an OFDM transmitter architecture, (c) a hybrid optical link example, (d) receiver architecture, and (e) coherent detector configuration. PBS/PBC: polarization beam splitter/combiner.
Fig. 11 The constellation diagrams for polarization-multiplexed 16-QAM (the aggregate data rate is 100 Gb/s) after 500 ps of DGD for $\sigma_x=0.1$, $\sigma_y=0.05$, and OSNR=50 dB observing the worst case scenario ($\theta=\pi/2$ and $\varepsilon=0$) without transmission diversity: (a) before PMD compensation, and (b) after PMD compensation. The corresponding constellation diagrams in the presence of PMD only (for DGD of 500 ps): (c) before PMD compensation, and (d) after PMD compensation.

We will further describe two concepts: (i) transmitter does not have any knowledge about the FSO link, and (ii) transmitter knows the FSO link properties. When the transmitter knows the FSO link properties we can employ the transmitter diversity concept. Before resuming our description of coded-OFDM receiver, in next section we provide more details about FSO and fiber-optic channel models. In simulation results shown in Figs. 11-12 we assume that PMD channel coefficients are known at the receiver, because they can easily be determined by pilot-aided channel estimation. On the other hand, the FSO channel may change significantly during the day time, and as such is difficult to estimate. To illustrate the efficiency of this scheme,
in Fig. 11(a,b) we show the constellation diagrams for aggregate rate of 100 Gb/s, corresponding to the $M=16$ QAM and the OFDM signal bandwidth of 12.5 GHz in the presence of atmospheric turbulence ($\sigma_X=0.1$ and $\sigma_Y=0.05$; $\sigma_X^2$ denotes the variance of the log-normally distributed amplitude, and $\sigma_Y^2$ is the variance of phase noise process introduced by scintillation), before (see Fig. 11(a)) and after (see Fig. 11(b)) PMD compensation, assuming the worst case scenario ($\theta=\pi/2$ and $\varepsilon=0$; $\theta$ denotes the polar angle, $\varepsilon$ denotes the azimuth angle for corresponding PMD model [J18]). The corresponding constellation diagrams in the presence of PMD only are shown in Fig. 10(c,d). The proposed coded-modulation scheme is able to compensate for the PMD with DGD of even 500 ps in the presence of atmospheric turbulence characterized by $\sigma_X=0.1$ and $\sigma_Y=0.01$.

In Fig. 12 we show the BER performance of the proposed scheme for both uncoded case (Fig. 12(a)) and LDPC-coded case (Fig. 12(b)). The OFDM system parameters were chosen as follows: the number of QAM symbols $N_{\text{QAM}}=4096$, the oversampling is two times, OFDM signal bandwidth is set to either 10 GHz ($M=32$) or 12.5 GHz ($M=16$), and the number of samples used in cyclic extension $N_0=64$. For the fair comparison of different $M$-ary schemes the OSNR on x-axis is given per information bit, which is also consistent with digital communication literature. The code rate influence is included in Fig. 12 so that the corresponding coding gains are net effective coding gains. The average launch power per OFDM symbol is set to -3 dBm, and the Gray mapping rule is employed. To generate the temporally correlated samples we used the Levinson-Durbin algorithm. From Figs. 11-12 it can be concluded that PMD can be successfully compensated even in the presence of atmospheric turbulence. The most of degradation is coming from FSO channel, as shown in Figs. 11-12. The 32-QAM case with aggregate data rate $R_D=100$ Gb/s performs 1.9 dB (at $BER=10^{-6}$) worse than 16-QAM (with the same aggregate rate) although the occupied bandwidth is smaller.

The net coded gain improvement (at $BER$ of $10^{-6}$) of LDPC-coded OFDM over uncoded-OFDM is between 11.05 dB ($M=16$, $\sigma_X=0.01$, $\sigma_Y=0.01$, corresponding to the weak turbulence regime) and 11.19 dB ($M=16$, $\sigma_X=0.1$, $\sigma_Y=0.01$, corresponding to the medium turbulence regime). The additional coding gain improvement due to transmission diversity with two lasers is 0.19 dB for 32-QAM based OFDM ($\sigma_X=0.01$ and $\sigma_Y=0.1$) at $BER$ of $10^{-6}$. On the other hand the improvement due to transmission diversity for uncoded case (at the same $BER$) is 1.26 dB. Therefore, in the regime of weak atmospheric turbulence, the improvements due to transmission diversity are moderate. On the other hand, in the moderate turbulence regime the use of transmission diversity is unavoidable. Otherwise, the uncoded BER error floor is so high (see $\sigma_X=0.5$, $\sigma_Y=0.1$ curve in Fig. 12(a)) that even the best LDPC codes are not able to handle, if the complexity is to be kept reasonable low. With transmission diversity, in moderate turbulence regime, we obtain
BER performance comparable to the case in the absence of turbulence regime, as shown in Fig. 12.

Fig. 12 BER performance of the proposed hybrid optical network scheme: (a) uncoded BER curves, and (b) LDPC-coded BERs. $R_D$ denotes the aggregate data rate, and $B_{\text{OFDM}}$ is OFDM signal bandwidth. TD $i$: transmission diversity of order $i$.

The strong turbulence regime is not considered here due to the lack of an appropriate temporal correlation model. The laser linewidths of transmitting
and local laser were set to 10 kHz, so that the atmospheric turbulence, PMD
and ASE noise are predominant effects. Notice that BER threshold required
to achieve BER=10^{-6} at the output of LDPC decoder is 1.96\cdot10^{-2}, and in this
region BER values for different laser linewidths are comparable.

**LDPC-Coded Turbo-Equalization**

In this section we describe an LDPC-coded turbo equalization scheme we
proposed in [J4] (see also [J1]), as a universal scheme that can be used simultaneoulsy for: (i) suppression of fiber nonlinearities, (ii) PMD
compensation, and (iii) chromatic dispersion compensation in multilevel
coded-modulation schemes. The LDPC-coded turbo equalizer is composed of
two ingredients: (i) the multilevel BCJR algorithm based equalizer, and (ii)
the LDPC decoder. The transmitter configuration, for multilevel coding
(MLC), is similar to Fig. 10(a), but for single carrier transmission case. The
receiver configuration of LDPC-coded turbo equalizer is shown in Fig. 13. The
outputs of upper- and lower-balanced branches, proportional to \(\text{Re}\{S_iL\}^*\)
and \(\text{Im}\{S_iL\}^*\) respectively, are used as inputs of multilevel BCJR equalizer,
where the local laser electrical field is denoted by \(L=|L|\exp(j\varphi_L)\) (\(\varphi_L\) denotes
the laser phase noise process of the local laser) and incoming optical signal
at time instance \(i\) with \(S_i\).

The multilevel BCJR equalizer operates on a discrete dynamical trellis
description of the optical channel. Notice that this equalizer is universal and applicable to any two-dimensional signal constellation such as \(M\)-ary PSK, \(M\)-
ary QAM or \(M\)-ary polarization-shift keying (Polsk), and both coherent and
direct detections. This dynamical trellis is uniquely defined by the following
triplet: the previous state, the next state, and the channel output. The state
in the trellis is defined as \(s_j=x_{j-m},x_{j-m+1},\ldots,x_{j+1},x_{j+m}\), where \(x_k\) denotes the index of the symbol from the following set of possible indices
\(X=\{0,1,\ldots,M-1\}\), with \(M\) being the number of points in corresponding \(M\)-ary
signal constellation. Every symbol carries \(l=\log_2 M\) bits, using the appropriate
mapping rule (natural, Gray, anti-Gray, etc.) The memory of the state is
equal to \(2m+1\), with \(2m\) being the number of symbols that influence the
observed symbol from both sides. An example trellis of memory \(2m+1=3\) for
4-ary modulation formats (such as QPSK) is shown in Fig. 14. The trellis has
\(M^{2m+1}=64\) states (\(s_0, s_1,\ldots, s_{63}\)), each of which corresponds to the different
3-symbol patterns (symbol-configurations).

The state index is determined by considering \((2m+1)\) symbols as digits
in numerical system with the base \(M\). For example, in Fig. 14, the
quaternary numerical system (with the base 4) is used. (In this system 18 is
represented by \((102)_4\).) The left column in dynamic trellis represents the
current states and the right column denotes the terminal states. The
branches are labeled by two symbols, the input symbol is the last symbol in
initial state (the blue symbol), the output symbol is the central symbol of
terminal state (the red symbol). Therefore, the current symbol is affected by both previous and incoming symbols. For the complete description of the dynamical trellis, the transition probability density functions (PDFs) \( p(y_j|x_j) = p(y_j|s), \ s \in S \) are needed; where \( S \) is the set of states in the trellis, and \( y_j \) is the vector of samples (corresponding to the transmitted symbol index \( x_j \)). The conditional PDFs can be determined from collected histograms or by using \textit{instanton-Edgeworth expansion} method. The number of edges originating in any of the left-column states is \( M \), and the number of merging edges in arbitrary terminal state is also \( M \).

Fig. 13 LDPC-coded turbo equalization scheme configuration.

![Diagram of LDPC-coded turbo equalization scheme configuration](image)

Fig. 14 A portion of trellis for 4-level BCJR equalizer with memory \( 2m+1 = 3 \).

