Amplified Signal in the Presence of ASE

N-Cascaded EDFAs in an Optical Link

EDFA NB Filter

Tx

B₀

Rx

| L |
Define: \[ P_m = m_p \ h \nu \] with \( m_p \) = spontaneous emission factor

(Not a power term, just a definitional convenience)

Amplified spontaneous emission noise power:
\[ P_N = 2m_p \ h \nu (G-1) B_o \]

- \( G \) = Amplifier gain
- \( B_o \) = optical BW (filter)

Assume that a PM photodiode is used:
\[ i = RG P_{op} \]

- \( P_e = i^2 R_e \) corresponding electrical signal power

\[ P_{op} \propto |\text{Field}|^2 = |E_{sig} + E_{noise}|^2 \]

There is some degree of coherence between the signal and noise fields. This produces noise components due to:

a) Signal - spontaneous beat noise
b) Spontaneous - spontaneous beat noise

d. In addition thermal + shot noise also exist

The different noise contributions at the receiver include:

\[ \sigma^2_{Th} = I_e^2 R_e \]
\[ \sigma^2_{Sh} = 2g R \left[ GP + P_m (G-1) B_o \right] R_e \]
\[ \sigma^2_{Si} = 4R^2 GP P_m (G-1) R_e \]
\[ \sigma^2_{Sp} = 2R^2 \left[ P_m (G-1) \right]^2 \left( 2B_o - R_e \right) R_e \]
\( \sigma_{\text{shot}}^2 + \sigma_{\text{quant}}^2 \) usually dominate thermal noise when the amplifier gain \( G > 10 \text{dB} \).

For BER ~ \( 10^{-9} - 10^{-15} \) the heat noise processes can be modeled as Gaussianprocesses.

Note that since the filter width \( R_0 \) can be controlled it is possible to reduce \( \sigma_{\text{quant}}^2 \)
do the limit \( R_0 \) can be made \( \approx 2 R_e \).

\( \sigma_{\text{quant}}^2 \) is the dominant noise source.

One can now quantify The ratio \( F_n = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} \) is the noise figure of the amplifier.

\[
\text{SNR}_{\text{in}} = \frac{(RP)^2}{2R_0P_e} \quad \{ \text{Assumes that only shot noise is present} \}
\]

\[
\text{SNR}_{\text{out}} \approx \frac{(RGP)^2}{4R^2G(G-1)M^2P_e} \quad \{ \text{Assumes that \( \sigma_{\text{quant}}^2 \) is dominant noise source} \}
\]

\[
F_n \approx \frac{2MP(G-1)}{G^2} = 2MP \frac{(G-1)}{G}
\]

At large \( G \) \( F_n \rightarrow 2MP \)

\( \text{shot noise} \quad m_{\text{sp}} = \frac{N_e}{N_e - N} \quad \text{can be} \quad 1 - 5 \)
Section 2: DISPERSION in OPTICAL FIBERS
**Problem with Dispersion:**
Short optical pulses enter a dispersive channel such as an optical fiber. After propagating through this channel they spread out into a much broader temporal distribution. If they broaden too much and the pulses are sent on very close intervals the data may not be retrievable.
MODAL DISPERSION

- Perhaps an easy way to visualize dispersion is to consider pulse propagation in a multi-mode fiber (MMF).

- Light launched into a MMF can effectively travel through several optical paths.

- The power in a narrow pulse signal will be distributed into many possible modes.

- Modal Dispersion is a function of the path length difference (PLD). Taking the PLD between the lowest and highest order modes

\[ \Delta L = \frac{L}{\cos \theta} - L \]

- The corresponding time delay between modes becomes

\[ \Delta \tau = \Delta L / (c / n_1) \]

- For MMF with NA = 0.20 and \( n_1 = 1.50 \),

\[ \Delta \tau \approx 45 \text{ nsec/km} \]
Phase and Group Velocity within Fibers:

Phase Velocity:
Propagation along the z axis

$$E(t, z) = E_0 \exp[j(\omega t - \beta z)]$$,

with $\omega = 2\pi v$, and $\beta = 2\pi n/\lambda$ (propagation constant along the z direction)

- Phase velocity describes the velocity of a constant phase value that propagates in the direction of power flow of the optical field (i.e. the Poynting vector).

  $$\omega t - \beta z = \text{constant}$$

  $$z = vt$$

  $$\omega t - \beta vt = \text{constant}$$

- In order for this to be true at all values of time

  $$v = \omega / \beta$$

- This is the phase velocity of the monochromatic wave component with frequency $v$.

