ECE 583
Lectures 15
RADAR
History and Basics
A BIT OF HISTORY

- RADAR -

The acronym - RADAR is an acronym for Radio Detection and Ranging

The Start:
The thought/concept of using propagating EM waves began almost with the start of experiments to understand EM waves. H. Hertz in 1886 performed experiments showing that reflected signals from incident EM waves could be measured from metallic and dielectric bodies. Huelsmeyer in Germany experimented with the detection of "radio waves" in 1903 and obtained a patent in 1904 for an obstacle detector and ship navigational device.
The 1920's:

* Marconi in 1922 in a speech delivered to a meeting of the Institute of Radio Engineers (later to become a part of IEEE) spoke of the use of a radar type device on a ship to "immediately reveal the presence and bearing of other ships in fog and thick weather."

* The US Navy, through the Naval Research Laboratory (NRL), began in 1922 investigating the use of radio waves for ship detection after noting signal interruptions in ship to ship and ship to shore communications. They used 60 MHz CW signals and separated transmitter and receiver, bistatic configuration, which was rather cumbersome and not as precise in ship location as later pulsed systems. As a result, acceptance was slow to catch on for field use of the CW-bistatic approach.
The first detection of an aircraft by radar occurred accidentally in 1930 with an NRL CW-bistatic system doing direction finding experiments and detecting an aircraft 2 miles away on the ground. By 1932 NRL demonstrated the detection of flying aircraft to distances of 50 miles. It was recognized that pulsed radar would be more effective for "target" detection and location, and a 60 MHz system attempt with pulses was made in 1935, but wasn't very effective. It was recognized that working at higher frequencies would be better, but higher frequency sources were yet to be developed with sufficiently high power. A 200 MHz system with 6 kW of power was finally demonstrated in 1938, being able to detect ships to 50 miles, and this system was then manufactured for the US Navy and placed on about 20 ships.
The 1930's (Continued):

The US Army also became interested in radar in about 1930 and pursued development through the Signal Corps. After visiting the NRL and seeing the effectiveness of the pulsed radar at 200 MHz, the Army developed their own version, with an antiaircraft battery version put in operation by 1938. An improved, long-range radar for "early warning" was developed by 1939 and then put in production for field use by 1940. Six of these systems were in operation in Hawaii around the Pearl Harbor area in December, 1941 at the time of the Japanese attack, and signals were observed of the incoming attack aircraft. However, methodologies/techniques for effective and accurate interpretation of radar "echoes" were in their "infancy" and the signals were dismissed as "unintelligible noise."
The 1930's (Continued):

The British development of radar began later than that of the US, starting in 1935, but they moved quickly to develop workable field systems, spurred by the impending threat of war. By 1936 they had a field deployable pulsed radar system operating at 25 MHz which could detect aircraft to a range of 90 miles. They proceeded with the development of the Chain Home (CH) series or radar stations along the coast of England, in place by 1938, which served to provide early warning of incoming German aircraft and were significant in turning the tide of the "Battle of Britain" in favor of the British.
The 1940's

The British and the US worked independently in radar development until the middle of 1940 when a British delegation came to the US to encourage more US support and, to help sway favor for such support, revealed their secret development of the cavity-magnetron, capable of multi-kW power output at frequencies in the GHz range (~ 3 GHz). This led to the establishment of a concerted crash program at MIT with the establishment of the Radiation Laboratory (the Rad Lab) which through the war years developed many different radar systems, in particular, aircraft-interception (AI) radar systems mounted on aircraft, made possible by operating at a high enough frequency to permit a small, aircraft viable antenna.
Besides the war efforts of the US and Britain, several other countries, including Germany, France, Russia, Italy and Japan also developed and implemented radars shortly before and during WW II. By the end of the war, radars had become quite complex and included features that could be attributed to developments from many countries.

In the US following the war, building on the great radar advances that had been made at the MIT Rad Lab, the field of radar meteorology came into being. Pioneers in this field, having served as "weathermen" in the military and having gone through MIT Rad Lab training, included Louis Batten (Prof. here at UA, now deceased) and David Atlas (long retired, but still alive). I had the pleasure and benefit of some mentoring by both of them.
The most common capability associated with RADARs is the ability to determine the range $R$ to target from the echo received back from a short pulse transmitted to the target.