As an illustration of the potential of the proposed scheme, the BER performance of an LDPC-coded turbo equalizer is given in Fig. 15 for the
dispersion map shown in Fig. 16 (launch power of 0 dBm and single channel transmission). EDFAs with a noise figure of 5 dB are deployed after every fiber section. The bandwidth of the optical filter is set to $3R_i$ and that of the electrical filter is set to $0.7R_i$, where $R_i = R_s/R$ with $R_s$ being the symbol rate and $R$ being the code rate (0.8).

![Graph showing BER performance](image)

Fig. 15 BER performance of LDPC-coded turbo equalizer in the presence of fiber nonlinearities for: (a) QPSK modulation format with aggregate data rate of 100 Gb/s, and (b) RZ-OOK modulation format at 40 Gb/s. For both simulations, dispersion map shown in Fig. 16 is used.

In Fig. 15(a), we present simulation results for QPSK transmission at the symbol rate of 50 Giga symbols/s. The symbol rate is appropriately chosen so that the effective aggregate information rate is 100 Gb/s. The figure
depicts the uncoded BER and the BER after iterative decoding with respect to the number of spans, which was varied from 4 to 84. The propagation was modeled by solving the nonlinear Schrödinger equation using the split-step Fourier method. It can be seen from Fig. 15(a) that when a 4-level BCJR equalizer of state memory $2m+1=1$ and an LDPC(16935,13550) code of girth-10 and column weight 3 are used, we can achieve QPSK transmission at the symbol rate of 50 Giga symbols/s over 55 spans (6600 km) with a BER below $10^{-9}$. On the other hand, for the turbo equalization scheme based on a 4-level BCJR equalizer of state memory $2m+1=3$ (see Fig. 15(a)) and the same LDPC code, we are able to achieve even 8160 km at the symbol rate of 50 Giga symbols/s with a BER below $10^{-9}$. Notice that in both cases the BCJR equalizer trellis detection depth was equal to the codeword length. The BER performance comparison of LDPC-coded TE against large-girth LDPC codes and turbo-product codes for RZ-OOK system operating at 40 Gb/s (in effective information rate) is given in Fig. 15(b), for different trellis memories. LDPC-coded TE with state memory $2m+1=7$ provides almost 12 dB improvement over the BCJR equalizer with state memory of $m=0$ at BER of $10^{-8}$.

The BER performance comparison of LDPC-coded TE against large-girth LDPC codes and turbo-product codes for RZ-OOK system operating at 40 Gb/s (in effective information rate) is given in Fig. 15(b), for different trellis memories. LDPC-coded TE with state memory $2m+1=7$ provides almost 12 dB improvement over the BCJR equalizer with state memory of $m=0$ at BER of $10^{-8}$.

Fig. 16 Dispersion map under study is composed of $N$ spans of length $L=120$ km, consisting of $2L/3$ km of D$+$_ fiber followed by $L/3$ km of D$-$ fiber, with pre-compensation of -1600 ps/nm and corresponding post-compensation. The fiber parameters are given in Table I.

Fig. 17 BER performance of LDPC(16935,13550)-coded PMD TE with trellis memory $2m+1=7$. 

![Dispersion map under study](image)
In order to apply the proposed multilevel turbo equalizations scheme to real 100 Gb/s systems, the practical circuit implementation study would be mandatory. It is evident from Fig. 14 that complexity of dynamic trellis grows exponentially, because the number of states is determined by $M^{2m+1}$, so that the increase in signal constellation leads to increase of the base, while the increase in channel memory assumption ($2m+1$) leads to the increase of exponent. We have shown in the case of QPSK transmission (see Fig. 15(a)), that even small state memory assumption ($2m+1=3$) leads to significant performance improvement with respect to the state memory $m=0$. For larger constellations and/or larger memories the reduced complexity BCJR algorithm is to be used instead. For example, instead of detection of sequence of symbols corresponding to the length of codeword $n$, we can observe shorter sequences. Further, we do not need to memorize all branch metrics but several largest ones. In forward/backward metrics’ update, we need to update only the metrics of those states connected to the edges with dominant branch metrics, and so on. Moreover, when $\max^*(x,y)=\max(x,y)+\log[1+\exp(-|x-y|)]$ operation, required in forward and backward recursion steps, is approximated by $\max(x,y)$ operation, the forward and backward BCJR steps become the forward and backward Viterbi algorithms, respectively.

The nonlinear ISI turbo equalizer described above can also be used as a PMD compensator. The results of simulations, for 10 Gb/s transmission and ASE noise dominated scenario, are shown in Fig. 17 for differential group delay (DGD) $\Delta \tau=100$ ps and girth-10 LDPC code of rate 0.81. RZ-OOK of a duty cycle of 33% is observed. The bandwidth of super-Gaussian optical filter is set to $3R_l$, and the bandwidth of Gaussian electrical filter to $0.7R_l$, with $R_l$ being the line rate. For DGD of 100 ps, the $R=0.81$ LDPC-coded turbo equalizer (for trellis memory $2m+1=7$) has penalty of only 2 dB with respect to the back-to-back configuration.

<table>
<thead>
<tr>
<th>Table I Fiber parameters</th>
<th>$D_+$ fiber</th>
<th>$D_-$ fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersion [ps/(nm km)]</td>
<td>20</td>
<td>-40</td>
</tr>
<tr>
<td>Dispersion Slope [ps/(nm$^2$ km)]</td>
<td>0.06</td>
<td>-0.12</td>
</tr>
<tr>
<td>Effective Cross-sectional Area [$\mu$m$^2$]</td>
<td>110</td>
<td>50</td>
</tr>
<tr>
<td>Nonlinear refractive index [m$^2$/W]</td>
<td>$2.6 \cdot 10^{-20}$</td>
<td>$2.6 \cdot 10^{-20}$</td>
</tr>
<tr>
<td>Attenuation Coefficient [dB/km]</td>
<td>0.19</td>
<td>0.25</td>
</tr>
</tbody>
</table>

In the rest of this Section we turn our attention to the experimental verification. The experimental setup for PMD compensation study by LDPC-coded turbo equalization is shown in Fig. 18(a). The LDPC-encoded sequence is uploaded into Anritsu pattern generator via GPIB card controlled by a PC.
Fig. 18 (a) Experimental setup for PMD compensation study by LDPC-coded turbo equalization, and (b) BER performance of the PMD compensator.

A zero-chirp Mach-Zehnder modulator is used to generate the NRZ data stream. The launch power is maintained at 0 dBm at the input of PMD emulator (with equal power distribution between states of polarization). The output of PMD emulator is combined with an ASE source immediately prior to the preamplifier. The ASE noise power is controlled by variable optical attenuator (VOA) in order to provide an independent optical signal-to-noise ratio (OSNR) adjustment at the receiver. A standard pre-amplified PIN receiver is used for direct detection and is preceded by another VOA to...
maintain a constant received power of -6 dBm. The sampling oscilloscope (Agilent), triggered by the data pattern, is used to acquire the received sequences, downloaded via GPIB card back to the PC which serves as an LDPC-coded turbo equalizer.

The experimental results for 10 Giga symbols/s (effective information rate) NRZ transmission are shown in Fig. 18(b), for different DGD values. The TE is based quasi-cyclic LDPC(11936,10819) code of code rate 0.906 and girth-10, with 5 outer and 25 sum-product decoding algorithm iterations. The OSNR penalty for DGD of 125ps is about 3dB at BER=10^{-6}, while the coding gain improvement over BCJR equalizer (with memory 2m+1=5) for DGD=125ps is 6.25 dB at BER=10^{-6}. Larger coding gains are expected at lower BERs.

**FPGA Implementation of Large Girth Quasi-Cyclic (QC) LDPC Codes**

The parity-check matrix of regular QC LDPC codes [11], [14], [10] can be represented by

\[
H = \begin{bmatrix}
I & I & I & \cdots & I \\
I & \mathbf{P}^{S[1]} & \mathbf{P}^{S[2]} & \cdots & \mathbf{P}^{S[c-1]} \\
I & \mathbf{P}^{2S[1]} & \mathbf{P}^{2S[2]} & \cdots & \mathbf{P}^{2S[c-1]} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
I & \mathbf{P}^{(r-1)S[1]} & \mathbf{P}^{(r-1)S[2]} & \cdots & \mathbf{P}^{(r-1)S[c-1]}
\end{bmatrix}, \tag{7}
\]

where \( I \) is \( pxp \) (\( p \) is a prime number) identity matrix, \( \mathbf{P} \) is \( pxp \) permutation matrix given by \( \mathbf{P}=(\mathbf{p}_{ij})_{pxp} \), \( \mathbf{p}_{i,i+1}=\mathbf{p}_{1,1}=1 \) (zero otherwise), and where \( r \) and \( c \) represent the number of block-rows and block-columns in (7), respectively. The set of integers \( S \) are to be carefully chosen from the set \( \{0,1,\ldots,p-1\} \) so that the cycles of short length, in the corresponding Tanner (bipartite) graph representation of (7), are avoided. The code rate of these QC codes, \( R \), is lower-bounded by

\[
R \geq \frac{|S|p-rp}{|S|p} = 1 - r/|S|, \tag{8}
\]

and the codeword length is \( |S|p \), where \(|S|\) denotes the cardinality of set \( S \). For a given code rate \( R_0 \), the number of elements from \( S \) to be used is \( \lfloor r/(1-R_0) \rfloor \). With this algorithm, LDPC codes of arbitrary rate can be designed.

