Types of Fiber Interferometers:

1. Does plane shift occur in transmission, reflecting, or between counter-propagating beams?

2. Does the sensor use resonance — trade off with dynamic range?

**Directional Coupler**

Field amplitudes $E_1, E_2, E_3, E_4$:

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \cos(KL_c) & j \sin(KL_c) \\ j \sin(KL_c) & \cos(KL_c) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

The intensity $I_3$ and $I_4$ vary with the phase difference in the coupler.

**Ideal 3 dB Coupler**

50% split to $E_3, E_4$.

$KL_c = \frac{\pi}{4}$, $K = \text{coupling coefficient}$.

When losses exist:

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

**Example**:

$E_3 = 1$

$E_2 = 0$

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \cos(KL_c) + j \sin(KL_c) \\ j \sin(KL_c) \cos(KL_c) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$
As discussed, the coupler operates on the fields and will split the ratio of the fields depending on the value of $\kappa L_c$.

For a 3 dB coupler, $\kappa L_c = \pi$.

Using a 3 dB coupler for both couplers in the interferometer, the output fields $E_{01}$ and $E_{02}$ can be computed using matrix optics:

$$
\begin{bmatrix}
E_{01} \\
E_{02}
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & j \\
1 & -j
\end{bmatrix} \begin{bmatrix}
\frac{e^{j\pi/4}}{\lambda_0} & 0 \\
0 & e^{j\pi/4}
\end{bmatrix} \begin{bmatrix}
1 & j \\
-1 & j
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

The output from either $E_{01}$ or $E_{02}$ can be used for the sensor.

$$
E_{01} = \frac{E_i}{2} \left\{ \exp\left[ j \frac{\pi}{2} \frac{m L_2}{\lambda_0} \right] - \exp\left[ j \frac{2 \pi m L_1}{\lambda_0} + \varphi_0 \right] \right\}
$$

$$
E_{02} = \frac{E_i}{2} \left\{ \exp\left[ -j \frac{\pi}{2} \frac{m L_2}{\lambda_0} \right] + \exp\left[ 2 \pi m L_1 \frac{L_2}{\lambda_0} + \varphi_0 \right] \right\}
$$

The corresponding output intensities become:

$$
I_{01} = \frac{I_i}{2} \left\{ 1 - \cos\left[ 2 \pi m \frac{(L_1 - L_2)}{\lambda_0} + \varphi_0 \right] \right\}
$$

$$
I_{02} = \frac{I_i}{2} \left\{ 1 + \cos\left[ 2 \pi m \frac{(L_2 - L_1)}{\lambda_0} + \varphi_0 \right] \right\}
$$

Let $m = \delta m \left[ \frac{L_1 - L_2}{\lambda_0} \right]$. Then

$$
I = 16 I_i^2
$$

with $I_{01} + I_{02} = I_i$.
For maximum sensitivity to phase changes let
\[ \Gamma = \pi [M + \frac{1}{2}] \] where \( M = \text{integer} \)
\[ \Rightarrow \Gamma = M\pi \] the sensitivity goes to zero.

To adjust the interferometer for maximum sensitivity one can change the path length difference between the arms of the interferometer.

* Environmental and Measurand Sensitivity:

Taking the output from channel 1 as the response
\[ T = \frac{F_0}{I_1} = \frac{1}{2} \left[ 1 - \cos(\Gamma + \phi_0) \right] \]

Differentiating \( T \):

**Measurand sensitivity:** \( S_m = \frac{\partial T}{\partial \phi_0} \frac{\partial \phi_0}{\partial m} = \frac{1}{2} \sin(\Gamma + \phi_0) \frac{d\phi_0}{dm} \)

**Environmental sensitivity:** \( S_{\phi_0} = \frac{\partial T}{\partial \phi_0} \frac{\partial \phi_0}{\partial \phi_e} = \frac{1}{2} \sin(\Gamma + \phi_0) \frac{d\phi_0}{d\phi_e} \)

* Both measurand and environmental sensitivity occur at the same biasing value for \( \Gamma \) (i.e. \( \frac{\pi}{2} \)).

