Lec # 8. Wire

1. The need for wire model

- wire material
  - n+, p+ diffusion layers
  - multiple layers of aluminum and copper
  - polysilicon

- wire parasitics
  1. propagation delay
  2. energy dissipation
  3. noise

We can only focus on the main effects!!

Not every effect!!

We have to simplify it!

How? Consider special cases!!
1. If $f$ is not high, cross section is small (especially for Aluminum)

\[ \omega L = 2\pi f L \]

\[ R \gg \omega L \rightarrow \text{neglect } L. \]

RC only model

2. Separation b/w wire is large \(\rightarrow\) no coupling caps!

3. If wires are short, but separation is also small \(\rightarrow\) all cap wire model.

Keep It Simple and Stupid (KISS)

A 3D extractor (fast cap, fast Henry) + first Ordel model

C. Basics for cap, resistance, inductance

\[ C_{\text{int}} = \frac{E\text{di}}{tdi} \]

\[ W L \]
\[ C_{\text{wire}} = C_{\text{int, wire}} + C_{\text{fringe}} + C_c \]

**WXH large \rightarrow W/H ratio constant**

- \( W \) can not increase due to area limitation.
- Large \( W/H \) \rightarrow parallel plate model
- \( W/H \to \frac{W}{tdi}, \) \( \frac{H}{tdi} \) the fringe part small.

**Now:**

\[ C_{\text{wire}} = C_{\text{int, wire}} + C_{\text{fringe}} + C_c \]

**Resistance:**

\[ R = \frac{PL}{A} = \frac{PL}{HW} \]

The model neglects f effect.

- \( f \) very high, \( \rightarrow \) skin effect
- \( R \) is frequency dependent
- Current flow at the surface of conductor

\[ s = \sqrt{\frac{f}{\pi \mu}} \]

permeability of dielectric

1GHz, Al, \( s = 2.6 \mu m \)
\[
R = \frac{PL}{A} = \frac{P}{2(W+H)} = \frac{P}{2(W+H)\sqrt{Lfr}} = \frac{\frac{\pi f_{up}}{2(W+H)}}{2(W+H)} \cdot L
\]

\[
\kappa(T) = \frac{R}{L} = \frac{\sqrt{\pi f_{up}}}{2(W+H)}
\]

**Inductance:** Low-resistive, high frequency (copper, over 5 GHz)

\[
\Delta V = L \frac{di}{dt} \quad \quad \quad \quad \quad \quad cL = 0.1 \mu \text{H}
\]

requires uniform dielectric

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- We just introduced hand calculation models, to give up fast soln for \( r, L, C \) per unit length.

- FastCap / FastHenry provide accurate results.

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3. **How to evaluate delays?**

- Different wire models lead to different strategy.

  - A short wire, and thick (cross section big) wires.

"** Equipotential "

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We will talk about driver model.
Example 1:

\[ \text{10kΩ} \]

\[ \text{10 cm long} \]

\[ \text{1 mm wide} \]

\[ \text{11 pF} \]

\[ \text{10 kΩ} \]

\[ V_{\text{out}} \]

\[ V_{\text{in}} \]

\[ \text{11 pF} \]

\[ C \frac{dV_{\text{out}}}{dt} + \frac{V_{\text{out}} - V_{\text{i}}}{R} = 0 \]

\( V_{\text{i}} \) is step input \( 0 \rightarrow V_{\text{dd}} \)

Transient response?

\[ SCV_{\text{o}} + \frac{1}{R} V_{\text{o}} - \frac{1}{R} \frac{V_{\text{dd}}}{s} = 0 \]

\[ V_{\text{o}} = \frac{1}{SC + 1} \frac{1}{s} V_{\text{dd}} \]

\[ V_{\text{o}} = \left( \frac{1}{s} - \frac{1}{sTRC} \right) V_{\text{dd}} \]

In time domain

\[ V_{\text{o}} = \left( 1 - e^{-\frac{t}{C}} \right) V_{\text{dd}} \]

This is one RC segment, how about many RC segments?

\[ \text{Diagram of multiple RC segments} \]
Elmore Delay: A way to hand estimate delay

Good for RC tree.