**Example 1:** By setting \( p=2311 \), the set of integers to be used in (7) is obtained as \( S=\{1, 2, 7, 14, 30, 51, 78, 104, 129, 212, 223, 318, 427, 600, 808\} \). The corresponding LDPC code has rate \( R_0=1-3/15=0.8 \), column weight 3, girth-10 and length \(|S|p=15\cdot2311=34665\). In the example above, the initial set of integers was \( S=\{1,2,7\} \), and the set of row to be used in
(7) is \{1,3,6\}. The use of a different initial set will result in a different set from that obtained above.

**Example 2**: By setting \( p=269 \), the set \( S \) is obtained as \( S=\{0, 2, 3, 5, 9, 11, 12, 14, 27, 29, 30, 32, 36, 38, 39, 41, 81, 83, 84, 86, 90, 92, 93, 95, 108, 110, 111, 113, 117, 119, 120, 122\} \). If 30 integers are used, the corresponding LDPC code has rate \( R_0=1-3/30=0.9 \), column weight 3, girth-8 and length \( 30 \cdot 269=8070 \).

For hardware implementation, we use the min-sum algorithm which is a further simplified version of the min-sum-with-correction-term algorithm [J1]. Among various alternatives, we adopted a partially parallel architecture in our implementation since it is a natural choice for quasi-cyclic codes. In this architecture, a processing element (PE) is assigned to a group of nodes of the same kind instead of a single node. A PE mapped to a group of bit nodes is called a bit-processing element (BPE), and a PE mapped to a group of check nodes is called a check-processing element (CPE). BPEs (CPEs) process the nodes assigned to them in a serial fashion. However, all BPEs (CPEs) carry out their tasks simultaneously. Thus, by changing the number of elements assigned to a single BPE and CPE, one can control the level of parallelism in the hardware. In Fig. 19, we depict a convenient method for assigning BPEs and CPEs to the nodes in a QC-LDPC code. This method is not only easy to implement but also advantageous since it simplifies the memory addressing.

The messages between BPEs and CPEs are exchanged via memory banks. In Table II, we summarize the memory allocation in our implementation where we used the following notation: MEM B and MEM C denote the memories used to store bit node and check node edge values, respectively; MEM E stores the codeword estimate; MEM I stores the initial log-likelihood ratios; and finally, MEM R holds the state of the random number generator needed for AWGN source, which is based on Mersenne Twister algorithm.

![Fig. 19 The assignment of bit nodes and check nodes to BPEs and CPEs, respectively.](image-url)
for i = 0 to p-1 do
  for each b = 0 to c-1 do
    - Read r data values from MEM B located in the range 
      \[b \times p \times r + i \times r, b \times p \times r + i \times r + r - 1\].
    - Sum them up and store the sum.
    - Update MEM E at location \((b \times p + i)\).
  for each k = 0 to r-1 do
    - Subtract from the computed sum the value 
      located at \((b \times p \times r + i \times r + k - 1)\) in MEM B.
    - Use this value to update MEM C at location 
      \((k \times p + ((p - k \times b) \mod p)) \times c + b)\).
end
end

Fig. 20 The pseudo code describing assignment of bit nodes and check nodes to BPEs and CPEs.

Table II Memory allocation of the implementation

<table>
<thead>
<tr>
<th>MEM Name</th>
<th>MEM B</th>
<th>MEM C</th>
<th>MEM E</th>
<th>MEM I</th>
<th>MEM R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data word (bits)</td>
<td>8</td>
<td>11</td>
<td>1</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>Address word (bits)</td>
<td>16</td>
<td>16</td>
<td>15</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Memory block size (words)</td>
<td>50805</td>
<td>50805</td>
<td>16935</td>
<td>16935</td>
<td>625</td>
</tr>
</tbody>
</table>

In our initial design [J1],[C4], we used the MitrionC hardware programming language, which is “an intrinsically parallel C-family language” developed by Mitrionics, Inc. Using MitrionC syntax, we provided a pseudo code in Fig. 20 showing how the data are transferred from MEM B to MEM C after being processed by BPEs. The code features three loop expressions of two types. The for loop sequentially executes its loop body for every bit node, i, in a BPE. On the contrary, the for each loop is a parallel loop, and hence, the operations in the loop body are applied to all the elements in its declaration simultaneously. To expatiate, due to the first for each loop, all BPEs perform their operations on their ith bit nodes in parallel. Since we are using a single memory in our implementation to store the edge values of all check nodes, the second for each loop causes a BPE to update its connections in MEM C in a pipelined fashion. As also shown in Fig. 20, we compute the memory addresses to read/write data from/to “on-the-fly” using the bit node ID \(i\), BPE ID \(b\) and CPE ID \(k\). This convenient
calculation of addresses is possible because of the quasi-cyclic nature of the code and the way we assigned BPEs and CPEs.

We tested our design on the FPGA Subsystem located at the High Performance Computing (HPC) Center at The University of Arizona. This FPGA Subsystem consists of SGI RASC RC1000 Blade having two Virtex 4 LX2000 FPGAs. In Fig. 20, we present BER performance comparison of FPGA and software implementations for a girth-10 quasi-cyclic LDPC (16935, 13550) code. We observe a close agreement between the two BER curves. Furthermore, the performance of the min-sum algorithm is only 0.2 dB worse than that of the min-sum-with-correction-term algorithm at the BER of $10^{-6}$ and the gap gets closer as the Q factor increases. The net coding gain of the min-sum algorithm for the same LDPC code at BER of $10^{-6}$ is found to be 10.3 dB.

The main problem in decoder implementation for large girth binary LDPC codes is the excessive codeword length, and a fully parallel implementation on a single FPGA is quite a challenging problem. To solve this problem, in the next section, we will consider large-girth nonbinary LDPC codes over $\text{GF}(2^m)$ [J1],[C3]. By designing codes over higher-order fields, we aim to achieve the coding gains comparable to binary LDPC codes but for shorter codeword lengths.

Fig. 21 BER performance comparison of FPGA and software implementations of the min-sum algorithm.

**Nonbinary QC LDPC Codes**

In this sub-section, we describe a two-stage design technique for constructing non-binary regular, high-rate LDPC codes. We show that the
complexity of the non-binary decoding algorithm over GF(4) used to decode this code is 1.1 times less complex compared to the min-sum-with-correction-term algorithm [J1], used for decoding a bit-length-matched binary LDPC code. Furthermore, we demonstrate that by enforcing the non-binary LDPC codes to have the same nonzero field element in a given column in their parity-check matrices, we can reduce the hardware implementation complexity of their decoders without incurring any degradation in the error-correction performance.

A $q$-ary LDPC code is a linear block code defined as the null space of a sparse parity-check matrix $H$ over a finite field of $q$ elements that is denoted by GF($q$) where $q$ is a power of a prime. Davey and MacKay devised a $q$-ary sum-product algorithm (QSPA) to decode $q$-ary LDPC codes, where $q = 2^p$ and $p$ is an integer. They also proposed an efficient way of conducting QSPA via fast Fourier transform (FFT-QSPA). A mixed-domain version of the FFT-QSPA (MD-FFT-QSPA) that reduces the computational complexity by transforming the multiplications into additions with the help of logarithm and exponentiation operations is adopted here.

In the first step of our two-stage code design technique, we design binary QC LDPC codes of girth-6 using the algebraic construction method based on the multiplicative groups of finite fields. Let $\alpha$ be a primitive element of GF($q$) and let $W = [w_{i,j}]$ be a ($q$-1)-by-($q$-1) matrix given as follows:

\[
W = \begin{bmatrix}
\alpha^0 -1 & \alpha -1 & \ldots & \alpha^{q-2} -1 \\
\alpha -1 & \alpha^2 -1 & \ldots & \alpha^{q-1} -1 \\
\ldots & \ldots & \ldots & \ldots \\
\alpha^{q-2} -1 & \alpha^{q-1} -1 & \ldots & \alpha^{2(q-2)} -1
\end{bmatrix}.
\]

We can transform $W$ into a quasi-cyclic parity-check matrix $H^{(1)}$ of the following form:

\[
H^{(1)} = \begin{bmatrix}
A_{0,0} & A_{0,1} & \ldots & A_{0,n-1} \\
A_{1,0} & A_{1,1} & \ldots & A_{1,n-1} \\
\ldots & \ldots & \ldots & \ldots \\
A_{m-1,0} & A_{m-1,1} & \ldots & A_{m-1,n-1}
\end{bmatrix},
\]

where every sub-matrix $A_{i,j}$ is related to the field element $w_{i,j}$ by

\[
A_{i,j} = \left[z\left(w_{i,j}\right) z\left(\alpha w_{i,j}\right) z\left(\alpha^2 w_{i,j}\right) \ldots z\left(\alpha^{q-2} w_{i,j}\right)\right]^T,
\]

wherein $z(q^t) = (z_0, z_1, ..., z_{q-2})$ is a ($q$-1)-tuple over GF(2) whose $i$th component $z_i = 1$ and all other $q$-2 components are zero. The parity-check matrix, $H^{(1)}$, given in (10), which is a ($q$-1)-by-($q$-1) array of circulant permutation and zero matrices of size ($q$-1)-by-($q$-1), has a girth of at least six. We use this quasi-cyclic, girth-6 parity-check matrix $H^{(1)}$ in the second stage.
If we simply choose $\gamma$ rows and $\rho$ columns from $H^{(1)}$ while avoiding the zero matrices, we obtain a $(\gamma, \rho)$-regular parity-check matrix whose null space yields a $(\gamma, \rho)$-regular LDPC code with a rate of at least $(\rho - \gamma)/\rho$. Instead of a simple, random selection, however, if we choose rows and columns of $H^{(1)}$ while avoiding performance-degrading short cycles, we can boost the performance of the resulting LDPC code. Hence, the first step in the second stage is to select $\gamma$ rows and $\rho$ columns from $H^{(1)}$ in such a way that the resulting binary quasi-cyclic code has a girth of eight. In the second step, we replace the 1’s in binary parity-check matrix with nonzero elements from GF($q$) either by completely random selection or by enforcing each column to have the same nonzero element from GF($q$) while letting the nonzero element of each column be determined again by a random selection. We denote the final $q$-ary $(\gamma, \rho)$-regular, girth-8 matrix by $H^{(2)}$.