If the total time for a pulse to reach the target and return from the target at range $R$ is $T_T$ then the radar range is given by

$$R = \frac{c(T_T)}{2}$$

Where $c$ is the speed of light ($3 \times 10^8$ m/s).
Radar Pulses

Radar pulses are usually transmitted at a pulse repetition frequency (PRF) of $f_p$. Thus, there is a maximum unambiguous range $R_{\text{max}}$, from which an echo can associated with a given pulse, given by:

$$R_{\text{max}} = \frac{c}{2f_p}.$$  

For example is $f_p = 1$ kHz then $R_{\text{max}} = 150$ km.
FIGURE 1.2
Range measurement through pulsing.

Total $T_T = \frac{2R}{c}$
Fig. 1.1. Block diagram of radar set

Operating Frequencies of Weather Radar

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Fig. 1.2. Various typical antenna reflectors and associated radar beams
Figure 1.2 Block diagram of a pulse radar.
Development of the Radar Point Target Detection Range Equation

Consider a pulsed radar system that transmits a rectangular pulse of peak power $P_t$ and pulse duration $t_P$.

If the pulse is transmitted uniformly or isotropically without loss in all directions, the transmitted power density, or irradiance at a distance $R$ from the transmitter, $S_t(R)$, would just be $P_t$ divided by the area of a sphere of radius $R$, $4\pi R^2$, or

$$S_t(R) = \frac{P_t}{4\pi R^2} \quad \text{for isotropic transmitter}$$
However, to have directional selectivity, radars use narrow beam antennas to sense in a given direction. This directional selectivity is characterized by the antenna gain $G$, which defines the added amount of power over an isotropic antenna that the antenna propagates in its main beam direction. Thus for a radar transmitter antenna with gain $G_t$, the transmitted power density at range $R$ in the antenna beam is given by

$$S_t(R) = \frac{P_t G_t}{4 \pi R^2}$$
If a scattering target of scattering area \( A_s \) is encountered by the transmitted radar pulse at range \( R \), a fraction of the power intercepted by the target will be backscattered to the radar receiver which is proportional to \( S \hat{r} A_s \). The amount of backscattering from the target is generally characterized in terms of the target radar backscattering cross section, \( \sigma_{rb} \).
It defines the required target area, assuming isotropic scattering, to yield the observed backscattered power density received back at the radar receiver, $S^\wedge$, due to backscattering from range $R$.

Thus,

$$S_r (R) = \frac{S_t (R) \sigma_{rb}}{4 \pi R^2}$$

or

$$S_r (R) = \frac{P_t G_t \sigma_{rb}}{(4 \pi)^2 R^4}$$
The radar receiving antenna has some receiver aperture area $A_r$, so the received power due to backscattering at range $R$ by a target of radar backscattering cross section $\sigma_{rb}$ is given by power backscattered from a point target at range $R$.

$$
P_r(R) = \frac{P_t G_t A_r \sigma_{rb}}{(4 \pi)^2 R^4}
$$

This is the point target radar range equation, defining the instantaneous
The same antenna is used to both transmit and receive for most radars, so terms can be rearranged in terms of gains or receiver areas using the basic gain and receiver aperture relation for an antenna:

\[
G = \frac{4 \pi A}{\lambda^2}
\]

Thus, the range equation may be written in terms of A,G or both, yielding three different representations.
These three different representations are:

\[ P_r (R) = \frac{P_t \, G \, A}{(4 \pi)^2} \, \sigma_{rb} \, R^4 \]

\[ = \frac{P_t \, G \, \lambda^2 \, \sigma}{(4 \pi)^3} \, R^4 \]

\[ = \frac{P_t \, A^2 \, \sigma}{(4 \pi) \lambda^2} \, R^4 \]