- The ratio of the speed of light to the phase velocity is the *refractive index*

  $$n = c/v$$

$\beta$ is bounded by the propagation constants within the core and the cladding of the fiber

$$k_1 > \beta > k_2$$

where $k_i = 2\pi n_i/\lambda_o$
GROUP VELOCITY:

- Consider an amplitude modulated analog signal

\[ E_{AM}(t, z = 0) = E(1 + m \cos \omega_1 t) \cos \omega_c t \]

Where \( \omega_1 \) is the modulation angular frequency and \( \omega_c \) is the carrier frequency, \( m \) is the modulation depth, \( z = 0 \) implies launching the signal at the entrance to the fiber.

- Using Euler's formula:

\[ E_{AM}(t, z = 0) = E \text{Re}\{ \exp(j\omega_c t) + (m/2)\exp[j(\omega_c - \omega_1)t] + (m/2)\exp[j(\omega_c + \omega_1)t] \} \]

- This implies that at the entrance to the fiber there are three different frequencies

\[ \omega_c ; \quad \omega_c - \omega_1 ; \quad \omega_c + \omega_1. \]

- Each component travels with its own phase velocity and accumulates its own phase shift.

- In general we must know \( \beta \) for each value of \( \omega \) to find the output field.

- Expanding \( \beta \) in a Taylor series

\[ \beta(\omega) = \beta_c + \dot{\beta} \Delta \omega + \frac{1}{2} \ddot{\beta}(\Delta \omega)^2 + \frac{1}{6} \dddot{\beta}(\Delta \omega)^3 + ... \]

\( \beta_c \) is the value of \( \beta \) at the carrier frequency \( \omega_c \);

\( \Delta \omega \) is the frequency difference between \( \omega \) and \( \omega_c \) and
\( \dot{\beta}, \ddot{\beta}, \dddot{\beta} \), are the derivatives of \( \beta \) with respect to \( \omega \) evaluated at \( \omega = \omega_c \)

\[
\Delta \omega = \omega - \omega_c
\]

\[
\dot{\beta} = \left. \frac{\partial \beta}{\partial \omega} \right|_{\omega = \omega_c}
\]

\[
\ddot{\beta} = \left. \frac{\partial^2 \beta}{\partial \omega^2} \right|_{\omega = \omega_c}
\]

\[
\dddot{\beta} = \left. \frac{\partial^3 \beta}{\partial \omega^3} \right|_{\omega = \omega_c}
\]

\( \dot{\beta} \) does not lead to *envelope distortion*, \( \ddot{\beta} \) and higher order derivatives do.

\[ \therefore \] \( \ddot{\beta} \) is known as the *first order dispersion coefficient*,

\( \dddot{\beta} \) is the *second order dispersion coefficient* etc.

- Neglecting second and higher order derivatives for the moment

\[
\beta(\omega) \cong \beta_c + \dot{\beta} \Delta \omega
\]
• The three propagation constants for the three different frequencies of the AM signal are:

\[ \beta = \beta_c - \Delta \beta, \text{ at } \omega_c - \omega_1 \]

\[ \beta = \beta_c, \text{ at } \omega_c \]

\[ \beta = \beta_c + \Delta \beta, \text{ at } \omega_c + \omega_1 \]

with \[ \Delta \beta \equiv \frac{\dot{\beta}}{\omega_c} \Delta \omega \]

\[ e^{j \left( \omega_c t - \beta_c z \right)} + \sum_{m=1}^{\infty} e^{j \left( (\omega_c - m\omega_1) t - (\beta_c - m\Delta \beta) z \right)} + \sum_{m=1}^{\infty} e^{j \left( (\omega_c + m\omega_1) t - (\beta_c + m\Delta \beta) z \right)} \]