Following the two-stage design we had discussed above, we generated (3,15)-regular, girth-8 LDPC codes over the fields GF($2^p$), where $0 \leq p \leq 7$. All the codes had a code rate ($R$) of at least 0.8 and hence an overhead OH = (1/$R$ − 1) of 25% or less. We compared the BER performances of these codes against each other and against some other well-known codes, namely the ITU-standard RS(255,239), RS(255,223) and RS(255,239)+RS(255,223) codes; and BCH(128,113)xBCH(256,239) TPC. We used the binary AWGN (BI-AWGN) channel model in our simulations and set the maximum number of iterations to 50. In Fig. 22(a), we present the BER performances of the set of non-binary LDPC codes discussed above. Using the figure, we can conclude that when we fix the girth of a non-binary regular, rate-0.8 LDPC code at eight, increasing the field order above eight exacerbates the BER performance. In addition to having better BER performance than codes over higher order fields, codes over GF(4) have smaller decoding complexities when decoded using MD-FFT-QSPA algorithm since the complexity of this algorithm is proportional to the field order. Thus, we focus our attention on non-binary, regular, rate-0.8, girth-8 LDPC codes over GF(4) in the rest of the sub-section.

In Fig. 22(b), we compare the BER performance of the LDPC(8430,6744) code over GF(4) discussed in Fig. 22(a) against that of the RS(255,239) code, RS(255,223) code, RS(255,239)+RS(255,223) concatenation code, and BCH(128,113)xBCH(256,239) TPC. We observe that the LDPC code over GF(4) outperform all of these codes with a significant margin. In particular, it provides an additional coding gain of 3.363dB and 4.401dB at BER of $10^{-7}$ when compared to the concatenation code RS(255,239)+ RS(255,223) and the RS(255,239) code, respectively. Its coding gain improvement over BCH(128,113)xBCH(256,239) TPC is 0.886dB at BER of $4 \times 10^{-8}$. Finally, we computed the NECG of the 4-ary, regular, rate-0.8, girth-8 LDPC code over GF(4) to be 10.784dB at BER of $10^{-12}$. We also presented in Fig. 22(b) a competitive, binary, (3,15)-regular, LDPC(16935,13550) code proposed in [C10].
Fig. 22 (a) Comparison of non-binary, (3,15)-regular, girth-8 LDPC codes over BI-AWGN channel, (b) Comparison of 4-ary (3,15)-regular, girth-8 LDPC codes; a binary, girth-10 LDPC code, three RS codes and a TPC code.
We can see that the 4-ary, (3,15)-regular, girth-8 LDPC(8430,6744) code beats the bit-length-matched binary LDPC code with a margin of 0.089dB at BER of $10^{-7}$. More importantly, the complexity of the MD-FFT-QSPA used for decoding the non-binary LDPC code is lower than the min-sum-with-correction-term algorithm [J1] used for decoding the corresponding binary LDPC code. When the MD-FFT-QSPA is used for decoding a $(\gamma,\rho)$-regular $q$-ary LDPC($N$/log$q$,K/log$q$) code, which is bit-length-matched to a $(\gamma,\rho)$-regular binary LDPC($N$,K) code, the complexity is given by $(M$/log $q$)$2pq$(log $q+1-1/(2\rho))$ additions, where $M=N-K$ is the number of check nodes in the binary code. On the other hand, to decode the bit-length-matched binary counterpart using min-sum-with-correction-term algorithm, one needs $M\rho(\rho-2)$ additions. Thus, a (3,15)-regular 4-ary LDPC code requires 91.28\% of the computational resources required for decoding a (3,15)-regular binary LDPC code of the same rate and bit length.

**Orbital angular momentum-based multi-channel communications**

Orbital angular momentum (OAM) is a property of light associated with the helicity of a photon's wavefront. Optical beams carrying OAM are usually called optical vortices, because they feature a phase discontinuity at their center. The momentum of a vortex field is proportional to the number of turns that this vector completes around the beam's axis after propagating a distance equal to one wavelength. This number is equal to the OAM state. The OAM state of a photon can take any integer value. This infinite set of OAM states forms an orthonormal basis. This property may be exploited in the context of optical communications. The orthogonality among beams with different OAM states allows the simultaneous transmission of information from different users, each on a separate OAM channel. Each orthogonal channel can be perfectly filtered and decoded at the receiver of an FSO communication link. OAM states may also be used for multilevel modulation.

For FSO applications, the orthogonality is not maintained in the presence of atmospheric turbulence. As a result of the random turbulence process, part of the energy launched into a single OAM state will be redistributed into other OAM states after turbulent propagation. Consequently, atmospheric turbulence induces a time-varying crosstalk among OAM channels.

Crosstalk is the power detected by a receive channel for which the transmitted signal is not intended. This crosstalk is represented by the elements of the crosstalk matrix $H$. Fig. 23 shows histograms of the OAM crosstalk found for turbulence parameters (a) $C_n^2 = 10^{-16}$ m$^{-2/3}$, (b) $C_n^2 = 10^{-15}$ m$^{-2/3}$, and (c) $C_n^2 = 10^{-14}$ m$^{-2/3}$. In each of the three cases, we present four examples corresponding to the crosstalk generated by transmitting on an isolated channel with OAM quantum number $m = 1, 5, 10, or 15$. Each of these subplots corresponds to a row in $H$ and therefore also includes the effect of reduced channel efficiency. In case (a) crosstalk
among channels is negligible. For instance, in the uppermost box of 23(a) for which the transmitted OAM state is \( m = +1 \), we obtain crosstalk levels of \(-24.6\) dB and \(-34\) dB of attenuation (decibels in the optical domain). For a transmit channel \( m = +10 \), we obtain crosstalk of \(-20.2\) dB and \(-27\) dB, respectively. It can be noted that, as expected from the trends seen in Fig. 23, crosstalk to adjacent channels increases with increasing OAM transmit state \(|m|\).

Each channel matrix \( H \) fully determines the communication limits for a fixed receiver noise power. Determining the maximum information rate (i.e., the channel capacity) for such a multiple input multiple-output channel is a complicated mathematical task. We instead take a simpler approach assuming that the system comprises a set of non-collaborative channels (for instance, the channels are used by independent users without knowledge of other channel’s statistics) for which we maximize the information rate, considering a uniform distribution of input symbols in each constituent channel and a uniform power policy among them. Furthermore, it is assumed that crosstalk from each channel is an independent Gaussian noise source that adds to the receiver noise.

For a given crosstalk matrix \( H \), we seek to find an optimum set of OAM channels in the sense of maximizing the system’s aggregated information rate. The optimum set, which we denote by \( O \), will depend on the number of channels \( M \), and is found by performing the maximization over all possible subsets of \( S \)

\[
O = \arg \max_{O \subset S} \sum_{m \in O} C(p_m)
\]
with \( C(p_m) = 1 + p_m \log_2 p_m + (1 - p_m) \log_2(1 - p_m) \) and \( p_m = \frac{1}{2} \text{erfc}(p/2) \), where \( C(p_m) \) is the capacity of a BSC with flip probability \( p_m \) and \( \text{erfc}(\cdot) \) is the complementary error function. This expression assumes orthogonal signal modulation, such as OOK or binary PPM. Fig. 24 depicts the optimum designs for several values of turbulence strength. In Fig. 24(a) the optimal OAM state numbers for a 7-channel system in a turbulent atmosphere with \( C_n^2 = 10^{-14} \text{ m}^{-2/3} \) are plotted. Note that at very low transmit power the optimal sets are simply \( O = \{-3, -2, -1, 0, +1, +2, +3\} \). Crosstalk among these states is clearly significant; however, for small values of \( P_{tx} \) the dominant noise is that of the detector. At \( P_{tx}/N_0 = 20 \text{ dB} \) the optimal set is \( O = \{-6, -3, -1, 0, +1, +3, +6\} \). As transmit power is increased greater loss can be tolerated and, therefore, we can afford to use channels with larger \( m \) resulting in optima with larger channel separations. For instance, at \( P_{tx}/N_0 = 32 \text{ dB} \) the optimal set is \( O = \{-10, -5, -2, 0, +2, +5, +10\} \). Also, note that increasing the transmit power beyond \( P_{tx}/N_0 = 36 \text{ dB} \) will not yield larger optimal channel separation for this turbulence condition, as aggregate capacity is near saturation (i.e., \( C \to 7 \)). It is promising to see that at this turbulence strength the largest optimal OAM mode needed is \( |m| = 10 \), corresponding to a receiver aperture of about 15 cm when other traditional design parameters are employed. Fig. 23(b) shows a plot of the optimal OAM state numbers for a system with \( M = 9 \text{ channels} \). The turbulence conditions are the same as those of Fig. 23(a).

