Lecture 9

The CMOS Inverter

1. The Static CMOS Inverter

\[ \text{Vin} = V_{\text{DD}} \quad \langle a \rangle \]

\[ \text{Vin} = V_{\text{SS}} \quad \langle b \rangle \]

- \( |V_{\text{gsp}}| = 0 < |V_{\text{tp}}| \rightarrow \text{PMOS is off} \)
- \( |V_{\text{gsp}}| = V_{\text{SS}} > |V_{\text{tp}}| \rightarrow \text{a finite on-resistance, NMOS is on} \)

If first PMOS is on (like \( \langle b \rangle \)) for long time, what is \( \text{Vout} \)?

If then PMOS is off, NMOS is on, for long time, what is \( \text{Vout} \)?

@ Our first conclusion: Voltage swing of this inverter is \( V_{\text{DD}} \).
(a) Logic level: the final voltage "0" "Vdd"
   logic "1" "Vd1"
   does not depend on Rs, Rp → does not depend on geometry size of nmos, pmos

(b) Low output impedance Rs Rp low in kΩ

(c) High input impedance resistance

\[ \frac{1}{\frac{C_{p} + C_{n}}{V_{dd}}} \]

\[ \Rightarrow \frac{1}{\frac{1}{2} (C_{p} + 2C_{n})} \]

\[ \Rightarrow \\text{no direct path between supply and ground rail} \]

\[ \Rightarrow \text{small static power consumption} \]

2. Non-switched model

\[ V_{in} = V_{dd} - V_{snp} = V_{po} + V_{gsn} \]

\[ -I_{Dn} \Rightarrow I_{DP} = 0 \Rightarrow I_{Dn} = -I_{DP} \]

\[ V_{out} = V_{dd} + V_{osp} \]

\[ V_{in} = 0 \]
Now identify $Q_L$

$$C_L = \frac{V_{dd}}{f}$$

$$C_L = C_{Dn} + C_{Dp} + C_{W} + C_{G}$$
reminder: \[ \text{Reg} \propto \frac{1}{B}. \]

\[ \beta = k \frac{W}{L} \]

\( L \rightarrow k_n \) or \( k_p \).

So if just switch model

\[ T = \text{Reg} C_L \rightarrow \text{time constant. Elmore delay} \]

\[ \text{Reg} \rightarrow T, \ C_L \rightarrow \tau \]

\[ \text{requires small } \text{Reg} \]

\[ \text{requires small } \text{Reg} \rightarrow \frac{W}{L} \]

However, \[ \frac{W}{L} = \alpha \]

\[ W = \alpha L \]

\[ \alpha \rightarrow C_L \text{ as well, so be careful!} \]
reminder: \( \text{Reg} \propto \frac{1}{\beta} \)

\[
\beta = k \frac{W}{L}
\]

\( L \equiv k_i \text{ or } k_p \)

So if just switch model

\[
T = \text{Reg} \cdot C_L \rightarrow \text{time constant, Elmore delay}
\]

\[
\text{Reg} \uparrow \rightarrow T \uparrow, \quad C_L \uparrow \rightarrow T \uparrow
\]

\[
\downarrow \text{ requires small \text{Reg}} \quad \rightarrow \frac{W}{L} \uparrow
\]

However, \( \frac{W}{L} = \alpha \), \( W = \alpha L \)

\( \alpha \uparrow \rightarrow C_L \uparrow \) as well, so be careful!
\( R_{en} C_L = R_{ep} C_L \)

\[ B_n = K_n' \left( \frac{W}{L} \right)_n \]
\[ B_p = K_p' \left( \frac{W}{L} \right)_p \]

\[ K_n' = \frac{\mu_n E_x}{t_{ox}} \]
\[ K_p' = \frac{\mu_p E_x}{t_{ox}} \]

Share the same poly

\[ \mu_p = 0.5 \mu_n \]

The mobility of p-type is half of the mobility of n-type (electrons)

\[ R_{en} = \frac{1}{\beta_n (V_{tox} - V_{in})} \]
\[ R_{ep} = \frac{1}{\beta_p (V_{tox} - V_{tp})} \]

\[ R_{en} = R_{ep} \quad V_{tn} = V_{tp} \quad \text{since} \quad \frac{E_x}{t_{ox}} \quad L_p = L_n \]

\[ \Rightarrow \quad \frac{1}{\beta_n} = \frac{1}{\beta_p} \quad \Rightarrow \quad \frac{B_p}{\beta_p} = 1 \quad \Rightarrow \quad \frac{\mu_p \left( \frac{W}{L} \right)_p}{\mu_n \left( \frac{W}{L} \right)_n} = 1 \]

\[ \Rightarrow \quad \frac{\mu_p}{\mu_n} = \frac{1}{2} \quad \Rightarrow \quad \frac{W_p}{W_n} = 2 \quad \Rightarrow \quad \mu_p = 2 \mu_n \]