• The output signal after propagating through \( z \) meters of fiber:

\[ E_{AM}(z,t) = E \text{Re} \left\{ j \left[ (\omega_c - \omega_1) t - (\beta_c - \Delta \beta) z \right] \right\} + \frac{m}{2} e^{j \left[ (\omega_c - \omega_1) t - (\beta_c - \Delta \beta) z \right]} + \frac{m}{2} e^{j \left[ (\omega_c + \omega_1) t - (\beta_c + \Delta \beta) z \right]} \]

• This equation can be rewritten into the form

\[ E_{AM}(z,t) = E \left[ 1 + m \cos(\omega_1 t - \Delta \beta z) \right] \cos(\omega_c t - \beta_c z) \]

Note that the phase shift accumulated by the carrier is \( \beta_c z \) and the phase shift accumulated by the modulation is \( \Delta \beta z \).
• The **group velocity** is defined as the *velocity that maintains constant phase of the envelope profile*

\[
\omega_1 t - \Delta \beta z = \text{const. for all } t, \\
\text{with } v_g = \frac{z}{t}, \\
\omega_1 t - \Delta \beta v_g t = \text{const. for all } t,
\]

this relation must also hold for the constant equaling 0 as well as other values.

• Therefore

\[
v_g = \frac{1}{\beta} = \frac{\partial \omega}{\partial \beta}
\]

• Both \(v_{ph}\) and \(v_g\) are determined by the shape of the \(\omega vs. \beta\) curve and in general will be different.

• Plotting \(\omega vs. \beta\) we can show the group and phase velocities directly.
• Re-writing the form for the AM signal to emphasize the difference between phase and group velocity

\[ E_{AM}(z,t) = E_0 \left[ 1 + m \cos \omega_1 \left( t - \frac{z}{v_{gr}} \right) \right] \cos \omega_c \left( t - \frac{z}{v_{ph}} \right) \]

• As indicated the group velocity is associated with the modulated information signal and the phase velocity with the carrier frequency.

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**GROUP VELOCITY DISPERSION**

• Even with SMF wavelength dependent effects eventually limit the maximum bit rate that can be transmitted.

• Effects are cumulative over the length of the fiber and are therefore specified per unit length of fiber.

• As discussed previously the group velocity is associated with the transmission of a pulse. Several frequencies are required to describe a pulse.

• The group velocity of the fundamental mode of a SMF is frequency dependent due to chromatic dispersion.

• Different spectral components of a pulse travel at slightly different group velocities. This is indicative of group velocity dispersion or GVD.
Two primary contributions to chromatic dispersion in SMF are:

a. material dispersion
b. waveguide dispersion

(Note that GVD is also referred to as Chromatic Dispersion.)

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**Group Velocity Dispersion**

For a SMF of length $L$ a spectral component of frequency $\omega$ reaches the end of the fiber after a time delay of

$$T = \frac{L}{\nu_g}$$

**Group Velocity:**

$$\nu_g \equiv \frac{d\omega}{d\beta}$$

Using

$$\beta = \bar{n}k = \bar{n} \frac{\omega}{c}$$

$$\nu_g = \frac{c}{\bar{n}_g}$$

**A Major Consequence of GVD:**

- Frequency dependence of $\nu_g$ leads to **pulse broadening**
- Different spectral components disperse during propagation and do not arrive simultaneously at the fiber output.
- This affect reduces the maximum bit rate that can be transmitted through the fiber system.
\[ \Delta T = \frac{dT}{d\omega} \Delta \omega = \frac{d}{d\omega} \left( \frac{L}{v_g} \right) \Delta \omega = L \frac{d^2 \beta}{d\omega^2} \Delta \omega = L \beta_2 \Delta \omega \]

The parameter
\[ \beta_2 = \frac{d^2 \beta}{d\omega^2} \]

is often referred to as the **GVD parameter**.