Fig. 24 Optimal OAM channel sets versus \( P_{tx}/N_0 \), for (a) \( M = 7 \) and \( C_n^2 = 10^{-14} \text{ m}^{-2/3} \), for (b) \( M = 9 \) and \( C_n^2 = 10^{-14} \text{ m}^{-2/3} \).

The optimal sets presented in the previous above were determined by maximizing the aggregate capacity of the collection of independent binary-symmetric channels. By plotting capacity as a function of \( P_{tx}/N_0 \) all power penalties caused by turbulence, including channel losses (consequence of beam wander and spreading) and crosstalk have been included. In Figure 25 we plot the aggregate capacity in bits/M-channels for (a) \( C_n^2 = 10^{-15} \text{ m}^{-2/3} \) with \( M = \{1, 2, \ldots, 10\} \), for (b) \( C_n^2 = 10^{-14} \text{ m}^{-2/3} \) with \( M = \{1, 2, \ldots, 10\} \), for (c) \( C_n^2 = 3 \times 10^{-14} \text{ m}^{-2/3} \) with \( M = \{1, 2, \ldots, 9\} \), and for (d) \( C_n^2 = 10^{-13} \text{ m}^{-2/3} \) with \( M = \{1, 2, \ldots, 8\} \).
Fig. 25 Aggregate capacity (in bits/M-channels) versus $P_{Tx}/N_0$ for (a) $C_n^2 = 10^{-15} \text{ m}^{-2/3}$, for (b) $C_n^2 = 10^{-14} \text{ m}^{-2/3}$, for (c) $C_n^2 = 3 \times 10^{-14} \text{ m}^{-2/3}$, and for (d) $C_n^2 = 10^{-13} \text{ m}^{-2/3}$.

Each curve is computed using the optimal channel set at every value of (per-channel) $P_{Tx}/N_0$. Therefore, the constituent channels in the optimal channel sets will change along each curve. On each curve we have marked the $P_{Tx}/N_0$ point at which the channel with the worst performance within the set has a BER=$10^{-5}$. This serves as a reference for quality comparisons among systems with different $M$ and different $C_n^2$. We see from Fig. 25(a), for which the crosstalk is weak, aggregate capacities show a steady growth with $P_{Tx}/N_0$ and $M$. From Fig. 24(b), we note that it takes an additional 9 dB of $P_{Tx}/N_0$ (for $M = 2$) to 18 dB (for $M = 10$) to reach the rates shown in (a), due to the lower efficiency of each constituent channel. At $C_n^2 = 3 \times 10^{-14} \text{ m}^{-2/3}$ [Fig. 25(c)], an additional 14.5 dB (for $M = 2$) over the $P_{Tx}/N_0$ in (b) is required to
achieve the same rate, and 12.5 dB if $M = 9$ (a curve with $M = 10$ cannot be completed for this turbulence level given the range of OAM states considered). The power levels required to achieve a large capacity at $C2n = 10^{-13}$ m$^{-2/3}$ [Fig. 25(d)] are very large. In this case, only systems with optical amplification at the receiver may be feasible, given this system design. However, by using more complex receiver models, capacity is likely to grow faster with transmit power than in the non-collaborative BSC model studied here. As a last note about Fig. 25, we observe that in all cases capacity has a monotonic growth with $M$.

**On trapping sets and guaranteed error correction capability of LDPC and GLDPC codes**

Iterative algorithms for decoding LDPC codes have been the focus of research over the past decade and most of their properties are well understood. Message passing decoders perform remarkably well which can be attributed to their ability to correct errors beyond the traditional bounded distance decoding capability. However, in contrast to bounded distance decoders (BDDs), the guaranteed error correction capability of iterative decoders is largely unknown.

The focus of this work is to establish lower and upper bounds on the guaranteed error correction capability of LDPC codes and generalized low-density parity-check (GLDPC) codes under the Gallager-B message passing algorithm, as a function of their column-weight and girth. For the case of GLDPC codes, we also find the expansion required to guarantee correction of a fraction of errors under the parallel bit flipping algorithm, as a function of the error correction capability of the sub-code. Our approach can be summarized as follows: (a) to establish lower bounds, we determine the size of variable node sets in a left regular Tanner graph which are guaranteed to have the expansion required by bit flipping algorithms, based on the Moore bound and (b) to find upper bounds, we study the sizes of smallest possible trapping sets in a left regular Tanner graph.

It is well known that a random graph is a good expander with high probability. However, the fraction of nodes having the required expansion is very small and hence the code length to guarantee correction of a fixed number of errors must be large. Moreover, determining the expansion of a given graph is known to be NP hard, and spectral gap methods cannot guarantee an expansion factor of more than 1/2. On the other hand, code parameters such as column weight and girth can be easily determined or are assumed to be known for the code under consideration. We prove that for a given column-weight, the error correction capability grows exponentially in girth.

To find an upper bound on the number of correctable errors, we study the size of sets of variable nodes which lead to decoding failures. A decoding
failure is said to have occurred if the output of the decoder is not equal to
the transmitted codeword. The conditions that lead to decoding failures are
well understood for a variety of decoding algorithms such as maximum
likelihood decoding, bounded distance decoding and iterative decoding on
the BEC. However, for iterative decoding on the BSC and AWGN channel, the
understanding is far from complete. Two approaches have been taken in this
direction, namely trapping sets and pseudo-codewords. We adopt the
trapping set approach to characterize decoding failures. Richardson
introduced the notion of trapping sets to estimate the error floor on the
AWGN channel. We define trapping sets with the help of fixed points for the
bit flipping algorithms (both serial and parallel). We then find bounds on the
size of trapping sets based on extremal graphs known as cage graphs,
thereby finding an upper bound on the guaranteed error correction
capability.

The Moore bound denoted by \( n_0(d, g) \) is a lower bound on the least
number of vertices in a \( d \)-regular graph with girth \( g \). Based on this bound,
we state and prove the following theorem: let \( G \) be a \( E \equiv 4 \)-left regular
Tanner graph \( G \) with \( g(G) = 2g' \). Then for all \( k < n_0(\gamma / 2, g') \), any set of \( k \)
variable nodes in \( G \) expands by a factor of at least \( 3E/4 \). With this theorem,
we show that the for such a code, the bit flipping algorithm can correct any
error pattern of weight less than \( n_0(\gamma / 2, g') / 2 \).

A \((d, g)\)-cage graph, denoted by \( G(d, g) \), is a \( d \)-regular graph with girth \( g \)
having the minimum possible number of nodes. We establish the relation
between trapping sets and cage graphs. Let \( T(\gamma, 2g') \) denote the size of a
smallest possible potential trapping set of an LDPC code \( C \) with \( E \)-left
regular Tanner graph \( G \) and girth \( 2g' \) for the bit flipping algorithm. Then, the
size of the trapping set is related to the size of the cage graph by
\[ |T(\gamma, 2g')| = n_c([\gamma / 2, g']). \]
In order to prove this, we first prove that
\[ |T(\gamma, 2g')| \geq n_c([\gamma / 2, g']) \]
and then exhibit a potential trapping set of size \( n_c([\gamma / 2, g']) \).

For GLDPC codes, we consider two bit flipping decoding algorithms and
then establish a relation between expansion and error correction capability.
Let \( C(G, S) \) be a GLDPC code with a \( E \)-left regular Tanner graph \( G \) and a
sub-code \( S \). Assume that the sub-code \( S \) has minimum distance at least
\( d_{\min} = 2t + 1 \) and is capable of correcting \( t \) errors. Let \( G \) be a \((\gamma, \rho, \alpha, \beta, \gamma)\)
expander where
\[ 1 > \beta > \frac{t + 2}{2(t + 1)} \]

Then the parallel bit flipping decoding algorithm will correct any \( \alpha \)
fraction of errors. We also show that this bound can not be improved when \( E \) is even.
Our approach can also be used to derive bounds on the guaranteed erasure recovery capability for iterative decoding on the Binary Erasure Channel (BEC) by finding the number of variable nodes which expand by a factor of $E/2$.

**Two-bit Message Passing Decoders for LDPC Codes Over the Binary Symmetric Channel**

The performance of various hard decision algorithms for decoding LDPC codes on the BSC, has been studied in great detail. The BSC is a simple yet useful channel model used extensively in areas where decoding speed is a major factor.

For iterative decoding over the BEC, it is known that avoiding stopping sets up to size $t$ in the Tanner graph of the code guarantees recovery from $t$ or less erasures. A similar result for decoding over the BSC is still unknown. The problem of guaranteed error correction capability is known to be difficult and in this work, we present a first step toward such result by finding three-error correction capability of column-weight-four codes. Column-weight-four codes are of special importance because under a fixed rate constraint (which implies some fixed ratio of the left and right degrees), the performance of regular LDPC codes under iterative decoding typically improves when the right and left degrees decrease.

In this work, we propose a class of message passing decoders whose messages are represented by two bits. We refer to these decoders as to two-bit decoders. The idea of using message alphabets with more than two values for the BSC was first proposed by Richardson and Urbanke. They proposed a decoder with erasures in the message alphabet. The messages in such a decoder have hence three possible values. The class of two-bit decoders that we propose is a generalization of their idea, since we consider four possible values for the decoder messages. In addition, for a specific two-bit decoder, we derive sufficient conditions for a code with Tanner graph of girth six to correct three errors.