Where it is assumed that \( A\hat{t}=A\hat{r}=A \) and \( G\hat{t}=G\hat{r}=G \).
Of particular importance in antenna design is the maximum directivity $D_0$, which is given by (3.19) with $F_n = 1$ in the numerator:

$$
\mathcal{G}_0 = D_0 = \frac{4\pi}{\iint \overline{F_n}(\theta, \phi) d\Omega} = \frac{4\pi}{\overline{\Omega}_p}, \quad \text{for no loss in antenna} \tag{3.21}
$$

where use has been made of (3.9). In terms of $D_0$, the directivity $D$ in any direction $(\theta, \phi)$ may then be written in the form

$$
D(\theta, \phi) = D_0 \overline{F_n}(\theta, \phi). \tag{3.22}
$$

For an antenna with a single main lobe pointing in the $z$-direction as shown in Fig. 3.5, the pattern solid angle $\overline{\Omega}_p$ is approximately equal to the product of the half-power beamwidths $\beta_{xz}$ and $\beta_{yz}$ (in radians):

$$
\overline{\Omega}_p \approx \beta_{xz} \beta_{yz},
$$

and therefore,

$$
D_0 = \frac{4\pi}{\overline{\Omega}_p} \approx \frac{4\pi}{\beta_{xz} \beta_{yz}}. \tag{3.23}
$$
Fig. 3.5 The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes, i.e., $\Omega_p \approx \beta_{xz} \beta_{yz}$. 
Fig. 3.3 Representative plots of the normalized radiation pattern of a microwave antenna in (a) polar form and (b) rectangular form.
Fig. 3.4 Three-dimensional antenna patterns. (a) Three-dimensional pencil-beam pattern of the AN/FPQ-6 radar antenna. Base is at $-40$ dB relative to the peak (from Skolnik, 1970). (b) Three-dimensional pattern of the 22.235-GHz Scanning Microwave Spectrometer (SCAMS) antenna flown on Nimbus 6. Plot is $\pm 100^\circ$ from axis and the base is at $-50$ dB relative to the peak (from Sissala, 1975).
Since the transmitted and received signals from two successive pulses are isolated in time, we can consider the received signal for a single pulse without concerning ourselves about the fact that it is a repetitive wave form. Let the transmitter pulse power be described by the amplitude and shape function $P_t(t)$. The received signal for a point target is simply a delayed replica of the transmitted signal, as indicated by

$$P_r(t) = A P_t(t - T) = A P_t \left( t - \frac{2R}{c} \right).$$  \hspace{1cm} (1.1)

Here $A$ is an amplitude factor that takes into account the change in signal power level associated with propagation through the atmosphere to the target, scattering from the target, and traveling back to the receiver. Typically $A$ is a very small number. The time delay, $T$, is related to the velocity of propagation of the wave, $c$, by the relationship $T = 2R/c$, where $R$ is the distance from radar to target, called the range. The factor 2 appears because the signal must travel the distance $R$ twice, once in going from radar to target and once in returning from target to radar.
Target Range Discrimination

The instantaneous backscattered power, \( P_r(R) \), from a point target at range \( R \) lasts for a time equal to the radar pulse duration, \( t_p \), and has the temporal shape of the transmitted radar pulse. The range increment, \( \Delta R \), associated with time \( t_p \) is

\[
\Delta R = \frac{c t_p}{2}
\]
Thus, given one target at range $R_1$, a second target at range $R_2$ cannot be unambiguously discriminated from the target at range $R_1$ unless they are separated by a range difference equal or greater than a $\Delta R_{\text{min}}$ of:

$$\Delta R_{\text{min}} \geq \frac{ct_p}{2}$$
The Many Scatterer Problem or The Radar Meteorology Range Equation

The radar equation as formulated is alright to apply to situations where a single major scatterer such as a ship, airplane, the earth’s surface, etc. is involved. However, in atmospheric remote sensing problems, there are typically many scatterers within the radar pulse. For such cases, it is necessary to properly formulate the scattering properties of the scatterers. In particular, many remote sensing problems are where the scattering properties of the scatterers are what are being remotely sensed.

For many scatterers within the radar pulse, each with a radar scattering (backscatter) cross section $\sigma_{rb_i}$ units of m2, the total radar signal is

$$P_r (R) = \frac{P_t A^2}{(4 \pi)^2 R^4} \sum_{i=1}^{N} \sigma_{rb_i}$$

for N scatterers within the “instantaneous” scattering volume of the radar pulse.

The instantaneous scattering volume is a volume equal to the pulse/receiver field-of-view cross section area at R times 1/2 the pulse length or depth.

It is the volume from which scattering can occur and still yield scattered power returning at the same instant t to the receiver.

