1. Assume a 25µm radius core fiber with $n_1 = 1.49$ and $n_2 = 1.47$ with $\lambda = 0.850\mu m$. There is a 10 µm lateral shift between fibers. Plot the power coupling loss between fibers as a function of angular misalignment from 0° to 10°. Make reasonable assumptions.

2. An LED with a circular radius of 40 µm and a Lambertian emission pattern of 125 W/cm²/sr is used to couple into two different optical fibers. The first has a core radius of 25 µm and an NA = 0.15. The second has a core radius of 31 µm and an NA = 0.12. How much optical power is coupled into each fiber if the LED is butt coupled against the end of each fiber neglecting Fresnel losses?

3. In this problem we are interested in the exact form of the coupling efficiency that results when two beams are displaced in translation. To solve this problem consider the overlap between modes from the launch fiber to the receiving fiber as a function of translation distance $\delta r$.
Calculate the power coupling efficiency as a function of $\delta r$ for $\delta r = 0 \to 2w_0$ for two identical fibers with $w_0 = 5$ µm at a wavelength of 1.3 µm. Assume that the fibers are parallel to each other but that the core axes are separated by $\delta r$ as shown in the figure below.
Problem #1

\[ a = 25 \mu m \quad \lambda = 0.85 \mu m \]

\[ n_1 = 1.49 \quad d = 10 \mu m \]

\[ n_2 = 1.417 \quad \text{angular misalignment } \Rightarrow 0^\circ \rightarrow 10^\circ \]

\[ \sqrt{\#} = \frac{2 \pi a}{\lambda} \cdot NA = \frac{2 \pi (25 \mu m)}{(0.85 \mu m)} \sqrt{1.49^2 - 1.417^2} = 41.96 \text{ \quad Multimode fiber} \]

\[ \text{Lateral} = -10 \log \left( \frac{2}{\pi} \left[ \cos^{-1} \left( \frac{d}{2a} \right) - \left( \frac{d}{2a} \right) \sqrt{1 - \left( \frac{d}{2a} \right)^2} \right] \right) \]

\[ \text{Angular} = -10 \log \left[ 1 - \left( \frac{n_2 \cdot \tan \theta}{n_1} \right) \right] \]

⇒ See Figure #1

In figure #1 we plot angular misalignment vs. total loss, where your total loss will be \( L_{\text{tot}} = L_{\text{ang}} + L_{\text{lat.}} \) in dB's.
Problem 2

\[ a = 40 \mu m \]

Lambertian = \( \frac{125}{\text{W m}^2 \cdot \text{s}r} \)

\[ \text{Fiber 1: } a = 25 \mu m \; ; \; \text{NA} = 0.15 \]

\[ \text{Fiber 2: } a = 31 \mu m \; ; \; \text{NA} = 0.12 \]

\[ P = \int L \cos \theta \, d\omega \, d\theta = A e L (2\pi) \left[ 1 - \cos^2 \theta \right] \]

\[ P = A e L (2\pi) \left[ \frac{\lambda a^2 \theta}{2} \right] \to A e L \pi \cdot (\text{NA})^2 \]

\[ \text{Source} > \text{Fiber} \]

\[ P = \pi^2 a^2 \cdot L \cdot \text{NA}^2 \]

\[ \text{for fiber #1} \]

\[ P = \pi^2 (25 \mu m)^2 (125 \times 10^{-4}) (0.15)^2 \]

\[ P = 173 \mu W \]

\[ \text{for fiber #2} \]

\[ P = \pi^2 (31 \mu m) (125 \times 10^{-4}) (0.12)^2 \]

\[ P = 170 \mu W \]

\[ \Rightarrow \]
Problem #3

Assuming a single mode fiber:

\[ w_0 = 5 \mu m \quad \lambda = 1.3 \mu m \quad dr \to 0 \to 2w_0 \]

\[ T = \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_1 \psi_2^* \ dx \, dy \right|^2 \]

where

\[ \psi_1 (x, y) = \sqrt{\frac{2}{\pi}} \frac{e^{-(x^2+y^2)/w_0^2}}{w_1} \quad \text{and} \quad \psi_2 (x, y) = \sqrt{\frac{2}{\pi}} \frac{e^{-(x^2+y^2)/(w_2^2)}}{w_2} \]

Solving the integral ("Introduction to Fiber Optics", Ajay Ghatak)

\[ T = e \]

See figure #2