We focus on a class of two-bit decoders that can be described using simple algebraic rules and the lookup table can be constructed from the algebraic description. The message alphabet is denoted by $M = \{\equiv S, \equiv W, W, S\}$ where $\equiv S$ denotes a strong “1”, $\equiv W$ denotes a weak “1”, $W$ denotes a weak “0” and $S$ denotes a strong “0” and $S, W \rightarrow R^+$. It should be noted that this representation can be mapped onto the alphabet $\{11, 01, 00, 10\}$, but we use the symbols throughout for the sake of convenience. The received value $r_v = \{0, 1\}$ on the channel of a variable node $v$ is mapped to $r_v = \{C, \equiv C\}$, $C \rightarrow R^+$ as follows: $1 \nrightarrow \equiv C$ and $0 \nrightarrow C$. It can be seen that each message is associated with a value and strength (strength of a message is an indication of its reliability). The class of two-bit decoders as a voting
scheme in the following way: every message has two components namely, the value (0 or 1) and strength (weak or strong). The sign of the message determines the value, whereas the values of \( W \) and \( S \) denote the number of votes. The received value is associated with \( C \) votes. To compute the outgoing message on the variable node side, the total number of votes corresponding to 0 and 1 are summed. The value of the outgoing message is the bit with more number of votes and the strength is determined by the number of votes. In the case of a tie, the outgoing message is set to the received value with a weak strength. Table III gives an example of message update for a column-weight-four code, when \( C = 2, S = 2 \) and \( W = 1 \). The message \( w_j(v, c) \) goes out of variable node \( v \), and is computed in terms of the three messages going into \( v \) from the neighboring check nodes different of \( c \).

Table III  Example of a message-update for a column-weight-four code when \( C=2, S=2 \) and \( W=1 \).

<table>
<thead>
<tr>
<th># incoming ( -S ) messages</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td># incoming ( -W ) messages</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td># incoming ( W ) messages</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td># incoming ( S ) messages</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( R_v )</td>
<td>(-C)</td>
<td>( C)</td>
<td>( C)</td>
<td>(-C)</td>
</tr>
<tr>
<td>( w_j(v, c) )</td>
<td>(-S)</td>
<td>( W)</td>
<td>( S)</td>
<td>(-W)</td>
</tr>
</tbody>
</table>

Fig. 26 FER versus the crossover probability \( \alpha \) for regular column-weight-four MacKay code with code rate 0.89 and the code length \( n = 1998 \).
Table IV Thresholds of different decoders for column-weight-four codes with row degree $\rho$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Rate</th>
<th>Gallager A</th>
<th>Gallager B</th>
<th>Algorithm E</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.0474</td>
<td>0.0516</td>
<td>0.0583</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.0175</td>
<td>0.0175</td>
<td>0.0240</td>
</tr>
<tr>
<td>32</td>
<td>0.875</td>
<td>0.00585</td>
<td>0.00585</td>
<td>0.00935</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Rate</th>
<th>Gallager A</th>
<th>Gallager B</th>
<th>Algorithm E</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.0467</td>
<td>0.0509</td>
<td>0.0552</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.0175</td>
<td>0.0165</td>
<td>0.0175</td>
</tr>
<tr>
<td>32</td>
<td>0.875</td>
<td>0.00585</td>
<td>0.00562</td>
<td>0.00486</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Rate</th>
<th>Gallager A</th>
<th>Gallager B</th>
<th>Algorithm E</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.0467</td>
<td>0.0567</td>
<td>0.0532</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.0175</td>
<td>0.0177</td>
<td>0.0168</td>
</tr>
<tr>
<td>32</td>
<td>0.875</td>
<td>0.00585</td>
<td>0.00587</td>
<td>0.00486</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Rate</th>
<th>Gallager A</th>
<th>Gallager B</th>
<th>Algorithm E</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.5</td>
<td>×</td>
<td>0.0467</td>
<td>0.0657</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>×</td>
<td>0.0218</td>
<td>0.0222</td>
</tr>
<tr>
<td>32</td>
<td>0.875</td>
<td>×</td>
<td>0.00921</td>
<td>0.00755</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Rate</th>
<th>Dynamic two-bit decoder with $S = 2$ and $W = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.0638</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.0249</td>
</tr>
<tr>
<td>32</td>
<td>0.875</td>
<td>0.00953</td>
</tr>
</tbody>
</table>

For the particular two-bit decoder with $(C, S, W) = (2, 2, 1)$, we find the sufficient conditions that guarantees correction of three errors. We consider only left-regular codes with column-weight-four. The problem of guaranteed error correction capability assumes significance in the error floor region. Roughly speaking, error floor is the abrupt degradation in the FER performance in the high SNR regime. The guaranteed error correction capability of column-weight-three LDPC codes under the Gallager A algorithm is now completely understood. For column-weight-four LDPC codes under the Gallager B algorithm, sufficient conditions to guarantee all error patterns with up to three errors have been derived by Chilappagari et al. The conditions derived in by them impose constraints on the least number of neighboring check nodes for a given set of variable nodes. The conditions that we derive are similar, but impose fewer constraints on the Tanner graph, thereby resulting in codes with higher rates for the same length.

The irregular expansion theorem which we state and prove in this work provides the sufficient conditions to guarantee correction of three errors. It
goes as follows. Let $G$ be the Tanner graph of a column-weight-four LDPC code with no 4-cycles, satisfying the following expansion conditions: each variable subset of size 4 has at least 11 neighbors, each one of size 5 at least 12 neighbors, each one of size 6 at least 14 neighbors, each one of size 8 at least 16 neighbors and each one of size 9 at least 18 neighbors. Then the two-bit decoder, with $C = 2$, $S = 2$ and $W = 1$, can correct up to three errors in the codeword within three iterations, if and only if these conditions are satisfied.

We also carry out asymptotic analysis of different two-bit decoders and determine the decoding thresholds for each decoder. Table IV summarizes the results. It can be seen that the two-bit decoders have better decoding thresholds than the one-bit Gallager A and B decoders. Fig. 26 shows the frame error rate performance of the $(2, 2, 1)$ two-bit decoder compared to Gallager B decoder. We observe better waterfall performance using the two-bit decoder, and about 1dB gain in the error-floor.

**Joint Message-Passing Symbol-Decoding of LDPC Coded Signals over Partial-Response Channels**

LDPC codes have been shown to achieve excellent error rate performance over channels with memory, such as magnetic storage and optical communication channels. For reasons of bandwidth efficiency, these inter-symbol interference (ISI) channels are equalized to a partial-response (PR) target with relatively small memory compared to the unequalized channel. Any LDPC decoder must cope with the controlled amount of ISI introduced by the PR. For an uncoded system, the channel input sequence is optimally detected in the presence of ISI by the Viterbi algorithm. For an LDPC coded system, good error rate performance can be achieved by using the turbo principle where information is iteratively passed back and forth between a soft-input soft-output (SISO) detector and a SISO LDPC decoder. The ISI in the channel output is eliminated by a detector using the channel information unknown to the decoder, and subsequently, the output is decoded by a LDPC decoder using the code structure information unknown to the detector. Since, Viterbi algorithm produces only hard decisions, algorithms like soft-output Viterbi algorithm (SOVA) and Bahl-Cocke-Jelinek- Raviv (BCJR) algorithm are used for SISO detection and the well-known sum-product algorithm (SPA) is used for SISO decoding. This iterative algorithm is known as **turbo equalizer**. Though sub-optimal, it is the best and most widely used decoder known today.

The performance can be improved beyond what is achieved by turbo-equalizer, if both channel and the code information are simultaneously used to make decisions on the channel inputs. This is referred to as joint detection and decoding or simply as joint decoding. A practical and popular approach in this direction has been to design message-passing (MP) decoding
algorithms that operate on a graph, which represents the constraints imposed by both the channel and the LDPC code. This problem has been addressed by Kurkoski et al., who introduced parallel bit-based and state-based MP algorithms for channel detection. However, the joint MP decoder worked well only for channels with unit memory lengths and for higher memory lengths, it achieved at best a performance as good as the turbo equalizer. The challenge in jointly using both the channel and code information arises from the fact that the channel imposes constraints on the channel output sequences, whereas the code imposes constraints on the channel input sequences. The idea motivating our approach is the observation that by imposing constraints on the channel input sequences, the code also imposes certain constraints on the noiseless channel output sequences.

In this work, we modify the LDPC MP decoder to produce information on the noiseless channel output symbols rather than on channel inputs. This enables us to combine it with a modified version of the state-based MP detector to design a joint decoder that significantly surpasses the performance of the turbo-equalizer. The joint decoder estimates a posteriori probabilities (APPs) of channel output symbols, from which APPs of channel inputs are derived. This algorithm can be used irrespective of the channel memory length, although it may perform relatively better for channels with small memory. Fig. 27 shows the system model where the channel response is denoted by \( h(D) \) or the corresponding vector \( \mathbf{h} \).

![Fig. 27 Block diagram of a PR system. Decoder is either turbo-equalizer (upper branch) or joint decoder (lower branch) and noise is modeled as additive white Gaussian.](image)

The trellis of a channel represents the constraints imposed on the range of noiseless channel output sequences. A channel trellis can be given as a factor graph shown in Fig. 28, where \( q_0, q_1, \ldots, q_{n+1} \) represents state (hidden) variables, \( x_0, x_1, \ldots, x_n \) represents channel inputs and \( y_0, y_1, \ldots, y_n \) represents noiseless channel outputs.
We modify the factor graph representing the channel constraints to remove the need for state variables and correspondingly alter the detection algorithm. The graph is now simply represented as a bipartite graph $G$ shown in Fig. 29.