If the antenna transmits the pulse with a full-angle half-power conical beam divergence of $\theta_t$, the instantaneous volume at range R is

$$V_{ins} = \pi \left( R \frac{\theta_t}{2} \right)^2 \frac{l_p}{2} = R^2 \Omega_T \frac{l_p}{2}$$

Where $l_p = c \tau_p$ and $\tau_p$ is the pulse duration.
Instantaneous Scattering

Volume

\[ \text{Orbi-scatterers in volume} \]

\[ \ell_p = \text{radar pulse length, } c \ell_p \]

Cross section of volume at range \( R \) is

\[ \pi \left( \frac{\ell_p}{2} \right)^2 = R^2 \Omega_L \]

\[ \Omega_L = \pi \left( \frac{\ell_p}{2} \right)^2 \]
If we assume the scatterers are uniformly distributed over the instantaneous scattering volume, or we just deal with an average, we may sum the per scatterer cross sections $\sigma_i$ over a unit volume to get the radar unit volume backscattering cross section, $\beta_{rb}$:

$$\beta_{rb} = \sum_{\text{unit vol}} \sigma_{rb_i} \left\{ \frac{\text{units of}}{m^2/m^3 - sr} \right\}$$

Sometimes called the radar reflectivity.

Then, $P_r(R)$ may be expressed by

$$P_R(R) = \frac{P_t A^2}{(4\pi)^2 R^4} \cdot \pi R^2 \left( \frac{\theta_t}{2} \right)^2 \frac{l_p}{2} \cdot \beta_{rb}$$

or

$$= \frac{P_t AG}{(4\pi)^2 R^2} \cdot \pi \left( \frac{\theta_t}{2} \right)^2 \frac{l_p}{2} \cdot \beta_{rb}$$

after cancelling terms and recalling that

$$\frac{A}{\lambda^2} = \frac{G}{4\pi}$$
Then also recalling that
\[ \Omega_i = \pi \left( \frac{\sigma_i}{2} \right)^2 = \frac{4\pi}{G} \]

\( P_r (R) \) becomes
\[ P_r (R) = \frac{P_t A l_p}{2 (4\pi) R^2} \beta_{rb} \]
or
\[ P_r (R) = \frac{P_t A l_p}{2 R^2} \cdot \frac{\beta_{rb}}{4\pi} \]

The term \( \frac{\beta_{rb}}{4\pi} \) defines the unit volume backscattering coefficient, \( \beta \):
\[ \beta \triangleq \frac{\beta_{rb}}{4\pi} \begin{cases} \text{units of} \\ m^{-1} \text{sr}^{-1} \end{cases} \]
Thus,

\[ P_r(R) = P_t \cdot \frac{A}{R^2} \cdot \frac{l_p}{2} \cdot \beta \]

\( w \quad \text{sr} \quad m \quad m^{-1} \text{sr}^{-1} \)

and

\[ P_r(R) = \text{amount of instantaneous power collected by receiver due to backscatter from instantaneous scattering volume with center point at range } R. \]

Note: This formulation of the RADAR/LIDAR many scatterer range equations assume that the scatterers may be treated independently insofar as their individual scattering contributions (i.e., given by a simple sum of the \( \sigma_{rb} \))
Radar Equation

\[ \begin{align*}
\text{Transmitter Gain} & \quad G \\
\text{Receiver Area} & \quad A \\
\text{Power Density Incident on Target} & \quad S_t = P_e G \left( \frac{1}{4\pi R^2} \right) \\
\text{Power Density Backscattered to Receiver} & \quad S_R = S_t \left( \frac{\sigma_{rb}}{4\pi R^2} \right) \\
\text{Total Received Power} & \quad P_R = \frac{P_e G A \sigma_{rb}}{(4\pi)^2 R^4}
\end{align*} \]

\( \sigma_{rb} = \text{target radar backscattering cross section; i.e., the area target would have if it scattered isotropically.} \)
For many targets in the beam, the received power becomes

\[ P_r = \frac{P_t AG}{(4\pi)^2 R^4} \sum_{i=1}^{N} \beta_{rb_i} \]

summation over \( V_{Inst} \)

\[ \sum_{i=1}^{N} \beta_{rb_i} = V_{Inst} \times \beta_{rb} \]

\( \beta_{rb} \) = unit volume radar backscatter coefficient or radar reflectivity

\[ V_{Inst} = \pi \left( \frac{R \theta_t}{2} \right)^2 \frac{L}{2} = R^2 \theta_t \frac{L}{2} \]