RC tree concept:

1. One input node
2. All cap to ground
3. No resistive loop

![Resistive loop](image)

Unique resistive path between two nodes:

- $S$ to 3: $R_{33} = R_1 + R_2 + R_3$
- $S$ to 4: $R_{44} = R_1 + R_2 + R_4$

Shared path resistance, for example $R_{43}$

$$R_{43} = R_1 + R_2$$

$$R_{12} = R_1$$

More assumptions:

1. Each node initially '0' charge → need to remember (important!!)
2. Step input. (not required for general case, however, Elmore delay was derived with this assumption)

At node $i$: 

$$T_{Di} = \sum_{k=1}^{N} C_k R_{ik}$$

at node $i$: 

$N$ nodes
Elmore delay is first-order time constant.

Example 2:

\[ \tau_{p4} = \frac{4}{k=1} C_k R_{4k} \]

\[ = C_1 R_{41} + C_2 R_{42} + C_3 R_{43} + C_4 R_{44} \]

\[ = R_1 C_1 + (R_1 + R_2) C_2 + (R_1 + R_2 + R_3) C_3 + (R_1 + R_2 + R_3 + R_4) C_4 \]

Example 3: Special case: \( \Omega RC \)-ladder

\[ V_{in} \quad \frac{R_1}{C_1} \quad \frac{R_L}{C_2} \quad \cdots \quad \frac{R_N}{C_N} \]

\[ \tau_{DN} = \frac{N}{k=1} C_k R_{kk} \]

\[ = R_1 C_1 + (R_1 + R_2) C_2 + (R_1 + R_2 + R_3) C_3 + \cdots + (R_1 + R_2 + \cdots + R_N) C_N \]

\( \Omega RC \) Line:

\[ R = r \frac{1}{N} \quad C = c \frac{1}{N} \]

\[ \tau_{DN} = \frac{N}{k=1} C_k R_{kk} = r \frac{1}{N} c \frac{1}{N} + 2r \frac{1}{N} c \frac{1}{N} + \cdots + Nr \frac{1}{N} c \frac{1}{N} \]
\[
\frac{N+1}{2N} \frac{r_c L^2}{N^2} = \frac{N+1}{2N} r_c L^2
\]

\[N \to \infty \quad R C \text{ line}\]

\[
\frac{N+1}{2N} r_c L^2 \xrightarrow{N \to \infty} \frac{r_c L^2}{2}
\]

Two properties:

a. delay is "quadratic" of length "L"

b. only half of RC lump model is required.

Lump model will be very inaccurate if RC line model is required.
What is Elmore delay?

\[ V_0 = \left(1 - e^{-\frac{t}{RC}}\right) V_{dd} \]

\[ V_0 = 50\% V_{dd} \]

\[ 0.5 V_{dd} = \left(1 - e^{-\frac{t}{RC}}\right) V_{dd} \]

\[ \frac{1}{2} = e^{-\frac{t}{RC}} \]

\[ \ln \frac{1}{2} = -\frac{t}{RC} \]

\[ t = \ln 2 \cdot RC \]

\[ \ln 2 = 0.69 \quad \text{so} \quad t_{50\%} = 0.69 RC \]

Similarly

\[ t_{10\%} = 0.22 RC \]

\[ t_{90\%} = 2.3 RC \]

\[ t = RC \quad t_1 = t = RC \]

\[ V_0 = \left(1 - e^{-\frac{t}{RC}}\right) V_{dd} = \left(1 - \frac{1}{e^{0.69}}\right) V_{dd} \]

\[ \approx 0.63 V_{dd} \]

So \[ t_{63\%} = t = RC \]

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<table>
<thead>
<tr>
<th>Voltage Range</th>
<th>Lumped RC (One RC Segment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 \rightarrow 50%</td>
<td>(0.69 RC)</td>
</tr>
<tr>
<td>0 \rightarrow 63%</td>
<td>(RC)</td>
</tr>
<tr>
<td>10% \rightarrow 90%</td>
<td>(2.2 RC)</td>
</tr>
<tr>
<td>0% \rightarrow 90%</td>
<td>(2.3 RC)</td>
</tr>
</tbody>
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Elmore Delay is the estimation of dominant pole in RC Circuit