![Fig. 28 Generalized factor graph representation of a PR channel trellis.](image)

We extend the graphical model described earlier to include constraints imposed by the parity checks of the LDPC code. The tripartite graph shown in Fig. 30 is obtained by including the parity check nodes to the channel graph of Fig. 29. Using this combined graph, a joint decoding algorithm is developed that estimates the symbol APPs using both the channel and the code information simultaneously.

We illustrate the performance of the joint symbol-decoding algorithm by simulating an LDPC coded PR system, where the channel is given by the impulse response $[1, 0, -1]$ (PR4), and the LDPC code is of length 1908 and rate 0.89. The channel output sequences are decoded by using both turbo-equalizer and the joint symbol-decoder. The performance comparison is shown in Fig. 31. At a signal-to-noise ratio (SNR) of 5.4 dB the BER obtained by the joint decoder is almost an order of magnitude better than the turbo-equalizer. Also, the figure suggests that the gain increases with increasing SNR.

![Fig. 29 A graph that represents constraints imposed by the channel on the noiseless channel output sequences.](image)
Fig. 30 A graph that represents constraints imposed by the channel and the parity checks of the LDPC code on the noiseless channel output sequences. Parity checks imposes certain constraints on the channel output sequences by imposing constraints on the channel input sequences.

Fig. 31 Bit error rate comparison of (1908, 212) random LDPC code on a PR4 channel when decoded using turbo-equalizer and the joint decoder.
3. Describe the opportunities for training and development provided by your project.

1. Three graduate students are working on the project.
2. A new course on free-space optical communications has been proposed and will discuss physical channels as well as novel modulation, multiplexing, and advanced error control coding techniques relevant to FSO.
3. A new course on quantum information processing and quantum error correction is developed and will be offered in Fall 2009 for the first time.
4. Advanced error control coding techniques, LDPC codes and iterative decoding are introduced into ECE 435, ECE 535, ECE 537, ECE 632, and ECE 638.

4. Describe outreach activities your project has undertaken.

The PIs has participated in organizing a couple of conferences:

a. Prof. Djordjevic is serving a Technical Program Committee Member for Optical Communications at the 22nd Annual Meeting of The IEEE Lasers & Electro-Optics Society (now Photonics Society), Belek-Antalya, Turkey, 4 - 8 October 2009.

b. Prof. Vasic is serving as the organizer of the workshop on Genomic Error Correction, Cergy- Pointoise, France, June 3-5, 2009.

c. Prof. Vasic served as a Technical Program Chair Member of 5th International Symposium on Turbo Codes & Related Topics 2008, Lausanne, Switzerland, Sept. 2-5, 2008.

Contributions

Now we invite you to explain ways in which your work, your findings, and specific products of your project are significant. Describe the unique contributions, major accomplishments, innovations and successes of your project relative to:

1. The principal discipline(s) of the project;

Adaptive Modulation and Coding for Communication over the Atmospheric Turbulence Channels

We proposed two adaptive modulation and adaptive coding schemes with RF feedback, which provide robust and spectrally efficient transmission over free-space optical channels, namely: (i) variable-rate and variable-power adaptation scheme, and (ii) truncated channel inversion scheme. It was
demonstrated that truncated channel inversion scheme is sufficient in the weak turbulence regime. The use of variable-power variable-rate scheme is needed in strong turbulence regime. We also proposed adaptive LDPC-coded modulation scheme, which is able to tolerate the deep fades in the order of 35 dB and above (at $P_b=10^{-6}$, $R/B=4$ bits/s/Hz) in the strong turbulence regime. With proposed adaptive coding scheme, the communication in saturation regime is possible.

**Communication over Hybrid Free-Space Optical - Wireless Fading Channels**

We proposed a hybrid FSO-RF system suitable for transmission at high rates and proposed two adaptation policies: (i) optimum variable-rate and variable-power adaptation scheme, and (ii) truncated channel inversion scheme. The optimum adaptation scheme is derived by maximizing the total channel capacity. The proposed scheme outperforms FSO optimum adaptation scheme by 3.39 dB at BER of $10^{-6}$ and spectral efficiency of 2 bits/s/Hz, for the strong atmospheric turbulence in FSO channel and for Rayleigh fading in RF channel. With LDPC coding, the proposed scheme outperforms corresponding LDPC-coded FSO scheme at BER of $10^{-6}$ in the strong atmospheric turbulence for $R/B=4$ bits/s/Hz as follows: 5.25 dB in Rayleigh RF fading, 5.51 dB in Nakagami $m=2$ RF fading and 5.63 in $\alpha=3$, $\mu=2$ RF fading. With proposed adaptive coding scheme even communication in saturation regime is possible. We also reported the spectral efficiencies for proposed hybrid FSO-RF system with adaptive modulation/coding and compare them against FSO system only. The proposed hybrid FSO-RF system significantly outperforms FSO system in terms of spectral efficiency.

**Polarization-Multiplexed Coded-OFDM**

We proposed a particular polarization multiplexed coded-OFDM scheme suitable for use in beyond 100 Gb/s optical transmission and 100 Gb/s Ethernet. By selecting the OFDM signal bandwidth to 10 GHz, and by using 32-QAM and polarization multiplexing the aggregate data rate of 100 Gb/s is achieved. The spectral efficiency of proposed scheme is 10 bits/s/Hz. For the same system parameters, the corresponding polarization diversity coded-OFDM would have the aggregate rate of 50 Gb/s, which is insufficient for 100 Gb/s Ethernet. Therefore, the proposed coded-modulation scheme achieves the aggregate rate of 100 Gb/s, while employing the mature 10 Gb/s technology. The proposed coded-modulation scheme is insensitive to PMD, while in corresponding single carrier systems the PMD introduces severe performance degradation. The results of simulations indicate that even 1200 ps of DGD, in polarization multiplexed coded-OFDM systems with aggregate data rate of 100 Gb/s, can completely be compensated with negligible penalty. In contrast to the PMD turbo equalization scheme whose
complexity grows exponentially as DGD increases (due to the exponential increase in complexity of BCJR algorithm), the complexity of the proposed scheme is essentially independent on the level of DGD as long as the guard interval is longer than maximum DGD. We also described how to determine the symbols log-likelihood ratios in the presence of laser phase noise.

**Coded-OFDM in Hybrid Optical Networks**

We proposed a particular polarization-multiplexed coded-OFDM scheme suitable for use in hybrid FSO – fiber-optics networks. This scheme is able simultaneously to deal with atmospheric turbulence, chromatic dispersion and PMD. We show that PMD can be compensated even in the presence of atmospheric turbulence. We have found that the most of degradation is coming from FSO channel. The proposed coded-modulation scheme supports 100 Gb/s per wavelength transmission and 100 Gb/s Ethernet. We compare the BER performance of two schemes with fixed aggregate rate of 100 Gb/s. The first scheme employs 32-QAM and occupies 10 GHz (with bandwidth efficiency of 10 bits/s/Hz), while the second scheme employs 16-QAM and occupies 12.5 GHz (with bandwidth efficiency of 8 bits/s/Hz). We have found that the 16-QAM scheme outperforms 32-QAM by 1.9 dB at $BER$ of $10^{-6}$, although it has a higher bandwidth, because larger constellation schemes are more sensitive to the atmospheric turbulence. The net coded gain improvement (defined at $BER$ of $10^{-6}$) of LDPC-coded 16-QAM OFDM, for $\sigma_X=0.1$ and $\sigma_Y=0.01$, over uncoded-OFDM 11.19 dB. The improvements due to transmission diversity are moderate in weak turbulence, and significant in moderate turbulence regime.

**LDPC-Coded Turbo Equalization**

We describe an LDPC-coded turbo equalization scheme, as a universal scheme that can be used simultaneously for: (i) suppression of fiber nonlinearities, (ii) PMD compensation, and (iii) chromatic dispersion compensation in multilevel coded-modulation schemes. The LDPC-coded turbo equalizer is composed of two ingredients: (i) the multilevel BCJR algorithm based equalizer, and (ii) the LDPC decoder. The proposed scheme employs large girth quasi-cyclic LDPC codes as channel codes. We show that even transmission distance of 8160 km with aggregate rate of 100 Gb/s, based on QPSK modulation, can be achieved without any countable error, when BJCR equalizer of state memory $m=1$ is employed and LDPC$(16935,13550)$ code is used as channel code. With polarization multiplexing even two channels of aggregate rate 100 Gb/s can be transmitted using the same wavelength channel. The proposed multilevel turbo equalizer can be used for 100 Gb/s upgrade using the installed 10 Gb/s optical transmission systems. By using the multilevel coded-modulation
scheme proposed here not only transmission but also all signal processing related to detection and decoding are effectively done at lower symbol rates (e.g., 50 GS/s), where dealing with nonlinear effects and PMD is more manageable, while keeping the aggregate rate at 100 Gb/s or above.