\( \theta_t = \frac{\theta_t}{R^2} \) and \( G = \frac{4\pi}{\theta_t} \)

Also, \( A = \frac{\lambda^2 G}{4\pi} \)
Thus,
\[ P_r = \frac{P_t A G}{(4\pi)^2 R^4} \times 2 R^2 - \frac{\ell}{2} \times \beta v b \]
or
\[ P_r = \frac{P_t A \ell}{2 R^2} (\frac{\beta v b}{4\pi}) \]

Unit volume backscattering coefficient \( \beta \) \((m^2/m^3\text{-sr})\)

\[ \beta = \frac{\beta v b}{4\pi} \]

and the lidar/radar equation is the result

\[ P(R) = \frac{P_t A \ell \beta}{2 R^2} \]

except transmission losses have been neglected.
Transmission Loss

To include medium transmission loss in the radar equation, a round-trip transmittance term $T^2(R)$ must be included:

$$T^2(R) = e^{- \int_0^R \alpha(R') dR'}$$

where $\alpha(R')$ is the unit volume extinction coefficient (units of $m^{-1}$) at any distance $R'$ from 0 to $R$. Also,

$$T^2(R) = e^{-2 \tau(R)}$$

where $\tau(R)$ is the partial optical depth through distance $R$.

Thus, including transmission loss, the lidar/radar equation becomes

$$P(R) = \frac{P_0 A L \eta T^2(R) \beta(R)}{2 R^2}$$

and the effect of the medium is included in $T^2(R) \beta(R)$, the attenuated backscatter from range $R$. 
Scattering Coefficients

incident flux density $S_t$

Scattered flux density at distance $R$ in direction $\theta \neq \phi$ is $S_s$

$$S_s = \frac{S_t \phi_s(\theta, \phi)}{R^2}$$

differential scattering cross-section

$$\phi_s(\theta, \phi) \triangleq \frac{R^2 S_s(\theta, \phi, R)}{S_t}$$

Units of $\phi_s(\theta, \phi)$ are $m^2 s^{-1}$

For $\phi$ symmetry, $\phi_s(\theta, \phi)$ only depends on $\theta$, the scattering angle
For backscatter and assuming symmetry so only $\phi$ is important,

$$0_s \rightarrow 0_s (180^\circ)$$

The unit volume backscattering coefficient $\beta$ is related to $0_s (180^\circ)$ by

$$\beta = \sum_{\text{vol}} \frac{0_s (180^\circ)}{\text{unit vol.}}$$

The radar backscattering coefficient $\beta_{rb}$ is related to $0_s (180^\circ)$ by

$$\beta_{rb} = 4\pi 0_s (180^\circ)$$

and the radar unit volume backscattering coefficient $\beta_{rb}$ is related to $\beta_{rb}$ by

$$\beta_{rb} = \sum_{\text{vol}} \frac{\beta_{rb}}{\text{unit vol.}}$$

$\beta_{rb}$ also known as the radar reflectivity
The total fraction of incident power scattered by a particle in all directions is given by integrating $\Omega_{\text{s}}(\theta, \phi)$ over all directions or over a sphere of $4\pi$ steradians. This defines the total scattering cross-section, $S_\text{s}$, by

$$S_\text{s} = \int_0^{4\pi} \int_0^{2\pi} \frac{\Omega_{\text{s}}(\theta, \phi)}{4\pi} \sin \theta \, d\theta \, d\phi \quad \text{units of} \quad \text{m}^{-2}$$

$$dw = \sin \theta \, d\theta \, d\phi$$

$$S_\text{s} \Omega_{\text{s}} = \begin{cases} \text{total power scattered in all directions due to flux density} & \\
\text{to flux density} & \\
\text{incident on scatterer} & \\
\end{cases}$$

In general, the scatterer will absorb some of the incident radiation with an absorption cross-section $\sigma_a$, and the total extinction cross-section $\sigma_{\text{ext}}$ of the scatterer is

$$\sigma_{\text{ext}} = \sigma_{\text{s}} + \sigma_a$$

$$S \sigma_{\text{ext}} = \begin{cases} \text{total power removed or} & \\
\text{attenuated by scatterer for} & \\
\text{incident flux density} & \\
\end{cases}$$