* Cannot reduce environmental sensitivity and simultaneously maximize the measurand sensitivity.
To adjust the interferometer to operate at maximum sensitivity, one can accurately equalize the path lengths in a fiber interferometer using an electrostrictive material such as a piezoelectric in cylindrical form with a radius controlled by the applied voltage.

\[ \Delta L = \frac{2\pi}{\lambda} \Delta R N \]

where \( N \) is the number of turns on the mandril.

\[ \Delta L = \frac{2\pi}{\lambda} \Delta R N = \left(\frac{2\pi}{\lambda}\right)^2 \Delta R N \]

For maximum sensitivity,

\[ \Gamma = \left( M + \frac{1}{2} \right) \frac{\pi}{\lambda} = \frac{2\pi}{\lambda} M \Delta L \]

Adjust length \( \Delta L = \frac{1}{2} \left( M + \frac{1}{2} \right) \frac{\lambda}{2\pi} \)
without sensor
in the interferometer
\[ \mathbf{I}(\Phi) = \frac{I_1}{2} \left\{ 1 + \cos \left( 2\pi [m_1 \Delta l + (-m_2 \Delta l) \right] \right\} \]

\[ = \frac{I_1}{2} \left\{ 1 + \cos \Phi_0 \right\} \]

where \( \Phi_0 = 2\pi \left[ m_1 \Delta l - m_2 \Delta l \right] \)

Add sensor:
To arm #2
Single pass
\[ \Phi_{\text{optimal}} = \Delta m \Delta n \]

{ 2 passes through optode
want to measure a change in refractive index in the optode

\[ \Phi = \Phi_0 + \Delta \Phi \]

Sensitivity to the change in refractive index
\[ S_m = \left( \frac{\partial \Phi}{\partial \Delta n} \right) \left( \frac{\partial \Delta \Phi}{\partial \Delta m} \right) \]

\[ = \frac{4\pi \Delta m}{\lambda_0} \sin \Phi \]

\[ \Delta \Phi = \sin \Phi \]

Sensitivity varies with \( \Phi \). Since \( \Delta \Phi \) is typically \( \Phi_0 \)

Can bias near optimum near \( \Phi_0 = \left( \frac{M+\frac{1}{2}}{M} \right) \pi \), \( M=\text{index} \)

Also \( \Phi = N\pi \), \( S_m = 0 \)

Note: There is also independence of \( S_m \) on \( \Delta n \).

Note: In our experiment, since the path length changes, the sensor sweeps through regions of low sensitivity and high sensitivity, i.e., sensitivity constantly changes.

Each fringe corresponds to a phase difference of \( 2\pi \).

The condition \( \Phi_0 = (m + \frac{1}{2})\pi \) of maximum sensitivity is referred to as having the interferometer in quadrature.

\[ S_m = \frac{1}{2} \left( \frac{4\pi \Delta m}{\lambda_0} \right) \left( \sin^2 \Phi \right) \]

\( S_m \) is a function of \( \sin^2 \Phi \), not \( \sin \Phi \).

\( T \) is a shifted 180° function so that it does not become \( \leq 0 \).
Note: At the point of optimum sensitivity, the measured intensity is

\[ I(P) = I_c \left[ 1 + \cos(\theta + \phi) \right] \]

Maximum change in sensitivity occurs at the inflection point.
* Fiber Gyro

* Consider photon propagating around a ring of radius R

* The ring rotates at an angular velocity \( \omega \).

* Light can propagate either clockwise or counter clockwise around the ring.

* Photons moving around the ring are traveling in an accelerated reference frame. Strictly speaking photon propagation should be described using general relativity.

If \( \omega = 0 \) the time for a photon to travel around the ring is

\[
t = \frac{2\pi R}{c} \]

When \( \omega \neq 0 \) photons traveling clockwise (CW) must travel a little farther to get to the starting point than photons traveling (CCW). \( \Delta L = R \omega t \)

The relative delay between CW and CCW propagating photons:

\[
\Delta T = \frac{2 \Delta L}{c} = \frac{2 R \omega t}{c} = \frac{2 R \omega R \omega t}{c^2} = \frac{2\pi R^2 \omega t}{c^2}
\]

* The delay \( \Delta T \) is a function of the area of the ring.

* The differential time delay is called the Sagnac Effect.

Light originating at the same point which experiences a time delay causes a phase shift between the recombining waves.
\[ \Delta \phi = \omega \Delta T = \frac{2\pi c/m}{\lambda_0 \lambda m} \Delta T = \frac{8\pi \omega A n^2}{\lambda_0 c} \]

The phase shift \( \Delta \phi \) can be increased by making the photons propagate through the ring \( N \) times.

\[ \Delta \phi = \frac{8\pi \omega A N m^2}{\lambda_0 c} \]

Example: Consider a SMF wrapped around a disk with area \( A = 500 \text{ cm}^2 \) (\( R \approx 12 \text{ cm} \))
Core index \( n = 1.5 \quad \lambda = 850 \text{ mm} \)
Angular rotation is \( 30^\circ/\text{hr} = \frac{30^\circ}{3600 \text{ sec}} = \frac{8.33 \times 10^{-3} \text{ deg/sec}}{1.46 \times 10^{-7} \text{ rad/sec}} \)