**FPGA Implementation of Large Girth Quasi-Cyclic (QC) LDPC Codes**

Due to their Shannon-limit-approaching performance and low-complexity LDPC codes have been used for forward error correction in a broad-range of communication and storage systems. In addition to its low-complexity, the iterative decoding algorithm used for decoding LDPC codes is inherently parallel. To exploit the parallelism at a larger extent, a significant amount of research has been directed toward field-programmable gate-array (FPGA) implementation of LDPC decoders in recent years. We proposed an alternative FPGA implementation of decoders for quasi-cyclic LDPC codes based on MitrionC hardware programming language. We provided FPGA results and compared them against the traditional software simulation results. We observed a close agreement between the hardware and software implementations. Furthermore, the performance of the min-sum algorithm is only 0.2 dB worse than that of the min-sum-with-correction-term algorithm at the BER of $10^{-6}$ and the gap gets closer as the Q factor increases. The net coding gain of the min-sum algorithm for the same LDPC code at BER of $10^{-6}$ is found to be 10.3 dB.

**Nonbinary QC LDPC Codes**

We showed first that it is not always possible to construct the parity-check matrix of a nonbinary (NB) QC-LDPC code by randomly assigning non-zero elements from the desired field to the 1s in the parity-check matrix of a corresponding B-QC-LDPC code. Then, we determined a necessary and sufficient condition in order for the null-space of a non-binary matrix obtained in this fashion to define an NB-QC-LDPC code. Using this theorem, we proposed a general scheme for constructing NB-QC-LDPC codes, and discussed some other schemes that might be employed under different design goals. With the help of simulations, we concluded that MD-FFTQSPA-based decoding yields almost optimum results when a maximum of 50 iterations is used. In addition, our simulations revealed that there is no performance difference between an NB-QC-LDPC code and a NB-LDPC code constructed using random element assignment scheme when both codes are based on the same B-QC-LDPC code. We also demonstrated that high-rate, regular NB-QC-LDPC codes are excellent candidates for high-rate applications due to their impressive coding gains, simpler encoders and comparably lower complexity decoders than non-quasi-cyclic NB-LDPC codes. Our future work includes a comprehensive study of the systematic
encoders for NB-QC-LDPC codes, hardware implementations of encoders and decoders for NB-QC-LDPC codes and the integration of these hardware elements into a real-world high-rate communication system.

**Orbital angular momentum-based multi-channel communications**

We have studied the feasibility of a multi-channel OAM terrestrial FSO link and have quantified the channel crosstalk as a function of turbulence strength, number of simultaneous channels, and signal-to-noise ratio. Simulations that were carried out based on numerical methods verified that optical turbulence induces OAM crosstalk and that the average crosstalk between channels grows with turbulence strength. For each transmitted OAM state we have (i) quantified the efficiency (% of power remaining in transmitted channel) of each channel in terms of the turbulence strength and (ii) quantified the average crosstalk observed on all channels in the studied range, in terms of turbulence strength. By modeling an OAM mode as a BSC channel with probability of flip error being a function of the channel efficiency, the crosstalk induced by the other constituent channels, and by detector noise, the optimal set of OAM mode numbers in the sense of maximizing the aggregate capacity were determined for a prescribed number of channels, at each value of SNR and turbulence strength considered.

**On trapping sets and guaranteed error correction capability of LDPC and GLDPC codes**

The relation between the girth and the guaranteed error correction capability of regular LDPC codes when decoded using the bit flipping (serial and parallel) algorithms is investigated. We derived lower bounds on the guaranteed error correction capability of LDPC and GLDPC codes by finding bounds on the number of nodes that have the required expansion. The bounds depend on two important code parameters namely: column-weight and girth. Since the relations between rate, column weight, girth and code length are well explored in the literature, bounds on the code length needed to achieve certain error correction capability can be derived for different column weights. An upper bound on the guaranteed error correction capability is established by studying the sizes of smallest possible trapping sets. The results are extended to generalized LDPC codes. It is shown that generalized LDPC codes can correct a linear fraction of errors under the parallel bit flipping algorithm when the underlying Tanner graph is a good expander. It is also shown that the bound cannot be improved when $E$ is even, by studying a class of trapping sets. A lower bound on the size of variable node sets which have the required expansion is established. The bounds presented in the paper serve as guidelines in choosing code
parameters in practical scenarios. The results can be extended to message passing algorithms as well. There is a considerable gap between the lower bounds and upper bounds on the error correction capability. Deriving lower bounds based on the sizes of trapping sets rather than expansion may possibly lead to bridging this gap. Our approach can be used to derive bounds on the guaranteed erasure recovery capability for iterative decoding on the BEC by finding the number of variable nodes which expand by a factor of $E/2$.

**Two-bit Message Passing Decoders for LDPC Codes over the Binary Symmetric Channel**

The problem of guaranteed error correction capability assumes significance in the error floor region of the performance of LDPC codes. Roughly speaking, error floor is the abrupt degradation in the FER performance in the high SNR regime. The error floor phenomenon has been attributed to the presence of a few harmful configurations in the Tanner graph of the code, variously known as stopping sets (for the BEC), near codewords, trapping sets (for iterative decoding on the BSC and the AWGN) and pseudo-codewords (for linear programming decoding). While girth optimized codes have been known to perform well in general, the code length and the degree distribution place a fundamental limit on the best achievable girth. Hence, additional constraints on the Tanner graph are required to ensure better error floor performance.

In this work, we proposed a class of two-bit decoders. We have focused on a specific two-bit decoder for which we have derived necessary and sufficient conditions on the Tanner graph for a code with Tanner graph of girth six to correct any three errors within three iterations. These conditions are weaker than the conditions for a code to correct three errors when it is decoded with Gallager B algorithm, which uses only one bit. We have computed thresholds for various two-bit decoders, and shown that the decoder, for which the previous conditions have been derived, has better thresholds than one-bit decoders, like Gallager A and B. Finally, we have compared the frame error rate performance of the two-bit decoder and Gallager B algorithm for decoding a column-weight-four code with high rate. The two-bit decoder performs better than Gallager B both in the waterfall and in the error-floor region. This illustrates that it is interesting to use two bits rather than one bit for decoding.

**Joint Message-Passing Symbol-Decoding of LDPC Coded Signals over Partial-Response Channels**

In this work, we consider the problem of joint detection and decoding of LDPC coded signals over partial response channels. In order to jointly use
the channel and code information, the LDPC decoder is modified to produce information on channel output symbols rather than on channel inputs. This is combined with a message-passing detector to develop a joint decoder that estimates channel input APPs by first estimating channel output symbol APPs. A method to graphically represent the constraints imposed by the channel and the code on the channel output sequence was introduced. This enabled the design of a detector and decoder that estimates a posteriori probabilities of noiseless channel output symbols rather than binary channel inputs. By running the sum-product algorithm (SPA) on this graph, a joint decoder is obtained that is shown to perform significantly better than the turbo-equalizer. The performance of this decoder is shown to significantly outperform that of the turbo-equalizer for a random LDPC code of rate 0.89.

2. Other disciplines of science or engineering;
None

3. The development of human resources;
None

4. The physical, institutional, or information resources that form the infrastructure for research and education; and
None

5. Other aspects of public welfare beyond science and engineering
such as commercial technology, the economy, cost-efficient environmental protection, or solutions to social problems.

Publications and Products

In this section, you will be asked to describe the tangible products coming out of your project. Specifically:

1. What have you published as a result of this work?

Journal publications


[J18] I. B. Djordjevic, "Coded-OFDM in Hybrid Optical Networks," IET Optoelectronics, accepted for publication.

Conference publications


Papers Submitted for Journal Publication


Books or other non-periodical, one-time publications


2. What Web site or other Internet site have you created?

http://www.ece.arizona.edu/~fso-comm/index.html

3. What other specific products (databases, physical collections, educational aids, software, instruments, or the like) have you developed?

Participants:

1. Ivan B. Djordjevic
   Over the past year, Dr. Djordjevic was working on hybrid and heterogeneous optical networks; the mitigation of both linear and nonlinear channel impairments in hybrid optical networks; the channel capacity evaluation of FSO, fiber-optics and hybrid optical networks; development of adaptive coding for hybrid FSO-RF communication; and quantum error correction.

2. Bane Vasic

3. Mark A. Neifeld

4. Murat Arabaci
   Murat Arabaci is working on the design and FPGA implementation of binary and nonbinary quasi-cyclic LDPC encoders and decoders suitable for high speed implementation; as well as on adaptive nonbinary LDPC-coded modulation for high-speed applications.

5. Lyubomir L. Minkov
   Lyubomir Minkov is working on experimental demonstrations of various coded-modulation scenarios and turbo equalization for both fiber-optics and free-space optical channels.

6. Shiva Planjery
   Shiva Planjery is working on trapping sets and guaranteed error correction capability of LDPC and GLDPC codes, two-bit message passing decoders, and more recently on joint message-passing symbol-decoding of LDPC coded signals over partial-response channels.

7. Ruchicka Verma (co-supervised with Prof. Akoglu)
   In spring 2008, she was working 10 hours per week on implementation of LDPC decoders in Verilog.
Have You had Other Collaborators or Contacts?

1. Lei Xu, NEC Labs, Princeton, NJ
2. Ting Wang, NEC Labs, Princeton, NJ
3. Franko Kueppers, Optical Sciences, University of Arizona
4. Milorad Cvijetic, NEC America, Inc.
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15. Ali Akoglu, Department of ECE, University of Arizona
16. Stojan Denic, Toshiba Wireless, Bristol, UK
17. Goran T. Djordjevic, University of Nis, Serbia