- \( \beta_2 \) determines how much the pulse will broaden with propagation through the fiber.

- It is often useful to determine the frequency spread \( \Delta \omega \) as a function of the range of wavelengths \( \Delta \lambda \) emitted by the source.

With \( \omega = 2\pi c/\lambda \) and \( \Delta \omega = -(2\pi c/\lambda^2) \Delta \lambda \) we can re-write pulse broadening as

\[ \Delta T = \frac{d}{d\lambda} \left( \frac{L}{v_g} \right) \Delta \omega = D L \Delta \lambda \]

\[ D = \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2 \]

- \( D \) is the dispersion parameter \([\text{ps/(km-nm)}]\).
**Effect of Dispersion on the Bit Rate B**

- The most general criteria is $B \Delta T < 1$ with $T_p = 1/B$. Usually the product is somewhat smaller than this i.e. $\Delta T \leq 1/(4B)$.

- Using the expression for $\Delta T$ the product becomes

$$BL|D|\Delta \lambda < 1$$

- For silica fibers near 1.3 μm, $D \sim 1$ps/(km-nm).

- A semiconductor laser with a spectral width $\Delta \lambda$ of 2-4 nm results in a *bit rate length product* of

$$BL > 100 \text{ (Gb/s)-km}$$

- With this $B-L$ product, a 1.3 μm system running at 2 Gb/s will have to have a repeater every 40-50 km. :: It is very important to reduce dispersion.

- $D$ can vary considerably when the wavelength differs from 1.3 μm.

- The wavelength dependence of $D$ is governed by the frequency dependence of the *mode index* $\bar{n}$

$$D = -\frac{2\pi c}{\lambda^2} \frac{d}{d\omega} \left( \frac{1}{v_g} \right) = -\frac{2\pi}{\lambda^2} \left( 2 \frac{d\bar{n}}{d\omega} + \omega \frac{d^2\bar{n}}{d\omega^2} \right)$$
• In general the *dispersion parameter* \( D \) can be written as the sum of two terms

\[
D = D_M + D_W
\]

• \( D_M \) is the *material dispersion* and \( D_W \) is the *waveguide dispersion*

\[
D_M = -2\pi \frac{\frac{dn_{2g}}{\lambda^2}}{d\omega} = \frac{1}{c} \frac{dn_{2g}}{d\lambda}
\]

\[
D_W = -2\pi \frac{n_{2g} \frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega} \frac{d(Vb)}{dV}}{\lambda^2}.
\]

\( n_{2g} \) is the *group index* of the cladding material of the fiber and \( V \) and \( b \) are

\[
V = k_o a \left( n_1^2 - n_2^2 \right)^{1/2},
\]

\[
b = \frac{\beta / k_o - n_2}{n_1 - n_2} = \frac{\bar{n} - n_2}{n_1 - n_2}.
\]

*Note these expressions for \( D_M \) and \( D_W \) assume that \( \Delta = \frac{(n_1^2 - n_2^2)}{2n_1^2} \) does not change with spectral frequency.*
- An additional term called *differential material dispersion* also exists. It has the form $d\Delta/d\omega \neq 0$. In practice it is quite small but may become significant as other dispersive and attenuation factors are reduced.
MATERIAL DISPERSION

- *Results from the dependence of the refractive index of silica on the optical frequency ω*

**Material Dispersion:**

- Fused silica has certain absorption resonance peaks.

- Away from these resonance frequencies $n(ω)$ can be described using the *Sellmeier Equation*

$$n^2(ω) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega^2 - \omega_j^2},$$

where $\omega_j$ are the *resonance frequencies* and $B_j$ is the *oscillator strength*.

- The refractive index $n$ can represent $n_1$ or $n_2$ to determine the properties of the core or the cladding.
- This sum extends over all material resonance frequencies that contribute in the frequency range of interest. These frequencies depend on the dopants used in the fused silica to fabricate the fiber.