This results in a phase shift:
\[ \Delta \phi = \frac{8\pi(500)(1.5)^2}{(0.85 \times 10^{-7})(13 \times 10^{-10})} (1.46 \times 10^{-7}) \]
\[ = 7.2 \times 10^{-7} \text{ rad} (1.5)^2 = 16.2 \times 10^{-7} \text{ rad} \{ \text{Not very large} \} \]

*To improve performance can use the fact that fiber has low attenuation losses (-1dB/km)*

\[ \sim N = 10^4 \rightarrow \Delta \phi \sim 16.2 \text{ mrad} \{ \text{This is much more reasonable} \} \]

For 2km of fiber can get \( \frac{1000 \text{ m}}{2 \pi R} = 1300 \text{ turns} \)
* 3dB coupler splits the beam and launches it into the same loop travelling in opposite direction.

* All variations due to temperature, stress, pressure, etc should be the same since "arms" of the interferometer is the same. (Reciprocal effect)

Now reciprocal effects like the Sagnac effect will result in a phase change for CW and CCW rotating beams.

**Total Phase Accumulation:** \( \Phi = \frac{2\pi m l}{\lambda} \)

For \( m = 1.5 \), \( l = 200 \text{ m} \), \( \lambda = 1 \text{ mm} \) \( \Rightarrow \Phi = 1.875 \times 10^9 \text{ rad} \)

For accurate positioning requirements, it may be necessary to resolve rotation of 0.01 - 0.001°/hr (1.25 × 10^{-5} - 1.25 × 10^{-6} rad/s)

This implies a sensitivity requirement of \( 10^{-14} \) (or 1 part in 10^{14})

With this degree of sensitivity, it is important to control other non-reciprocal effects.

1. One important effect in this category is the Kerr effect:

   \[ \text{Kerr Effect} \quad m_{\text{eff}} = m_0 + m_1 \]

   Effectively, a normally isotropic material becomes birefringent under the influence of a strong electric field, in this case the optical field.
Total Phase Error Resulting from the Optical Kerr Effect

\[ \Phi_k = \alpha (1 - 2k) \left( \frac{\langle I_0(t) \rangle}{\langle I_0(t)^2 \rangle} - \langle I_0(t) \rangle^2 \right) \]

only non-zero when light incoherent.

Ideally \( k = \frac{1}{2} \), but this is hard to achieve.

Alternatively, if partially coherent light is used, \( \langle I^2 \rangle = \langle I \rangle^2 \) except within the coherence length.

In this case \( \Phi_k = 0 \). For this purpose can use a super luminescent diode or an Amplified Spontaneous Emission source. (Note the same concept is applied to OCT)

To optimize sensitivity need to bias the interferometer for a phase shift of \( \frac{\pi}{2} \). Can use a phase modulator to introduce a phase difference in the counter propagating beams.

Light passing the modulator in one direction is advanced and propagates in the opposite direction is retarded.

The modulator also removes the D.C. phase shift and \( \frac{\pi}{2} \) modulator.

Detection Noise Limitations - A change in the output intensity due to noise cannot be distinguished from a signal

A fundamental limit is shot noise

The uncertainty in the phase signal

\[ \delta \phi = \frac{\text{photon shot noise (} \delta \phi \text{)}}{\text{Erinage Slope (} \delta \phi \text{)}} \]
Fringe slope is maximum at $\phi = \frac{\pi}{4}$

\[ I = \frac{I_{\text{max}}}{2} (1 + \cos \phi) \]

\[ \frac{\partial I}{\partial \phi} = \frac{I_{\text{max}}}{2} \sin \phi \]

The detector current at this phase is

\[ i_{\text{det}} = R \frac{I_{\text{max}}}{2} \cos \phi \]

\[ \frac{(g I_{\text{max}} B)^{1/2}}{I_{\text{max}}^{1/2}} = \frac{2(g B)^{1/2}}{\sqrt{I_{\text{max}}}} \]

Using this in the fiber gyro sensitivity relation (to obtain a minimum detectable angular velocity)

\[ \Delta \phi = \frac{8\pi \left( g B \right) \Delta \phi_{\text{det}}}{\lambda_0 c} \]

\[ \Delta \phi = \frac{\lambda c}{8\pi ANm^2} \Delta \phi = \frac{\lambda c}{8\pi AN} \frac{(g B)^{1/2}}{\sqrt{I_{\text{max}}}} \]

In this case, the performance of an optical gyro is limited by shot noise.

The uncertainty increases with $\sqrt{\frac{N_{\text{ph}} B}{I_{\text{max}}}}$

Increasing the bandwidth will increase the noise level and decrease sensitivity.