PRÁCTICE PROBLEM SET 6
Logic Optimization, Part 1

1. Which of the following equations are in sum-of-minterms form?
   (a) \( F(a, b, c) = 1 \)
   (b) \( F(a, b, c) = \sum m(1, 2, 3) + \sum d(6) \)
   (c) \( F(a, b, c) = ab + a'b' \)
   (d) \( F(a, b, c) = abc \)

2. Implement the following equations using gates (do not optimize the equations). What is the size and area for each circuit?
   (a) \( F = a' \)
   (b) \( F = ab + c' \)
   (c) \( F = a(b'c' + d) + bd' \)

3. Plot the following design options on the following graph. Which of the following solutions represent a pareto points?
   (a) Option A \{area = 10, delay = 1\}
   (b) Option B \{area = 2, delay = 2\}
   (c) Option C \{area = 4, delay = 8\}
   (d) Option D \{area = 8, delay = 9\}
   (e) Option E \{area = 1, delay = 8\}
   (f) Option F \{area = 5, delay = 4\}
   (g) Option G \{area = 1, delay = 5\}
   (h) Option H \{area = 9, delay = 6\}

4. Use DeMorgan’s Law to find the inverse of the following equations. Provide your answer in sum-of-products form.
   (a) \( F(a, b, c, d) = a' + b + c'd \)
   (b) \( F = a'bc' + ab \)

5. Given \( F(a, b, c) = ab + ac + a'b'c \)
   (a) List the variables in \( F \)
   (b) List the literals in \( F \)
   (c) List the product terms in \( F \)
   (d) List the minterms in \( F \)

6. Consider the equation \( F(a, b, c, d) = \sum m(4, 5, 7, 12, 14, 15) \). Which of the following product terms are prime implicants of the equation (Hint: draw a K-map).
   (a) \( a'bc'd' \)
   (b) \( ab'c \)
   (c) \( ad' \)
   (d) \( bc'd' \)
   (e) There are no prime implicants in this equation.
7. Using the K-map provided in Fig. 1, identify
(a) minterms
(b) implicants
(c) prime implicants
(d) essential prime implicants

Fig. 1: K-map used in Problem 7.

8. Given the constraint matrix where columns correspond to prime implicants and rows correspond to minterms in the on-set, what type of reductions can be made to simply the matrix?
(a) Dominating rows can be removed, dominating columns can be removed.
(b) Dominating rows can be removed, dominated columns can be removed.
(c) Dominated rows can be removed, dominated columns can be removed.
(d) Dominated rows can be removed, dominating columns can be removed.
(e) None of the above.

9. What does the complete sum of a function tell us?
(a) The minimum cover of the function.
(b) The essential prime implicants of the function.
(c) The prime implicants of the function.
(d) The on-set, the off-set, and don’t care set.
(e) The implicants of the function.

10. Which of the following equations represent a complete sum for $F(a, b, c) = \sum m(3, 4, 5) + \sum d(1, 6)$?
(a) $F(a, b, c) = ab' + a'c$
(b) $F(a, b, c) = ac' + ab' + b'c + a'c$
(c) $F(a, b, c) = a'bc + ab'c' + ab'c$
(d) $F(a, b, c) = a'bc + ab' + ac + b'c$
(e) none of the above

11. Using the Shannon expansion theorem, $F(a, b, c) = a'c + b + ab'e'$ expanded with respect to $c$ is equal to which of the following expressions?
(a) $F = c'(b + ab') + c(a' + b)$
(b) $F = c'(a + b + ab') + c(c + b' + a'b)$
(c) $F = c'(1) + c(0)$
(d) $F = c'(a') + c(b'c')$
(e) $F = a'(c + b) + a(b + b'c')$

12. What is the difference between an exact algorithm and a heuristic?

13. Provide an overview of the Quine-McCluskey algorithm. What are the possible methods to implement each step?

14. Perform two-level logic optimization for $F(a, b, c, d) = a'b'c'd' + a'c'd' + a'c'd + bcd' + acd'$
(a) Using K-maps, express your solution in sum-of-products form
(b) Using Boolean Algebra, express your solution in sum-of-products form

15. Perform two-level logic optimization for $F(a, b, c, d) = \sum (0, 2, 6, 8, 9, 10, 13, 15)$.

16. Using Quine-McCluskey (tabular minimization method) minimize $F(a, b, c) = \sum m(1, 2, 3, 4, 6, 7)$. 
17. Using Quine-McCluskey (tabular minimization method) minimize \( F(a, b, c) = \sum m(0, 2, 3, 4, 5, 7) \).

18. Using Petrick’s method, determine the minimum cover of the prime implicants chart provided in Fig. 2.

19. Petrick’s method utilizes the absorption property \((X + XY = X)\) to determine the minimum cover. Prove the absorption property works \(\text{NOTE: THIS DOES NOT NEED TO BE A FORMAL PROOF}\).

20. How could you change Petrick’s method if the size of the prime implicants was taken into consideration (i.e. Size of \(a'b'c'\) is 6 transistors vs. \(ab\) is only 4 transistors).

21. Using the Shannon expansion theorem, expand \( F(a, b, c) = b'c + a'b' + abc' \) with respect to variable \(c\).

22. Using the Shannon expansion theorem, expand \( F(a, b, c, d) = a'c + bcd' + ab'c'd' + ac \) with respect to variables \(a\) and \(d\).

23. Is \( F(a, b, c, d) = a'c'd + a'cd' + ad \) a complete sum? If not, find the complete sum of \(F\).

24. Using Quine-McCluskey (tabular minimization method) minimize \( F(a, b, c) = \sum m(0, 6) + \sum d(2, 3, 7) \). Be sure to show your work.

25. Using Quine-McCluskey (tabular minimization method) minimize \( F(a, b, c, d) = \sum m(1, 2, 3, 4, 5) + \sum d(12, 13, 14, 15) \).

26. Demonstrate two methods to find all the primes of \( F(a, b, c) = a'b'c' + a'b'c + a'bc + a'bc' + ab'c \).

27. Using the Iterated Consensus Method, determine the complete sum of \( F(a, b, c, d) = ab'c' + a'bc' + abc' + a'b' \). Why is the complete sum important?

28. Using the Recursive Consensus Method, determine the complete sum of \( F(a, b, c, d) = ab'c' + a'bc' + abc' + a'b' \). Note: this is the same equation as the previous problem so you should get the same complete sum.

29. Can the row/column dominance method be utilized to minimize \( F(a, b, c) = \sum m(0, 1, 2, 5, 7, 6) \)? If so, provide the minimized equation. If not, explain why and provide alternative methods (you do not need to actually do these alternative methods, just list your options).

30. Can the row/column dominance method be utilized to minimize \( F(a, b, c, d) = \sum m(1, 3, 4, 6, 7, 12, 13) + \sum m(8, 9, 10, 11) \)? If so, provide the minimized equation. If not, explain why and provide alternative methods (you do not need to actually do these alternative methods, just list your options).

31. Using the Row/Column Dominance Method, minimize \( F(a, b, c, d) = \sum m(0, 1, 2, 3, 9, 10, 11) + \sum d(12, 14, 15) \).

32. Provide an example of row dominance. Explain why the dominating row (versus the dominated row) can be removed.

33. Does the order in which variable are expanded effect the efficiency (number of iterations required) of the recursive consensus method? Provide an example illustrating your answer. If variable selection does make an impact, discuss possible methods to select the splitting variable such that the number of iterations required to complete the recursive consensus algorithm are reduced.