- For pure fused silica $M = 3$
  
  $B_1 = 0.6961663 \text{ at } \lambda_1 = 0.0684043 \ \mu\text{m}$
  
  $B_2 = 0.4079426 \text{ at } \lambda_2 = 0.1162414 \ \mu\text{m}$
  
  $B_3 = 0.8974794 \text{ at } \lambda_3 = 9.896161 \ \mu\text{m}$

  Where $\lambda_j = 2\pi c / \omega_j$ with $j = 1 - 3$

The group refractive index $n_g = n + \omega (dn/d\omega)$ can be obtained using the *Sellmeier Equation* with these parameters.
• Plot shows the \( \lambda \) dependence of \( n_g \) and \( n \) in the range 0.5 – 1.6\( \mu m \).

• **Material Dispersion** is related to the slope of \( n_g \) by the relation

\[
D_M = \frac{1}{c} \left( \frac{dn_g}{d\lambda} \right)
\]

• \( \frac{dn_g}{d\lambda} = 0 \) at \( \lambda = 1.276 \mu m \). This is the zero dispersion wavelength \( \lambda_{ZD} \).

• \( D_M = 0 \) at \( \lambda_{ZD} = 1.276 \mu m \)

• For \( \lambda < \lambda_{ZD} \), \( D_M < 0 \); Similarly for \( \lambda > \lambda_{ZD} \), \( D_M > 0 \)

• For 1.25 \( \mu m \) < \( \lambda \) < 1.66 \( \mu m \)

\[
D_M \approx 122(1 - \frac{\lambda}{\lambda_{ZD}})
\]

• Note that \( D_M = 0 \) at \( \lambda_{ZD} = 1.276 \mu m \) only for pure fused silica.

• 1.27 \( \mu m \) < \( \lambda_{ZD} \) < 1.29 \( \mu m \) for fused silica that is doped to vary the index.
WAVEGUIDE DISPERSION:

Results from slight changes to the propagation path for a mode due to different matching conditions with different wavelengths.

Waveguide Dispersion:

- The contribution of waveguide dispersion to the total dispersion parameter $D$ is

$$D_w = -\frac{2\pi}{\lambda^2} \left[ \frac{n_{2g}^2}{n_2 \omega} \frac{V d^2 (V_b)}{dV^2} + \frac{dn_{2g}}{d\omega} \frac{d(V_b)}{dV} \right]$$

note the dependence on the $V#$ of the fiber.
- The elements of $D_w$ are $d(Vb)/dV$ and $Vd^2(Vb)/dV^2$. The figure shows how they change with $V$.

- Both $d(Vb)/dV$ and $Vd^2(Vb)/dV^2$ are positive, therefore $D_w < 0$ for $0 \mu m < \lambda < 1.6 \mu m$

- In contrast $D_M < 0$ for $\lambda < \lambda_{ZD}$ and $D_M > 0$ for $\lambda > \lambda_{ZD}$

- The Figure shows $D_M$, $D_w$, and their sum $D = D_w + D_M$ for a typical SMF

- The primary effect of $D_w$ is to shift $\lambda_{ZD}$ by 30-40 nm to make $\lambda_{ZD} \approx 1.31 \mu m$

- $D$ is also reduced from $D_M$ in the range $1.3 - 1.6 \mu m$. This is important for OC applications.

- $D \sim 15-18 \text{ ps/(km-nm)}$ near $1.55 \mu m$. This is the low loss region for fused silica optical fibers. High values of $D$ limit performance of $1.55 \mu m$ systems.
Dispersion Shifted, Flattened, and Decreasing Fiber

- Waveguide dispersion coefficient $D_w$ depends on the $V$#
- The $V$# is a function of the core radius ($a$) and the refractive index difference.

- Therefore it is possible to design fibers that shift $\lambda_{ZD}$ near 1.55 $\mu$m.
- This is the basis for dispersion shifted fiber design.
- Can also tailor $D_w$ so that the total dispersion $D$ is reduced over selected wavelength bands. These fibers are referred to as dispersion flattened fibers.
- Design of DS and DF fibers typical focus on control of the refractive index of the core and cladding materials.

- Waveguide dispersion has also been used to produce fibers with GVD values that decrease along the fiber length. This is produced by axially varying the core radius. These fibers are referred to as dispersion decreasing fibers. They are of interest for soliton applications.
## Typical Dispersion Characteristics of Several Commercial Fibers

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>NA</th>
<th>Δ (%)</th>
<th>2w (μm)</th>
<th>λzd (μm)</th>
<th>GVD Slope-S [ps/(km-n m²)] at λzd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corning SMF-28</td>
<td>0.13</td>
<td>0.36</td>
<td>9.3</td>
<td>1.312</td>
<td>0.90</td>
</tr>
<tr>
<td>Corning SMF-DS</td>
<td>0.17</td>
<td>0.90</td>
<td>8.1</td>
<td>1.550</td>
<td>0.075</td>
</tr>
<tr>
<td>LITSPEC DSM-15</td>
<td>0.17</td>
<td>0.90</td>
<td>9.0</td>
<td>1.555</td>
<td>0.072</td>
</tr>
<tr>
<td>AT&amp;T TrueWave</td>
<td>0.16</td>
<td>0.75</td>
<td>8.4</td>
<td>1.530</td>
<td>0.095</td>
</tr>
</tbody>
</table>

Higher order dispersive effects are governed by the dispersion slope,

\[ S = \frac{\partial^2}{\partial \lambda^2} \]

The effective value of the dispersion parameter is \( D = S \Delta \lambda \)
**POLARIZATION MODE DISPERSION**

- Pulse broadening effect related to fiber birefringence.

- Variations from perfect cylindrical symmetry result in birefringence – this causes different mode indices for orthogonal polarized field components of the fundamental fiber mode.

- If both polarization states of the fundamental mode are excited at the input they will disperse as a function of propagation distance due to their different group velocities. This is the source of PMD.

- PMD can have a large impact on the performance of periodically amplified fiber systems.

- The magnitude of pulse broadening resulting from PMD can be estimated from the time delay $\Delta T$ between the two polarization components after propagating over some length of fiber $L$.

$$\Delta T = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| = L \left| \beta_{1x} - \beta_{1y} \right| = L \Delta \beta_1$$

$x$ and $y$ refer to the orthogonal polarization states and $\Delta \beta_1$ is related to the fiber birefringence $\beta_1 = \partial \beta / \partial \omega$.

- $\Delta T / L$ is a measure of PMD similar to intermodal dispersion of MMF.

- For polarization preserving fiber $\Delta T / L \sim 1 \text{ ns/km}$ when both polarization components are excited at launch, but can be reduced to $\sim 0$ when launched along one axis.
This expression does not account for random perturbations in birefringence and coupling between the polarization modes in standard telecommunication fibers.

- Coupling between the between modes tends to equalize the propagation times for the two polarization components.

- \textbf{PMD} is characterized by the RMS value of $\Delta T$ after averaging over random perturbations

\[ \sigma_T^2 = \langle (\Delta T)^2 \rangle = \frac{1}{2} \Delta \beta_1^2 h^2 \left[ \frac{2L}{h} - 1 + \exp \left( - \frac{2L}{h} \right) \right], \]

where \textit{h} is the de-correlation length (1-10 m range).

- Polarization preserving fibers have an infinitely large (h) and $\sigma_T$ increases linearly with fiber length.

- With $h \ll L$

\[ \sigma_T \approx \Delta \beta_1 \sqrt{hL} = D_p \sqrt{L} \]

$D_p$ is the \textit{PMD} parameter with values $D_p = 0.1-1$ ps/$\sqrt{\text{km}}$

- \textbf{PMD} pulse broadening is much smaller than GVD effects since PMD effects vary with $\sqrt{\text{L}}$.

- The effects of PMD become significant for long propagation distances near $\lambda_{ZD}$.

\textit{Combine this effect with other dispersion effects.}